

SSC FOUNDATION



सम्यान बैच

समय से तैयारी, समय से सफलता

MATHEMATICS

ALGEBRA (बीजगणित)

सबसे सटीक, सबसे बेहतर



LIVE STREAM : 11:00 AM

BY - SHASHI SIR

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ALGEBRA (बीजगणित) (CLASS-03) [Algebraic Identity]

Concept – 6

$$\cancel{x} \quad \begin{matrix} k_1 & \times & q \\ & \text{---} & \end{matrix} \quad \begin{matrix} p \\ k_1 \end{matrix} \quad \begin{matrix} p \\ \text{---} \end{matrix}$$

$$\cancel{x} \quad (a^2 + ab + b^2)(a^2 + b^2 - ab) = a^4 + a^2b^2 + b^4$$

$$\cancel{x} \quad \frac{a^4 + a^2b^2 + b^4}{a^2 + ab + b^2} = (a^2 + b^2 - ab)$$

$$\cancel{x} \quad \frac{a^4 + a^2b^2 + b^4}{a^2 + b^2 - ab} = (a^2 + b^2 + ab)$$

$$\begin{aligned} a^2 + ab + b^2 &= 4 \\ a^2 - ab + b^2 &= 2 \end{aligned}$$

$$2ab = 2$$

$$ab = \frac{2}{2} = 1$$

1. यदि $a^4 + a^2b^2 + b^4 = 8$ और $a^2 + ab + b^2 = 4$ है, तो ab का मान क्या है?

If $a^4 + a^2b^2 + b^4 = 8$ and $a^2 + ab + b^2 = 4$ then what is the value of ab ?

(a) -1

(b) 0

(c) 2

(d) 1

$$(a^2 + ab + b^2)(a^2 - ab + b^2) = a^4 + a^2b^2 + b^4$$

$$4 \times 2 = 8$$

$$K = 2$$

$$ab = \frac{P_1 - P_2}{2}$$

$$a^2 + b^2 = \frac{P_1 + P_2}{2}$$

$$xy = \frac{7-5}{2} = \textcircled{1}$$

$$x^2 + y^2 = \frac{7+5}{2} = \textcircled{6}$$

$$\begin{aligned} x+y &= \sqrt{x^2+y^2+2xy} \\ &= \sqrt{6+2} \\ &= \sqrt{8} = 2\sqrt{2} \end{aligned}$$

2. यदि $x^4 + x^2y^2 + y^4 = 35, x^2 + xy + y^2 = 7$ और $x, y > 0$ हैं तो $x^3 + y^3$ का मान बताओ।

If $x^4 + x^2y^2 + y^4 = 35, x^2 + xy + y^2 = \textcircled{7}$ and $x, y > 0$, then find the value of $x^3 + y^3$

- (a) ~~$10\sqrt{2}$~~
- (b) $22\sqrt{2}$
- (c) $16\sqrt{2}$
- (d) C.N.D.

$$x^2 - xy + y^2 = \frac{35}{7} = \textcircled{5}$$

$$\begin{aligned} x^3 + y^3 &= (x+y)(x^2 + y^2 - xy) \\ &= 2\sqrt{2}(6-1) \\ &= 2\sqrt{2} \times 5 = \textcircled{10\sqrt{2}} \end{aligned}$$

3. यदि $x^8 + x^4y^4 + y^8 = 144$ और $x^4 + x^2y^2 + y^4 = 18$ है, तो $x^2 + y^2$ का मान बताओ।

[If $x^8 + x^4y^4 + y^8 = 144$ and $x^4 + x^2y^2 + y^4 = 18$ then find the value of $x^2 + y^2$?

