Machine Learning Predictions as Regression Covariates

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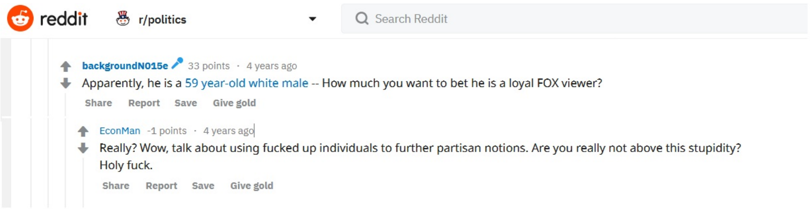
March 1, 2019

Studying Uncivil Political Dialogue



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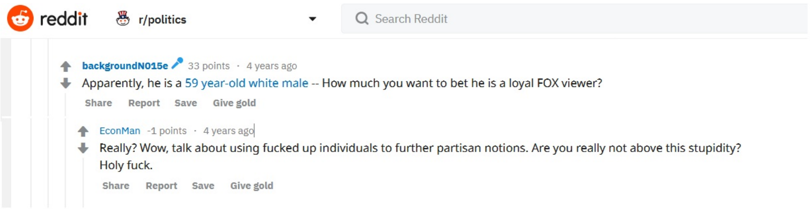

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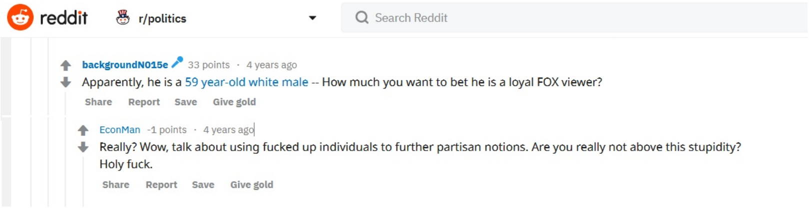

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- **Solution?** hand-code a subset of comments,then predict x_1 with z_1 (using algorithm) for the remaining millions

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- Satellite imagery:

  Jean et al. (2016)

- Congressional pictures with constituents:

  Anastosopoulos et al. (2017)

- Supreme Court audio recordings:

  Knox and Lucas (2018)

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  - If just ER and $\mathrm{E}(x_2(x_1 - z_1)') = 0$, then Naive Estimator is substantially attenuated

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- These are substantial biases that limit hypothesis tests of $b = 0$

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  - Accounts for overfitting z to x in training data
  - (Asymptotically) more efficient than just using data where x_1 is coded

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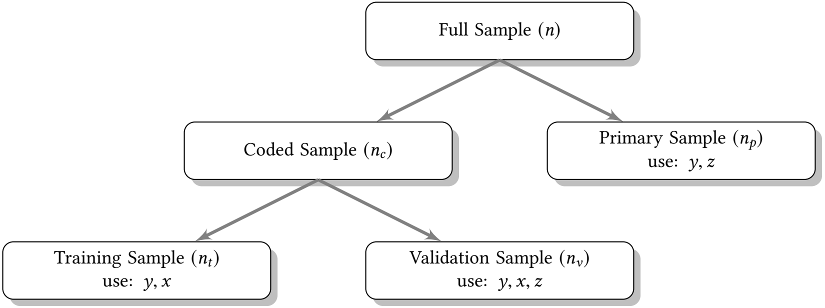

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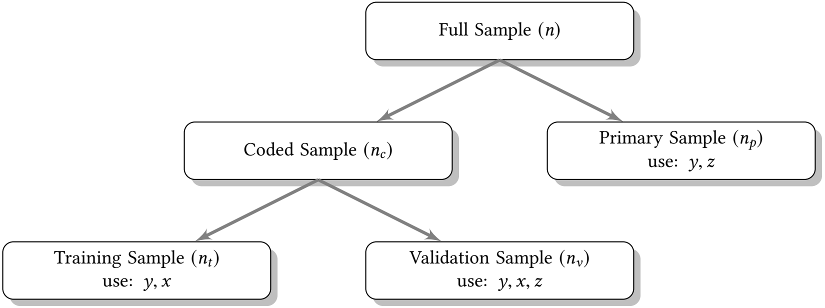

$\rightsquigarrow$ Training sample to fit z-algorithm, but then hide its z values from Steps 2 & 3 to avoid overfitting bias

Step 2: First-Stage Regression

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- Compactly write $x' = z'\Lambda_0 + \eta'$ with $\text{E}(z\eta') = 0$

Step 2: First-Stage Regression on Validation Sample

- Given Exclusion Restriction, plug in $z'\Lambda_0$ for x':

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- e.g.,

$$\hat{\lambda}_k = \left(\sum_{i=1}^{n} v_i z_i z_i'\right)^{-1} \left(\sum_{i=1}^{n} v_i z_i x_{ik}\right) \tag{1}$$

