Online Appendix A: Computation of the parameter MV

This appendix presents the formulation for computing the value of the parameter MV, which is an upper bound on the number of sites, in addition to the depot, that can be included in any route whose maximal duration is L. The notation follows from Section 3:

$$Max \sum_{i \in N} v_i \tag{18}$$

$s.t.$

$$\sum_{j \in P} x_{0j} = 1 \tag{19}$$

$$\sum_{i \in D} x_{i0} = 1 \tag{20}$$

$$\sum_{j \in N^0} x_{ij} = \sum_{j \in N^0} x_{ji} \qquad \forall i \in N \tag{21}$$

$$v_i = \sum_{j \in N^0} x_{ij} \qquad \forall i \in N \tag{22}$$

$$\sum_{i \in N^0} \sum_{j \in N^0, i \neq j} \ell_{ij} x_{ij} + \sum_{i \in N} p_i v_i \leq L \tag{23}$$

$$u_i - u_j + (|N^0| + 1) x_{ij} \leq |N^0| \qquad \forall i \in N, j \in N, i \neq j \tag{24}$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in N^0 \tag{25}$$

$$v_i \in \{0,1\} \qquad \forall i \in N \tag{26}$$

The objective function (18) maximizes the number of sites that are visited. Constraints (19)-(22), (23)-(26) are equivalent to constraints (2)-(5), (12)-(15) in the MILP for the H-PDSP in Section 3. They make sure that a valid route is obtained, i.e., that each site is visited at most once; that the depot is followed by a pickup site and preceded by a delivery site; that the total traveling time is not exceeded; and that no subtours are included in the solution.

Although optimal values to this integer program were obtained within several minutes to the instances from all of the datasets described in Section 8, the problem can be shown to be NP-Hard through a reduction from the TSP.

Online Appendix B: The Lorenz Curve and the Gini Index

Many inequality measures are based on the Lorenz curve, initially developed by Lorenz (1905) to represent the inequality of wealth distribution. Each point (x, y), $x, y \in [0,1]$ on the curve (see Figure 2) indicates that 100x% of the population with the least amount of wealth has $100y$% of the wealth. Naturally, the points (0,0) and (1,1) are on the plot. A perfectly equal distribution of wealth occurs when every person has the same amount of wealth, which is represented by the straight line $y = x$, also called the *Line of Equality*. By contrast, in a perfectly non-equitable distribution, one person has all the wealth, thus creating a Lorenz curve where $y = 0\ \forall x \neq 1$ and $y = 1$ for $x = 1$.

In his original paper, Gini (1912) defined the *Coefficient of Concentration* based on the area between the Lorenz curve and the Line of Equality. This area, referred to by Gini as the *Area of Concentration*, ranges between 0 and 0.5, and the coefficient was defined to be twice this area (to scale it to values between 0, representing perfect equality, and 1, representing perfect inequality). Since then, other approaches for the calculation of the Gini coefficient have been suggested, based on more algebraic interpretations of the measure, as described next.

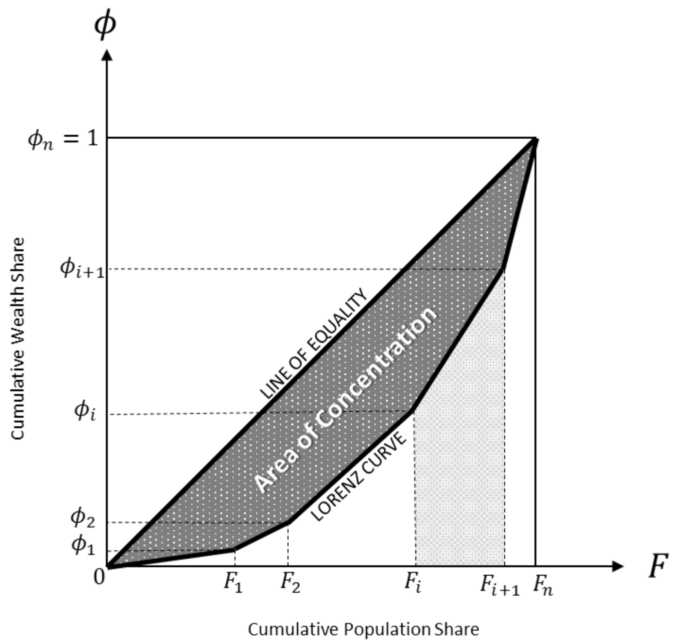

Figure 2: The Lorenz Curve

Suppose we consider some allocation to N individuals $\vec{Y} = (Y_1, Y_2, \dots, Y_N)$ such that the individuals are labeled in non-decreasing order of their wealth: $Y_1 \leq Y_2 \leq \dots \leq Y_N$. Let $\bar{Y}$ be the mean allocation: $\bar{Y} = \frac{\sum_{i=1}^{N} Y_i}{N}$. Let F_i be the cumulative population share up to individual i and ϕ_i be the cumulative wealth share up to individual i. We also define $F_0 = \phi_0 = 0$. Thus, $F_i = \frac{i}{N},\ \phi_i = \frac{\sum_{j=1}^{i} Y_j}{N\bar{Y}}$, $i = 0,1, \dots, N,$ and $F_i - F_{i-1} = \frac{1}{N},\ \phi_i - \phi_{i-1} = \frac{Y_i}{N\bar{Y}},\quad i = 1,2, \dots, N.$

Kendall and Stuart (1963) were the first to suggest that the Gini coefficient could also be calculated algebraically as one half of the *Relative Mean Difference* of the individual allocations $Y_1, Y_2, \dots, Y_N$, denoted by $\Delta = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}|Y_i - Y_j|}{N^2\bar{Y}}$.