(a) 23

(b) $\sqrt{23}$

(c) 5

(d) $\sqrt{18}$

$$\begin{aligned} x^2 + y^2 &= \sqrt{x^4 + y^4 + 2x^2y^2} \\ &= \sqrt{13 + 2 \times 5} \\ &= \sqrt{13 + 10} \\ &= \sqrt{23} \end{aligned}$$

$$\begin{aligned} (x^4 + x^2y^2 + y^4)(x^4 - x^2y^2 + y^4) &= x^8 + x^4y^4 + y^8 \\ 18 \times 8 &= 144 \end{aligned}$$

$$x^4 + y^4 = \frac{18+8}{2} = \frac{26}{2} = 13$$

$$x^2y^2 = \frac{18-8}{2} = \frac{10}{2} = 5$$

★ CONCEPT - 07 ★

If $x + \frac{1}{x} = K$ then

✓ (i) $x^2 + \frac{1}{x^2} = K^2 - 2$

✓ (ii) $x^4 + \frac{1}{x^4} = [K^2 - 2]^2 - 2 = P$ (let)

✓ (iii) $x^8 + \frac{1}{x^8} = P^2 - 2$

✓ (iv) $x^3 + \frac{1}{x^3} = K^3 - 3K$

✓ (v) $x^5 + \frac{1}{x^5} = (x^2 + \frac{1}{x^2})(x^3 + \frac{1}{x^3}) - (x + \frac{1}{x})$

96 $x + \frac{1}{x} = 3$

① $x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$

② $x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$

③ $x^8 + \frac{1}{x^8} = 47^2 - 2$
 $= 2309 - 2$
 $= 2307$

$K^3 - 3K$

$$x^7 = x^3 \times x^4 = x^1$$

$$(v) x^7 + \frac{1}{x^7} = (x^3 + \frac{1}{x^3})(x^4 + \frac{1}{x^4}) - (x + \frac{1}{x})$$

$$\text{IF } x - \frac{1}{x} = P \text{ then}$$

$$(i) x^2 + \frac{1}{x^2} = (P^2 + 2)$$

$$(ii) x^4 + \frac{1}{x^4} = (P^2 + 2)^2 - 2$$

$$(iii) x^3 - \frac{1}{x^3} = (P)^3 + 3P$$

$$(iv) x^5 - \frac{1}{x^5} = (x^2 + \frac{1}{x^2})(x^3 - \frac{1}{x^3}) - (x - \frac{1}{x})$$

$$x + \frac{1}{x} = 5$$

$$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 125 - 15 = 110$$

Q. If $x + \frac{1}{x} = 5$ find $x^4 + \frac{1}{x^4} =$

$$\rightarrow x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

$$\rightarrow x^4 + \frac{1}{x^4} = 23^2 - 2$$

$$= 529 - 2 = \boxed{527}$$

$x + \frac{1}{x} = \underline{\underline{3}}$

$$x^5 + \frac{1}{x^5} = (x^3 + \frac{1}{x^3})(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x})$$

$$= (3^3 - 3 \times 3)(3^2 - 2) - 3$$

$$18 \times 7 - 3 = 126 - 3 = \underline{\underline{123}}$$

$$x - \frac{1}{x} = k$$

11
15

$$x^2 + \frac{1}{x^2} = k^2 + 2$$

11
15

$$= 15^2 + 2$$

$$= 225 + 2$$

$$= 227$$

4. यदि $x - \frac{1}{x} = 15$ हो तो $x^2 + \frac{1}{x^2}$ का मान ज्ञात करें।

If $x - \frac{1}{x} = 15$ then Find the value of $x^2 + \frac{1}{x^2}$

- (a) 227 ✓
(b) 225
(c) 223
(d) 220

$$x - \frac{1}{x} = k$$

$$x + \frac{1}{x} = \sqrt{k^2 + 4} =$$

$$x - \frac{1}{x} = 5$$

$$\begin{aligned} x + \frac{1}{x} &= \sqrt{5^2 + 4} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

