On the Link Between Intergenerational Mobility and Inequality: Are They Truly Distinct?

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Abstract

Income inequality and income intergenerational mobility are negatively associated empirically across countries and across time. There is also a known mechanical relationship between measures of income inequality and intergenerational mobility. This paper tests whether the mechanical relationship explains the empirical association. We find that this relationship alone explains at least 64% of the variance in mobility across 36 countries. We also show that the mechanical relationship accords well with income inequality data across time for the United States. This suggests that policy aiming to achieve more equal outcomes will likely lead to more equal opportunities and vice versa. Yet, these findings also imply that validating empirically causal mechanisms for links between mobility and inequality require being over and above the mechanical relationship.

Keywords: Intergenerational mobility, Income inequality, Steady state
JEL Codes: D6, E2, J6

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1 Introduction

The distinction between equality of outcomes and equality of opportunities is crucial both from normative and policy perspectives. In light of the rise in income inequality across the Globe in recent decades, this distinction becomes even more salient for positive economic research and for the political and public debate on inequality and social mobility.

Equalities of opportunities and of outcomes may indeed be distinct, and a decrease in social mobility may not accompany the rising income and wealth inequalities. If "economic success is largely unpredictable on the basis of observed aspects of family background, [...] we can reasonably claim that society provides equal opportunity." (Stokey, 1998) For policy, this distinction is important. It can motivate, for example, progressive taxation as a redistributive tool, which could mitigate inequality of outcomes. Alternatively, public services such as universal healthcare and public education systems, could promote equality of opportunities. However, if the link between these equalities is strong, policy aiming to achieve an effect on one, would also affect the other.[1]

This paper addresses the distinction between equality of outcomes and equality of opportunities. It studies the mechanical relationship between canonical measures of income inequality and social mobility. It then tests whether this relationship can explain the existing empirical evidence on links between inequality and mobility. We find that this evidence can be largely explained by the mechanical relationship.

We also study the steady state assumption in the mechanical relationship. This relates to the "movie vs. snapshot" debate (Hart, 1981; Kanbur and Stiglitz, 1986; Jenkins, 1987; Kanbur, 2019; Nybom and Stuhler, 2019): "The conventional justification for moving from income distribution to intergenerational mobility analysis is that it is a move from static to dynamic, from outcome to process, indeed from snapshot to movie." (Kanbur, 2019) We find that inequality measures can be well-approximated by estimating intergenerational mobility measures. It follows that the distinction between inequality of outcomes and opportunities is blurred.

A vast literature has studied the close relationship between inequality and intergenerational mobility over the last half a century. Andrews and Leigh (2009); Björklund and Jäntti (2011) have presented early empirical evidence for the positive relationship between inequality and intergenerational persistence. This relationship across countries is sometimes referred to as the *Great Gatsby curve* (Krueger, 2012; Corak, 2016), displayed in Fig. 1. As noted by Guner (2015), "the Great Gatsby curve can result from simple statistical mechanics. Imagine that persistence across gener-

[1] In the context of this paper we consider intergenerational mobility measures as a proxy of equality of opportunity. As described by Chetty et al. (2014a): "Studies of intergenerational mobility seek to measure the degree to which a child's social and economic opportunities depend on his parents' income or social status. Because opportunities are difficult to measure, virtually all empirical studies of mobility measure the extent to which a child's income (or occupation) depends on his parents' income (or occupation)." Equality of opportunity and its relationship with standard measures of intergenerational mobility have been widely discussed in recent literature (see, for example, Björklund and Jäntti (2011); Brunori, Ferreira and Peragine (2013); Roemer and Trannoy (2015); Bourguignon (2018); Hufe, Kanbur and Peichl (2018); Adermon, Brandén and Nybom (2019)).

ations follows a first-order autoregressive process. Then, the stationary distribution has a higher variance when persistence is higher." This paper addresses this point exactly. Inequality and mobility are quantified using measures that are related by design. Thus, we study the extent to which the empirical evidence for their relationship fits the mechanical links between them.