Claim B.1: The Gini coefficient of the allocation $\vec{Y}$ is equal to $\frac{\Delta}{2}$.

Proof (based on Anand, 1983): Because each pair of indices is counted twice in the summation, the following is true:

$$\frac{\Delta}{2} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}|Y_i - Y_j|}{2N^2\bar{Y}} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{i}|Y_i - Y_j|}{N^2\bar{Y}}$$

The individuals are ordered by their amount of wealth; hence, $|Y_i - Y_j| = Y_i - Y_j$ for $j \le i$. Therefore,

$$\frac{\Delta}{2} = \frac{1}{N^2\bar{Y}}\sum_{i=1}^{N}\sum_{j=1}^{i}(Y_i - Y_j) = \frac{1}{N^2\bar{Y}}\sum_{i=1}^{N}\left(i \cdot Y_i - \sum_{j=1}^{i} Y_j\right) = \frac{1}{N^2\bar{Y}}\sum_{i=1}^{N}(i \cdot Y_i - N\bar{Y}\phi_i) =$$

$$= \sum_{i=1}^{N}\left(\frac{i}{N} \cdot \frac{Y_i}{N\bar{Y}} - \frac{\phi_i}{N}\right) = \sum_{i=1}^{N}\left[F_i \cdot (\phi_i - \phi_{i-1}) - \frac{\phi_i}{N}\right] =$$

$$= \sum_{i=1}^{N}\left[\phi_i\left(F_i - \frac{1}{N}\right) - F_i\phi_{i-1}\right] = \sum_{i=1}^{N}[\phi_i F_{i-1} - F_i\phi_{i-1}] =$$

$$= \sum_{i=0}^{N-1}[\phi_{i+1}F_i - F_{i+1}\phi_i] = \sum_{i=0}^{N-1}[\phi_{i+1}F_i - F_{i+1}\phi_i] + 1 - \sum_{i=0}^{N-1}[\phi_{i+1}F_{i+1} - F_i\phi_i]$$

The last step is correct since we add to the first expression 1 and subtract $\sum_{i=0}^{N-1}[\phi_{i+1}F_{i+1} - F_i\phi_i] = F_N\phi_N - F_0\phi_0 = 1$. So:

$$\frac{\Delta}{2} = 1 - \sum_{i=0}^{N-1}[\phi_{i+1}F_{i+1} - F_i\phi_i - \phi_{i+1}F_i + F_{i+1}\phi_i] = 1 - \sum_{i=0}^{N-1}(F_{i+1} - F_i)(\phi_{i+1} + \phi_i) =$$

$$= 2 \cdot \left[\frac{1}{2} - \frac{1}{2}\sum_{i=0}^{N-1}(F_{i+1} - F_i)(\phi_{i+1} + \phi_i)\right]$$

Now note that $\frac{1}{2}(F_{i+1} - F_i)(\phi_{i+1} + \phi_i)$ is the area of the light trapezoid shown in Figure 2 such that $\frac{1}{2} - \frac{1}{2}\sum_{i=0}^{n-1}(F_{i+1} - F_i)(\phi_{i+1} + \phi_i)$ is the area between the Lorenz curve and the Line of Equality, which means that $\frac{\Delta}{2}$ is equal to the Gini coefficient, by definition. ■

It is also worth mentioning that the definition provided here for the Gini coefficient relies on the assumption that there are n *individuals*, but in our problem there are n groups (agencies) of different sizes sharing the wealth. This problem requires an additional adaptation of the definition, proposed by Mandell (1991). The definition is based on the fact that if group i is made up of n_i individuals ($\sum_{i=1}^{|D|} n_i = N$), then each of them receives $\frac{Y_i}{n_i}$ units. The Gini coefficient is therefore

$$G = \frac{\sum_{i=1}^{|D|}\sum_{j=1}^{|D|} n_i n_j \left|\frac{Y_i}{n_i} - \frac{Y_j}{n_j}\right|}{2N^2\left(\frac{\sum_{i=1}^{N} Y_i}{N}\right)} = \frac{\sum_{i=1}^{|D|}\sum_{j>i} n_i n_j \left|\frac{n_j Y_i - n_i Y_j}{n_i n_j}\right|}{N\sum_{i=1}^{N} Y_i} = \frac{\sum_{i=1}^{|D|}\sum_{j>i}\left|n_j Y_i - n_i Y_j\right|}{N\sum_{i=1}^{N} Y_i} =$$

$$= \frac{\sum_{i=1}^{|D|}\sum_{j>i}\left|\frac{n_j Y_i - n_i Y_j}{N}\right|}{\sum_{i=1}^{N} Y_i} = \frac{\sum_{i=1}^{|D|}\sum_{j>i}\left|q_j Y_i - q_i Y_j\right|}{\sum_{i=1}^{N} Y_i}$$

where we recall that $q_i = \frac{n_i}{N}$ is the proportion of the population served by agency i.