5. यदि $x - \frac{1}{x} = 5$ हो तो $x + \frac{1}{x} = ?$

If $x - \frac{1}{x} = 5$ then $x + \frac{1}{x} = ?$

- ☒ (a) $\sqrt{29}$
- (b) $4\sqrt{2}$
- (c) $5\sqrt{3}$
- (d) $4\sqrt{5}$

$$3x - 2 = \frac{3}{x}$$

$$3x - \frac{3}{x} = 2$$

$$x - \frac{1}{x} = \frac{2}{3}$$

$$x^2 + \frac{1}{x^2} = \left(\frac{2}{3}\right)^2 + 2$$

$$= \frac{4}{9} + 2 = \frac{22}{9}$$

6. यदि $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$ है तो $x^2 + \frac{1}{x^2}$ का मान क्या है?

[If $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$ then find the value of $x^2 + \frac{1}{x^2}$?

(a) $\frac{19}{9}$

(b) $\frac{22}{9}$

(c) $-\frac{14}{9}$

(d) $\frac{28}{9}$

$$\frac{a^2}{ab} + \frac{b^2}{ab} = \frac{5ab}{ab}$$

$$\frac{a}{b} + \frac{b}{a} = 5$$

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 5^2 - 2 = \boxed{23}$$

7. यदि $a^2 + b^2 = 5ab$ है, तो $\frac{a^2}{b^2} + \frac{b^2}{a^2}$ का मान क्या है?

If $a^2 + b^2 = 5ab$ then find the value of $\frac{a^2}{b^2} + \frac{b^2}{a^2}$?

(a) 27

(b) 32

✓ (c) 23

(d) -23

$$x - \frac{1}{x} = k = 5$$

$$x^3 - \frac{1}{x^3} = k^3 + 3k$$

$$5^3 + 3 \times 5 = 125 + 15 \\ = 140$$

8. If $x - \frac{1}{x} = 5$ then $x^3 - \frac{1}{x^3} = ?$
 यदि $x - \frac{1}{x} = 5$ then $x^3 - \frac{1}{x^3} = ?$

(a) 110

(b) 140

(c) 125

(d) 105

$$\begin{aligned}
 x &= 3 + \sqrt{8} \\
 \frac{1}{x} &= 3 - \sqrt{8} \\
 \hline
 x + \frac{1}{x} &= 6
 \end{aligned}$$

$$\begin{aligned}
 x^2 + \frac{1}{x^2} &= ? \\
 &= 6^2 - 2 \\
 &= 36 - 2 \\
 &= 34
 \end{aligned}$$

$x + \frac{1}{x} = \square$

9. If $x = 3 + \sqrt{8}$ then $x^2 + \frac{1}{x^2} = ?$

यदि $x = 3 + \sqrt{8}$ हो तो $x^2 + \frac{1}{x^2} = ?$

- (a) 38
- (b) 36
- ✓ (c) 34
- (d) 30

$$x + \frac{1}{x} = 6$$

$$x - \frac{1}{x} = 2\sqrt{8}$$

10. If $x = \sqrt{3} + \sqrt{2}$ then the value of $x^3 + \frac{1}{x^3} = ?$

यदि $x = \sqrt{3} + \sqrt{2}$ हो तो $x^3 + \frac{1}{x^3} = ?$

(a) $10\sqrt{2}$

(b) $30\sqrt{2}$

✓ (c) $18\sqrt{3}$

(d) $24\sqrt{3}$

$$x + \frac{1}{x} = 2\sqrt{3}$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= (2\sqrt{3})^3 - 3 \times 2\sqrt{3} \\ &= 24\sqrt{3} - 6\sqrt{3} \\ &= 18\sqrt{3} \\ &= \end{aligned}$$

Reverse Case :