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- 2SLS on the primary & validation samples:

$$\hat{\beta}_2 \text{ such that } g_2(\hat{\beta}_2) = 0$$

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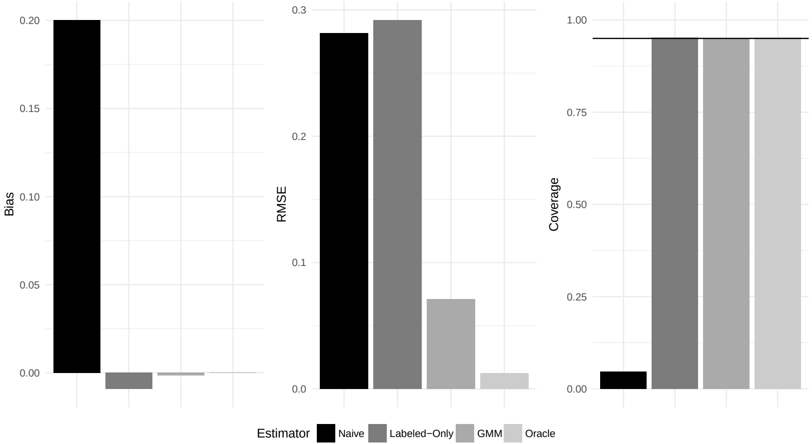
- We derive W that makes $\hat{\beta}_{\text{GMM}}$ more efficient than $\hat{\beta}_1, \hat{\beta}_2$

Data-Based Simulation: GMM is Equal to Coding 16 Times More Posts



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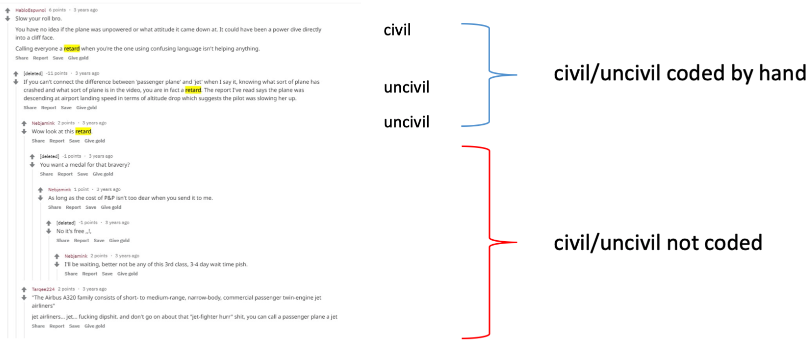

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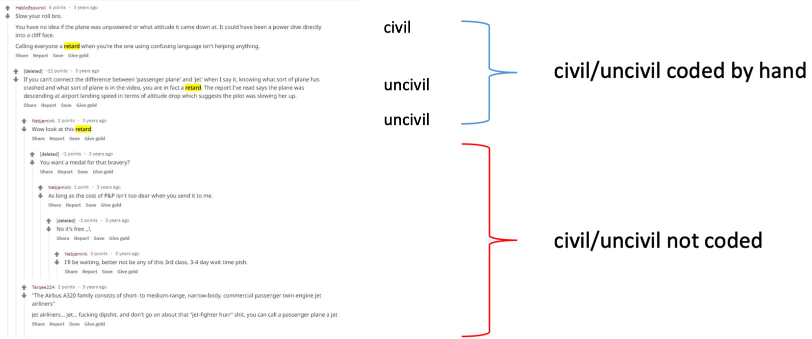

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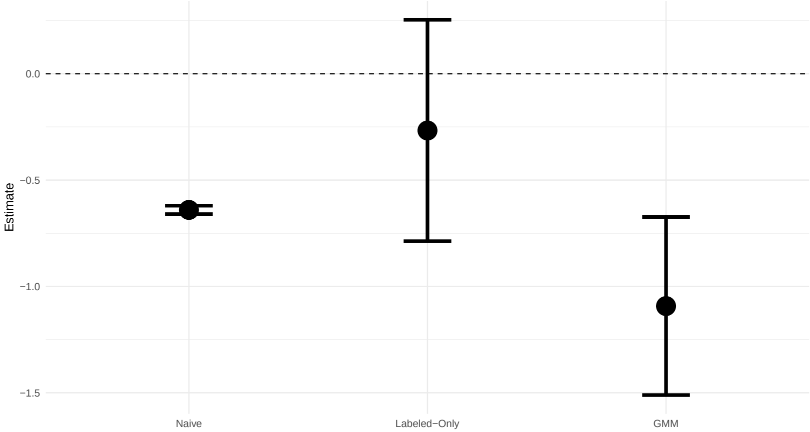
- 5,000 most common words + LASSO $\implies$ z

GMM Corrects for Naive Estimator's Attenuation Bias



GMM: uncivil posts on average have ≈ 1.1 (0.2) lower score

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- Nonparametric methods that do not lean on ER (Chen, Hong, & Tamer 2005) are not RMSE-competitive in realistic scenarios (appendix)

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- Many practitioners are plugging-in algorithmic predictions or using ad-hoc adjustments
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- No functional form or homoskedasticity assumptions
- More efficient than just using hand-coded data!