Online Appendix C: Alternative Objective Functions

As stated in the literature review, several other functions have been suggested (in various settings) to address the question of how to model the trade-off between effectiveness and equity. We briefly present two of the more commonly used schemes and subsequently discuss their adaptation to the setting of the H-PDSP. For this purpose, we denote an allocation vector for n identical agents by $\vec{x} = (x_1, x_2, \dots, x_n)$.

(1) **The α-fairness social welfare function** – This function, first suggested by Atkinson (1970), assumes that a central decision maker maximizes the total social utility, which consists of the sum of the individual utilities, while considering an *inequity aversion* parameter $\alpha > 0$. This approach is similar to the risk aversion concept from the field of decision theory. Let $U_\alpha^i(x_i)$ denote the utility function of individual i. Thus, each individual considers only the amount allocated to himself, regardless of his "envy" of other individuals' allocations. The α-fairness utility functions are suggested to be the following:

$$U_\alpha^i(x_i) = \begin{cases} \dfrac{x_i^{1-\alpha}}{1-\alpha}, & \alpha > 0, \alpha \neq 1 \\ \ln(x_i), & \alpha = 1 \end{cases}$$

These functions are known in the economic literature as iso-elastic functions. They are increasing and concave functions and thus exhibit a diminishing marginal utility. The function to be maximized is the total social utility, denoted by $U_\alpha(x_1, x_2, \dots, x_n)$, which consists of the sum of the individual utilities, i.e., $U_\alpha(x_1, x_2, \dots, x_n) \equiv \sum_{i=1}^{n} U_\alpha^i(x_i)$.

This scheme includes as special cases several important solution concepts: (a) For $\alpha = 0$, the scheme represents the utilitarian principle, where the central decision maker is inequality neutral and consequently maximizes the total units allocated; (b) for $\alpha = 1$, we obtain the *proportional fairness,* which is a generalization of the Nash solution for a two-player game (Nash, 1950) that was also considered as a "fair" objective function (e.g., Bertsimas et al., 2012); and (c) for $\alpha \to \infty$, the function becomes *max-min fairness*.

The α-fairness function was first applied as an objective function by Mo and Warland (2000) in a data network resource allocation problem, and has since been used in several allocation problems: Bertsimas et al. (2012) studied the efficiency-fairness trade-off for this function and demonstrated its use in air traffic flow management problems; McCoy and Lee (2014) incorporated it in a humanitarian healthcare application; and Iancu and Trichakis (2014) applied it in a joint optimization of the trading activities and cost splitting of multiple portfolios. Adapting the function to the setting of the H-PDSP, where the agents are of different size, leads to the following objective function:

$$\text{Maximize } \; U_\alpha\big(Y_1, Y_2, \dots, Y_{|D|}\big) = \begin{cases} \displaystyle\sum_{i \in D} n_i \cdot \frac{\left(\frac{Y_i}{n_i}\right)^{1-\alpha}}{1-\alpha}, & \alpha > 0, \alpha \neq 1 \\ \displaystyle\sum_{i \in D} n_i \cdot \ln\!\left(\frac{Y_i}{n_i}\right), & \alpha = 1 \end{cases}$$

However, the disadvantages of using this function in our setting are the following:

1. The inequality-averseness parameter α may not have a conclusive value, and different measurement approaches might yield diverse estimates. From a decision theory point of view, the parameter can be assessed by "behavioral" experiments (see Wakker (2008) and the references therein). Alternatively, Bertsimas et al. (2012) use bounds on the "price of fairness" and the "price of efficiency" such that the value chosen for the parameter α guarantees a bound on the maximal degradation in fairness or efficiency, respectively. As another option, the authors propose choosing the value of α that balances the prices of fairness and efficiency.
2. If the “appropriate” value for α is larger than 1, the function may not be defined for welfare agencies that receive zero allocation. We note that such a case is quite probable in realistic instances of the H-PDSP because it may not be feasible or optimal to visit all sites under the time limitation. Ignoring delivery sites that receive nothing may be misleading because the utility of a certain agency may have a negative value.
3. Using this function requires maximizing a non-linear term over a discrete solution space (the routing part of the problem), which imposes computational difficulties.

   Nevertheless, because the α-fairness function is widely accepted in the decision theory community, we next investigate the differences between the solutions obtained by this function and our suggested function.

(2) **Lexicographic max-min (LMM)** – This function, suggested as a solution concept by Ogryczak (2007) and used by Luss (2012) and Orgyczak (2014) in location, data network and resource allocation problems, is a generalization of the well-known max-min function. The latter aims to maximize the minimum value allocated to all agents; thus, its solution only takes into account one extreme value. The lexicographic max-min function starts with the max-min function and then repeats the process for the next lowest value, etc. Hence, another possible objective function for the H-PDSP is the following: Lexicographically Maximize $\min_{i\in D}\{\frac{Y_i}{n_i}\}$.