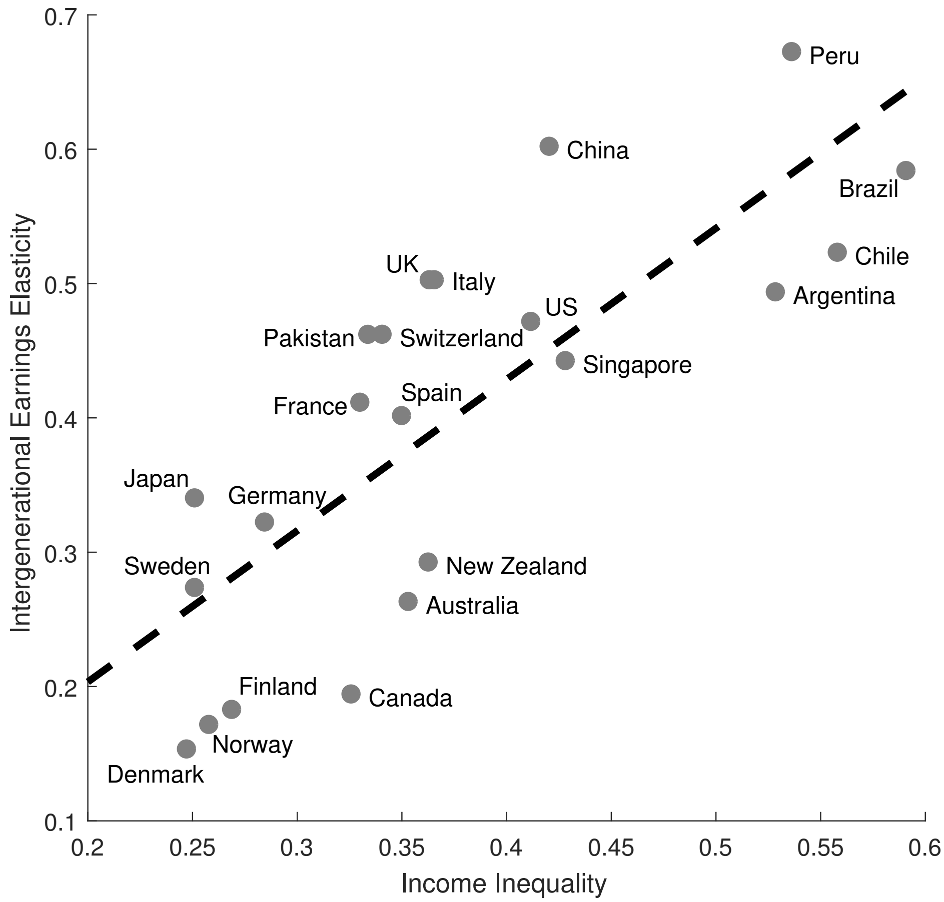

Figure 1: The Great Gatsby curve. The intergenerational earnings elasticity is measured as the elasticity between paternal earnings and a son's adult earnings, using data on a cohort of children born during the early to mid-1960s and measuring their adult outcomes in the mid- to late-1990s in various countries. The income inequality is measured as the Gini coefficient, referring to income distributions of 1985, and based on OECD reports. The dashed black curve is the linear least-squares best-fit for these data ($R^2 = 0.61$). This figure is reproduced from Corak (2013).

Prior to the empirical literature on the topic, a large body of theoretical literature has developed, pioneered by Becker and Tomes (1979). Classic structural theories of intergenerational mobility[2] have emphasized the role of human capital accumulation and parental investment, complementarities in the formation of skills, and the effect of inequality on redistributive preferences and policy. Based on a similar framework, Bratsberg et al. (2007) showed that "microfounded models of parental

[2]See, for example, Becker and Tomes (1979); Loury (1981); Becker and Tomes (1986); Mulligan (1997); Bénabou and Ok (2001); Solon (2004); Hassler, Mora and Zeira (2007); Cunha, Heckman and Schennach (2010); Becker et al. (2018).