① $x + \frac{1}{x} = p$

$\hookrightarrow x - \frac{1}{x} = \sqrt{p^2 - 4}$

② $x - \frac{1}{x} = k$

$\hookrightarrow x + \frac{1}{x} = \sqrt{k^2 + 4}$

$x + \frac{1}{x} = \textcircled{4}$

$x - \frac{1}{x} = \boxed{\sqrt{12}} = 2\sqrt{3}$

Q. 9b $x + \frac{1}{x} = \textcircled{5}$

$x^2 - \frac{1}{x^2} =$

$\sqrt{\left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)}$

$5 \times \sqrt{21} = 5\sqrt{21}$

$x^3 - \frac{1}{x^3} = ?$

$(\sqrt{5})^3 + 3\sqrt{5}$

$5\sqrt{5} + 3\sqrt{5}$
 $= \underline{8\sqrt{5}}$

(Q) $x + \frac{1}{x} = 3$

$\downarrow$
 $x - \frac{1}{x} = \sqrt{5}$

$$(a-3) - \frac{1}{a-3} = 6-3 \Rightarrow (a-3)^3 - \frac{1}{(a-3)^3} =$$

$$= 3^3 + 3 \times 3$$

$$= 27 + 9 = 36$$

11. यदि $a - \frac{1}{a-3} = 6$ है, तो $(a-3)^3 - \frac{1}{(a-3)^3} = ?$

If $a - \frac{1}{a-3} = 6$ then तो $(a-3)^3 - \frac{1}{(a-3)^3} = ?$

(a) 27

(b) 18

(c) 36

(d) 25

$$x^{16} + \frac{1}{x^{16}} = 2\sqrt{10}$$

$$x^{16} - \frac{1}{x^{16}} = \sqrt{(2\sqrt{10})^2 - 4}$$

$$= \sqrt{40 - 4} = \sqrt{36} = 6$$

$$\frac{(x^{16})^3 - \frac{1}{(x^{16})^3}}{(x^{16})^3} = 6^3 + 3 \times 6$$

$$= 216 + 18 = 234$$

12. यदि $(x^{16})^3 + \frac{1}{x^{16}} = 2\sqrt{10}$ है, तो $x^{48} - \frac{1}{x^{48}}$ का मान ज्ञात कीजिए।

[If $x^{16} + \frac{1}{x^{16}} = 2\sqrt{10}$ then $x^{48} - \frac{1}{x^{48}}$

(a) 216

(b) 198

(c) 234

(d) 110

$$\frac{x^{24}}{x^{12}} + \frac{1}{x^{12}} = 7$$

$$x^{12} + \frac{1}{x^{12}} = 7$$

$$\begin{aligned} \frac{x^{72} + 1}{x^{36}} &= x^{36} + \frac{1}{x^{36}} = 7^3 - 3 \times 7 \\ &= 343 - 21 \\ &= \underline{322} \end{aligned}$$

13. यदि $\frac{x^{24} + 1}{x^{12}} = 7$ है तो $\frac{x^{72} + 1}{x^{36}}$ का मान क्या है?

If $\frac{x^{24} + 1}{x^{12}} = 7$ then what is the value of $\frac{x^{72} + 1}{x^{36}}$

(a) 364

✓ (b) 322

(c) 352

(d) 358

14. यदि $a^2 + \frac{1}{(a-3)^2} - 6a = 9$ है तो $(a-3) - \frac{1}{(a-3)}$ का मान क्या है?

If $a^2 + \frac{1}{(a-3)^2} - 6a = 9$ then $(a-3) - \frac{1}{(a-3)} = ?$

$$(a^2 - 6a + 9) + \frac{1}{(a-3)^2} = 9 + 9$$

$$(a-3)^2 + \frac{1}{(a-3)^2} = 18$$

(a) 4

(b) $2\sqrt{3}$

(c) 3

(d) Imaginary Number (काल्पनिक संख्या)

$$(a-3) - \frac{1}{(a-3)} = \sqrt{18-2} = \sqrt{16} = 4$$

$$(a-3) + \frac{1}{a-3} = \sqrt{18+2}$$

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