The main disadvantage of this approach is that improving the minimal value only slightly may require a considerable sacrifice in terms of the effectiveness of the allocation. The lexicographic max-min allocation is considered to be "the most equitable solution" (Kostreva and Ogryczak, 1999), but this is not necessarily desirable for the H-PDSP because no consideration to the effectiveness objective is given. In addition, this approach requires sequential solution procedures, which may prove to be intractable even for medium-sized instances.

Despite the mentioned disadvantages of the abovementioned alternative functions, they do satisfy the following important property:

Claim C.1: The α-fairness and LMM objective functions are component-wise increasing and satisfy the principle of transfers.

Proof: Both properties are satisfied for the LMM by definition of the lexicographic order. $U_\alpha(Y_1, Y_2, \dots, Y_{|D|})$ is strictly increasing in Y_i $\forall \alpha > 0, i \in D$ by definition. To prove the principle of transfers, consider an allocation $\overrightarrow{Y^1}$ in which a certain agency $i \in D$ is better off than agency $j \in D$, and suppose $Y_i^1 + Y_j^1 = S$. Now suppose a transfer is to be made from agency i to agency j. Because the amounts allocated to all other agencies remain unchanged, the change in the value of U_α following this transfer results from changes to agencies i and j only. We prove that the principle of transfers is satisfied by proving an even stronger result, which is based on finding the best transfer size:

- For $\alpha > 0, \alpha \neq 1$:

$$U_\alpha(Y_i) = U_i(Y_i) + U_j(S - Y_i) = n_i \frac{\left(\frac{Y_i}{n_i}\right)^{1-\alpha}}{1-\alpha} + n_j \frac{\left(\frac{S-Y_i}{n_j}\right)^{1-\alpha}}{1-\alpha}.$$

$$\frac{dU_\alpha}{dY_i} = \left(\frac{Y_i}{n_i}\right)^{-\alpha} - \left(\frac{S-Y_i}{n_j}\right)^{-\alpha} = \left(\frac{Y_i}{n_i}\right)^{-\alpha} \cdot \left(\frac{S-Y_i}{n_j}\right)^{-\alpha} \cdot \left[\left(\frac{S-Y_i}{n_j}\right)^{\alpha} - \left(\frac{Y_i}{n_i}\right)^{\alpha}\right]$$

$$\frac{dU_\alpha}{dY_i} = 0 \to \left(\frac{S-Y_i}{n_j}\right)^{\alpha} - \left(\frac{Y_i}{n_i}\right)^{\alpha} = 0 \to \frac{S-Y_i}{n_j} = \frac{Y_i}{n_i} \to Y_i^* = \frac{n_i}{n_i + n_j} S$$

$$\frac{d^2U_\alpha}{dY_i^2} = -\alpha \frac{1}{n_i}\left(\frac{Y_i}{n_i}\right)^{-\alpha-1} - \alpha \frac{1}{n_j}\left(\frac{S-Y_i}{n_j}\right)^{-\alpha-1} < 0$$

- For $\alpha = 1$:

$$U_\alpha(Y_i) = U_i(Y_i) + U_j(S - Y_i) = n_i \ln\left(\frac{Y_i}{n_i}\right) + n_j \ln\left(\frac{S-Y_i}{n_j}\right).$$

$$\frac{dU_\alpha}{dY_i} = \frac{n_i}{Y_i} - \frac{n_j}{S-Y_i}$$

$$\frac{dU_\alpha}{dY_i} = 0 \to \frac{n_i}{Y_i} - \frac{n_j}{S-Y_i} = 0 \to \frac{S-Y_i}{n_j} = \frac{Y_i}{n_i} \to Y_i^* = \frac{n_i}{n_i + n_j} S$$

$$\frac{d^2U_\alpha}{dY_i^2} = -\frac{n_i}{Y_i^2} - \frac{n_j}{(S-Y_i)^2} < 0$$

It follows that U_α is higher when a transfer is performed, and that the closer the allocation is to the proportional allocation, the higher its value is. Hence, the principle of transfers is satisfied. ∎

A direct result of Claims 5.6 and C.1 is the following:

Corollary C.1: Consider a variant of the H-PDSP, in which the objective function is replaced with the α-fairness or the LMM objective function. The RH algorithm solves to optimality the ASP sub-problem of these problems.

Building on this result, we conducted an experiment aimed at analyzing how our objective function balances effectiveness and equity, compared with the frequently used α-fairness objective function. An important question is how far off a central decision maker (the food bank), who believes in the α-fairness function, would be if our proposed objective function would be used instead of the α-

fairness function. Another important question is what degree of "inequality-averseness" is expressed by the solution obtained when using our function. For the purpose of answering these questions, we denote by $Z(\vec{Y})$ and $U_\alpha(\vec{Y})$ the values of the allocation vector $\vec{Y}$ under the Z objective function (our function) and the α-fairness objective function with a specific value of the parameter α, respectively. We refer to the following allocation vectors: $\vec{Y}^{Z^*}$ is the optimal allocation vector under the Z objective value; $\vec{Y}^{U_\alpha^*}$ is the optimal allocation vector under the α-fairness function for a specific value of α; and $\vec{Y}^{Z_b}$ is the best allocation vector, in terms of the Z objective value, among the set of allocation vectors that are optimal for some value of the parameter α, i.e., $\vec{Y}^{Z_b} = \arg\max_{\vec{Y}^{U_\alpha^*}}\{Z(\vec{Y}^{U_\alpha^*})\}$ (note that this vector may not be unique). Thus, a relevant measure for the analysis of the first question is $\frac{U_\alpha(\vec{Y}^{Z^*})}{U_\alpha(\vec{Y}^{U_\alpha^*})}$. Values close to 1 establish the similarity of the solutions that both functions choose. The second question can be answered by the range of values for the parameter α that corresponds to the set of all vectors which satisfy the definition of $\vec{Y}^{Z_b}$.

Hence, for the purpose of this experiment, we were required to obtain the optimal allocations for given instances under both objective functions. To avoid the ambiguity of the α-fairness objective value with respect to zero allocations, the parameter α was only tested with the values $0.01i$ for $i = 0,1,\ldots,99$. Because of the nonlinear nature of the α-fairness function, the solutions with respect to this objective function were obtained by a complete enumeration of all routes of the considered instances. For each feasible route, the optimal allocation was found with the RH algorithm, and its objective value $\vec{Y}^{U_\alpha^*}$ was computed for each value of α. However, this approach restricted us to instances that were small enough such that all routes could be generated. The HFB dataset was used for this purpose because the number of sites included in it is quite limited. In addition, we recall that the feasibility of routes is determined by the value of the maximal traveling time parameter (L). Hence, the value chosen for this parameter was small enough such that a full enumeration could be conducted, but not so small that the combinatorial nature of the problem would not lose its essence (there were still over 11 million routes, approximately 700,000 of them feasible). Because the value of L was fixed (which left us with 4 instances from the HFB dataset), we created 16 more instances based on the HFB's real data, leading to a total of 20 instances.

The results are shown in Table 5. For each instance, we report several measures: (1) the average proportion $\frac{U_\alpha(\vec{Y}^{U_\alpha^*})}{U_\alpha(\vec{Y}^{Z^*})}$ over all values of α; (2) the proportion $\frac{Z(\vec{Y}^{Z_b})}{Z(\vec{Y}^{Z^*})}$; and (3) the range of values of α $[\alpha^{low}, \alpha^{high}]$ that correspond to $\vec{Y}^{Z_b}$. We note several important observations: For the first measure, the lowest proportion obtained in any instance for any value of α was 87%, and the average value was 95.65%. For the second measure, in only 3 of the 20 instances $\vec{Y}^{Z_b}$ was not an optimal solution under our objective function, and even in these cases, the lowest proportion obtained was 96.7% (average proportion: 99.68%). Finally, the range of α values corresponding to $\vec{Y}^{Z_b}$ tended to be quite extensive, (with average interval length of 0.49) and quite central (e.g., only one instance did not include the value 0.5). Two key conclusions can be derived from these results. The first is that the objective functions are quite interchangeable. In other words, a food bank considering which objective function should be incorporated into the H-PDSP model can use our objective function even if it believes the α-fairness objective function is the "true" one, and vice versa. The second conclusion is that our objective function tends to balance effectiveness and equity in a manner that does not apportion an exaggerated amount of weight to any of these two components.

Table 3: Comparison of the results from the Z objective function and α-fairness scheme

Instance	$\underset{\alpha}{\mathrm{avg}} \frac{U_\alpha(\vec{Y}^{U_\alpha^*})}{U_\alpha(\vec{Y}^{Z^*})}$	$\frac{Z(\vec{Y}^{Z_b})}{Z(\vec{Y}^{Z^*})}$	α^{low}	α^{high}
1	96.53%	100.00%	0.15	0.61
2	97.89%	100.00%	0.25	0.81
3	96.48%	100.00%	0.13	0.61
4	96.61%	100.00%	0.03	0.51
5	95.38%	100.00%	0.44	0.76

6	93.41%	100.00%	0.48	0.89
7	97.05%	100.00%	0.43	0.90
8	96.18%	100.00%	0.10	0.54
9	98.33%	100.00%	0.27	0.92
10	97.64%	100.00%	0.11	0.72
11	96.33%	100.00%	0.20	0.76
12	96.52%	96.76%	0.36	0.92
13	95.20%	100.00%	0.42	0.77
14	93.27%	100.00%	0.17	0.51
15	89.85%	96.86%	0.52	0.91
16	91.92%	100.00%	0.14	0.53
17	94.77%	100.00%	0.44	0.92
18	95.18%	100.00%	0	0.59
19	96.70%	100.00%	0.11	0.62
20	97.80%	99.94%	0.27	0.93

Online Appendix D: NP-Hardness Proof for the H-PDSP

The decision version of the TSP is defined as follows: "Given a complete graph $G = (V, E)$ with distance matrix d_{ij}, is there a tour that visits each node exactly once with total distance at most b?"

We define the decision version of the H-PDSP with the same input as the optimization version presented in Section 3. The question to be answered is as follows: "Is there a feasible solution, i.e., a feasible route and an allocation that corresponds to this route, such that the value of the objective function is at least β?".

Let us define the following instance of the H-PDSP: $P = \{1\}, D = \{2, \dots, |V|\}$, and the depot is in the same location as site 1; that is, $\ell_{ij} = d_{ij}, \ell_{0j} = d_{1j}, \ell_{j0} = d_{j1} \;\; \forall i, j \in N$. The supply in the pickup site is $S_1 = Q > 0$, the population in each delivery site $i \in D$ is $n_i = 1$ and the vehicle capacity is infinite. There are no loading/unloading times, i.e., $p_i = 0 \;\; \forall i \in N$, and the time limit imposed is $L = b$, and $\beta = Q$.

Because site 1 is the only pickup site and because no time is required to travel to it from the depot, surely the first stop visited by the vehicle will be site 1. Therefore, if it is possible to visit all delivery sites and return to the depot under the time limitation, then the optimal allocation is $Y_i = \frac{1}{|D|} Q \;\; \forall i \in D$, i.e., a perfectly equitable allocation (with $1 - G = 1$), and its objective value would be Q. Otherwise, the objective value would necessarily be lower than Q. Hence, the answer to the TSP is "Yes" if and only if the answer to the H-PDSP is "Yes".

Note that a similar reduction applies for variants of the H-PDSP, in which the objective function is one of the alternatives presented in Appendix C, with the following changes:

- $\boldsymbol{U_\alpha}$: $\beta = |D| \frac{\left(\frac{Q}{|D|}\right)^{1-\alpha}}{1-\alpha}$ for $\alpha > 0, \alpha \neq 1$ or $|D| \ln \left(\frac{Q}{|D|}\right)$ for $\alpha = 1$.
- **LMM:** Because this objective function is not defined by a single value, the decision version of the problem becomes "Is there a feasible solution, i.e., a feasible route and an allocation that corresponds to this route, such that the value of the maximal wealth of any agency is at least β_1, the second highest wealth of any agency is β_2, and so on for all β_i, $\forall i \in D$?". Consider the same instance described above except that $\beta_i = \frac{Q}{|D|} \;\; \forall i \in D$. Therefore, the answer to the TSP is "Yes" if and only if the answer is "Yes" to the instance of the H-PDSP.

Online Appendix E: Pseudocode of the Robin Hood Algorithm for the allocation sub-problem

Algorithm RH

$B_{[g]} = \frac{S_{[g]}}{NS_{[g]}}, \;\; I_{[g]} = 0$

For $i = g - 1$ to 1:

  If $\frac{S_{[i]}}{NS_{[i]}} \leq B_{[i+1]}$:

    $B_{[i]} = \frac{S_{[i]}}{NS_{[i]}}$

    continue

  $supply = S_{[i]}$

  $r = min${the index of the left-most blocking segment, the last segment + 1}

  $J = \underset{k \in \{i+1,\dots,r-1\}}{\operatorname{argmin}} \{B_k\}$ //the set of poorest segments

  $j = \max(J)$ //segments i+1,…,j are the sub-sequence of poorest segments

  $B_{[i]} = B_{[j]}$

  $supply = supply - B_{[i]} \cdot NS_{[i]}$

  While $supply > 0$:

    // First bound for maximal increase: the supply that is still available

    $\Delta_1 = \frac{supply}{\sum_{k=i}^{j} NS_{[k]}}$

    // Second bound for maximal increase: the wealth of next sub-sequence

    $\Delta_2 = \begin{cases} \infty, & if \;\; j = r - 1 \\ B_{[j+1]} - B_{[j]}, & otherwise \end{cases}$

    // Third bound for maximal increase: the wealth increase that causes blockage

    $\Delta_3 = \min_{k=i+1,\dots,j} \left\{ \frac{C - I_{[k]} - S_{[k]}}{\sum_{x=k}^{j} NS_{[x]}} \right\}$

    $\Delta = \min\{\Delta_1, \Delta_2, \Delta_3\}$

    $B_{[k]} = B_{[k]} + \Delta \;\; \forall k = i, i+1, \dots, j$

    $supply = supply - \Delta \cdot \sum_{k=i}^{j} NS_{[k]}$

    $I_{[k]} = I_{[k]} + \Delta \cdot \sum_{x=k}^{j} NS_{[x]} \;\;\; \forall k = i+1, i+2, \dots, j$

    // update r,J,j

    $r = min${the index of the left-most blocking segment, the last segment+1}

    $J = \underset{k \in \{i,\dots,r-1\}}{\operatorname{argmin}} \{B_k\}$ //the segments that can be considered for an improving transfer

    $j = \max(J)$ //segments i+1,…,j are the sub-sequence of poorest segments

Online Appendix F: Details on the Computation of $UB5$

The computation of the bound relies on three optimization problems as a backbone, each solved as a MILP. If any of them cannot be solved to optimality due to limited computational resources, the upper bound from the MILP solver can be used as a valid bound.

MILP 1 - Obtaining MS

We recall that MS represents the maximal number of segments in a route whose maximal duration is L. It can be obtained by solving the following MILP (the notation follows from Section 3):

$$Max\ \ Z = \sum_{i\in P}\sum_{j\in D} x_{ij} \tag{27}$$

$s.t.$

$$\sum\nolimits_{j\in P} x_{0j} = 1 \tag{28}$$

$$\sum\nolimits_{i\in D} x_{i0} = 1 \tag{29}$$

$$\sum\nolimits_{j\in N^0} x_{ij} = \sum\nolimits_{j\in N^0} x_{ji} \qquad \forall i \in N \tag{30}$$

$$\sum\nolimits_{i\in N^0}\sum\nolimits_{j\in N^0, i\neq j} \ell_{ij}x_{ij} + \sum\nolimits_{i\in N} p_i v_i \leq L \tag{31}$$

$$u_i - u_j + (MV+1)x_{ij} \leq MV \qquad \forall i \in N, j \in N, i \neq j \tag{32}$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \in N^0 \tag{33}$$

$$u_i \geq 0 \qquad \forall i \in N \tag{34}$$

The objective function (27) maximizes the number of times the vehicle moves from a pickup site to a delivery site, which defines a segment. Constraints (28)-(34) are identical to constraints (1)-(3), (12)-(14), (16) from the formulation in Section 3. This problem can be shown to be NP-Hard through a reduction from the TSP.

MILP 2 - Obtaining $MV(ms)$

$MV(ms)$ can be obtained by adding the following constraint to the MV problem presented in Appendix A:

$$\sum_{i\in P}\sum_{j\in D} x_{ij} = ms$$

Clearly, this problem is also NP-Hard.

MILP 3 - The Segment Sequencing Sub-problem

We recall that the input of the Segment Sequencing sub-problem (SSP) includes a subset of pickup (delivery) sites that need to be visited, denoted by $S_p \subseteq P$ ($S_d \subseteq D$), and the number of segments that are defined by these sites, denoted by ms. We can formulate the sub-problem as follows:

$$Max\ \ Z = \sum_{i\in P} Y_i - \sum_{i\in D}\sum_{j\in D, j>i} E_{ij} \tag{35}$$

$s.t.$

$$\sum_{j\in(S_p\cup S_d)} x_{ji} = 1 \qquad \forall i \in S_p \cup S_d \qquad (36)$$
$$\sum_{j\in(S_p\cup S_d)} x_{ij} = 1 \qquad \forall i \in S_p \cup S_d \qquad (37)$$
$$\sum_{j\in S_p} x_{0j} = 1 \qquad (38)$$
$$\sum_{i\in S_d} x_{i0} = 1 \qquad (39)$$
$$Q_i \le S_i \qquad \forall i \in S_p \qquad (40)$$
$$I_{0j} = 0 \qquad \forall j \in S_p \cup S_d \qquad (41)$$
$$Q_i = \sum_{j\in(S_p\cup S_d\cup\{0\})} I_{ij} - \sum_{j\in(S_p\cup S_d\cup\{0\})} I_{ji} \qquad \forall i \in S_p \qquad (42)$$
$$Y_i = \sum_{j\in(S_p\cup S_d\cup\{0\})} I_{ji} - \sum_{j\in(S_p\cup S_d\cup\{0\})} I_{ij} \qquad \forall i \in S_d \qquad (43)$$
$$Y_i = 0 \qquad \forall i \in N\backslash S_d \qquad (44)$$
$$I_{ij} \le C\cdot x_{ij} \qquad \forall i,j \in S_p \cup S_d \cup \{0\} \qquad (45)$$
$$\sum_{i\in S_p}\sum_{j\in S_d} x_{ij} = ms \qquad (46)$$
$$E_{ij} \ge q_iY_j - q_jY_i \qquad \forall i,j \in D \qquad (47)$$
$$E_{ij} \ge q_iY_j - q_jY_i \qquad \forall i,j \in D \qquad (48)$$
$$x_{ij} \in \{0,1\} \qquad \forall i,j \in S_p \cup S_d \cup \{0\} \qquad (49)$$
$$Q_i \ge 0 \qquad \forall i \in S_p \qquad (50)$$
$$Y_i \ge 0 \qquad \forall i \in D \qquad (51)$$
$$I_{ij} \ge 0 \qquad \forall i,j \in S_p \cup S_d \cup \{0\} \qquad (52)$$

This objective function (35) is identical to that of the H-PDSP, and similarly the related constraints (47)-(48). Constraint (46) is the same constraint as the one used in MILP 2, restricting the number of segments to a given value. All other constraints are identical to those that appear in the formulation of the H-PDSP, except that the sets S_p and S_d replace the sets P and D, respectively. Note that a sub-tour elimination constraint is not required in this formulation since there are no cost or time considerations. This problem can be shown to be NP-Hard by a reduction from the Partitioning Problem.

Online Appendix G: Comparison of the Food Bank Benchmark Algorithm and the LNS

Table 4: Comparison of the results obtained per day

	FB			LNS			
Day	F_{FB}	$1-G_{FB}$	Z_{FB}	F_{LNS}	$1-G_{FBLNS}$	Z_{LNS}	$\frac{Z_{LNS}-Z_{FB}}{Z_{FB}}$
0	16,806.71	0.17	2,856.10	16,806.71	0.35	5,886.32	106.10%
1	7,558.11	0.12	925.34	7,558.11	0.22	1,642.65	77.52%
2	8,137.555	0.11	857.73	8,137.55	0.14	1,131.80	31.95%
3	3,013.38	0.09	266.74	3,101.37	0.20	617.94	131.66%
4	1,555.25	0.14	212.43	1,555.25	0.25	396.19	86.50%
5	13,370.93	0.07	956.45	13,370.93	0.10	1,287.42	34.60%
6	4,617.512	0.09	418.74	4,724.22	0.12	565.29	35.00%
7	20,351.81	0.12	2,521.18	20,351.81	0.36	7,276.95	188.63%
8	35,801.83	0.10	3,693.66	35,801.83	0.16	5,770.43	56.23%
9	24,723.55	0.11	2,730.00	24,723.55	0.13	3,183.45	16.61%
10	8,444.161	0.18	1,515.72	8,444.16	0.38	3,171.90	109.27%
11	5,355.028	0.11	565.60	5,355.03	0.18	966.28	70.84%
12	8,679.625	0.03	218.14	8,679.62	0.12	1,039.03	376.31%
13	4,495.2	0.09	383.88	4,495.20	0.37	1,652.74	330.54%
14	5,224.977	0.06	316.23	5,224.98	0.16	825.86	161.16%
15	7,767.471	0.06	501.97	7,767.47	0.25	1,910.39	280.58%
16	6,035.341	0.13	765.47	6,035.34	0.14	847.64	10.73%
17	18,186.52	0.15	2,691.86	18,186.52	0.17	3,050.83	13.34%
18	6,256.472	0.07	415.60	6,256.47	0.19	1,166.26	180.62%
19	7,681.255	0.09	665.04	7,681.25	0.23	1,750.17	163.17%

Table 5: Comparison of the results across agencies

			FB		LNS					FB		LNS	
Agency	n_i	w_i	**# Visits**	**Total Allocation**	**# Visits**	**Total Allocation**	**Agency**	n_i	w_i	**# Visits**	**Total Allocation**	**# Visits**	**Total Allocation**
9	71	1	3	1,983.95	3	192.47	40	270	2	2	3,605.96	2	710.76
10	150	1	2	476.19	2	264.34	41	3,500	4	6	7,715.51	6	28,435.19
11	500	3	3	4,281.01	4	3,672.77	42	45	1	1	45.66	2	82.12
12	600	3	4	4,149.58	4	1,687.21	43	172	1	2	1,228.23	2	1,058.89
13	912	4	6	16,259.97	6	17,765.03	44	350	2	3	6,137.22	3	4,559.99
14	550	3	4	5,508.50	4	4,557.22	45	480	3	4	2,091.48	4	1,604.59
15	300	2	3	2,823.32	3	3,047.80	46	600	3	4	13,927.73	4	4,717.31
16	620	3	4	4,538.71	4	5,999.50	47	600	3	4	4,760.77	4	5,294.74
17	120	1	3	794.73	3	851.16	48	1,200	4	6	11,790.21	6	15,806.23
18	5,000	4	7	6,452.86	7	21,735.19	49	160	1	1	642.85	1	161.59
19	140	1	2	541.44	2	413.68	50	250	2	2	212.83	2	214.50
20	400	3	4	2,231.33	4	3,027.73	51	360	2	2	761.84	2	876.16
21	250	2	2	1,365.55	2	704.43	52	260	2	3	6,829.83	3	1,107.46
22	70	1	1	642.85	2	102.95	53	60	1	1	822.25	2	529.12
23	30	1	2	2,826.16	2	153.82	54	250	2	3	1,137.42	3	999.95
24	125	1	1	1,620.58	2	363.73	55	230	2	3	1,676.70	3	960.22
25	210	2	2	2,152.61	2	1,851.91	56	100	1	2	434.20	2	402.94
26	350	2	3	3,587.38	3	1,557.82	57	120	1	0	0.00	2	349.18
27	250	2	3	1,142.93	3	1,837.59	58	370	2	3	2,068.07	2	664.88
28	200	2	2	213.48	2	319.98	59	400	3	4	4,994.46	4	3,287.90
29	300	2	3	2,557.77	3	2,697.31	60	350	2	2	2,190.82	2	1,049.45
30	300	2	2	1,180.52	3	2,205.11	61	120	1	2	541.44	2	312.38
31	130	1	1	79.68	2	161.60	62	400	3	4	2,284.08	4	1,022.91
32	200	2	2	2,183.96	3	337.44	63	160	1	2	818.14	2	134.77
33	4,000	4	5	9,089.42	6	26,067.50	64	220	2	3	2,404.08	3	607.49
34	1,000	4	6	13,965.35	7	9,991.16	65	245	2	1	275.06	2	522.80
35	380	2	3	2,613.48	3	1,118.57	66	100	1	2	894.39	2	114.38
36	250	2	3	4,064.73	3	1,006.09	67	350	2	2	2,005.51	2	851.82
37	1,250	4	6	9,554.68	6	10,298.97	68	200	2	3	1,123.23	3	2,020.64
38	500	3	4	4,547.81	4	4,409.80	69	1,000	4	6	7,742.56	6	4,452.50
39	330	2	2	2,973.13	2	214.87	70	250	2	3	6,496.47	3	2,729.76
									Sum	**184**	**214,062.69**	**197**	**214,257.38**
									$1-G$		**0.55**		**0.70**
									Z		**116,153.68**		**150,984.01**

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