investment in children could predict either a concave or convex relationship between the children's income and parental income, depending particularly on the nature of credit constraints." (Kanbur, 2019) Galor and Tsiddon (1997) get similar results. They show that "in periods of major technological inventions, a decline in the relative importance of initial conditions raises inequality [and] enhances mobility." A strand of reduced-form models has evolved around the same time (Conlisk, 1974, 1977), yet "the Becker-Tomes model remains the main building block of economic research on intergenerational mobility." (Mogstad, 2017)

This paper discusses the interrelationship between intergenerational mobility and income inequality without any structural theoretical presumptions. We show that standard mobility and inequality measures can be derived from one another. Thus, canonical measures of mobility and inequality may quantify similar, rather than distinct, underlying properties of the income distribution. For these reasons, empirical attempts to show causal links between mobility and inequality will have to be over and above this mechanical relationship.

We begin by looking at the definition of a canonical measure of mobility, the intergenerational earnings elasticity (IGE). We get closed-form expressions for standard inequality measures such as the variance of logarithms and the Gini coefficient directly from the IGE definition. We then test the validity and practical relevance of these expressions using data from Corak (2013), Chetty et al. (2017) and Narayan et al. (2018). We find that the closed-form expressions better explain the cross-country data presented in the Great Gatsby curve than a naïve linear relationship. The mechanical relationship alone explains at least 64% of the variance in intergenerational mobility across 36 countries and for different birth cohorts. This suggests that reducing income inequality will lead to higher mobility and vice versa. Yet, this also implies that to draw conclusions about the causal links between mobility and inequality in the sense described in the theoretical literature other mobility measures may be necessary.

We also find that the evolution of inequality measures over time in the United States is well-approximated by the output of intergenerational mobility measurement. We also find the opposite – measuring inequality allows estimating the IGE with high confidence. This proximity, between the empirical evidence and the theoretical results, suggests that despite the evolution of the income distribution over time, changes in inequality between one generation to another can be thought of as changes between one steady state to another, in practice. It follows that for the measures of inequality and intergenerational mobility used, one could move from measuring mobility to inequality and vice versa. This means that the effect of parents on their children's incomes is rapidly absorbed in the children income distribution. Mobility is high enough for this to occur within less than 30 years.

The importance of steady state assumptions in intergenerational mobility research was discussed by Atkinson and Jenkins (1984). This work was expanded by Nybom and Stuhler (2019) and applied to recent empirical literature. We show that estimated intergenerational elasticities inform how fast the convergence to a steady state occurs. Thus, it becomes possible to interpret whether

the steady state assumption is valid or not in studies of intergenerational mobility. We find that for the relationship between mobility and inequality it is justified for intergenerational elasticities of less than 0.6. Thus, the steady state assumption is valid in almost all countries.

We note that previous works have already presented functional connections between mobility and inequality. As noted above, the existence of a mechanical relationship between the IGE and measures of inequality is not a new insight in and of itself. It goes back to the works of Gibrat (1931); Kalecki (1945); Champernowne (1953); Creedy (1974); Hart (1976).[3] For this reason, among others, the empirical literature is moving away from using the IGE to Spearman's rank correlation (or rank-rank slope) (Chetty et al., 2014a). Yet, whether the IGE have been increasing in recent decades, while the rank correlation remained stable (Chetty et al., 2014b) is debatable. For example, Aaronson and Mazumder (2008); Davis and Mazumder (2019) suggest that the rank correlation and the IGE have been increasing together in the United States. Furthermore, both measures are linearly dependent in steady state (see details in Appendix B, also supported empirically by Acciari, Polo and Violante (2019)). Across generations, therefore, it is plausible to assume that the IGE and the rank correlation co-move.

Our contribution is thus fourfold. First, from an empirical perspective, the primary contribution of this paper is to show that the current evidence on the relationship between the IGE and measures of economic inequality is mainly mechanical. They can be explained, to a large extent, by the definition of the IGE. This includes, in particular, the Great Gatsby curve.

Second, our results show that income inequality measures converge rapidly with respect to the intergenerational impact on income. Income distributions are constantly evolving. Yet, in practice, changes in inequality between one generation to another can be thought of as changes between one steady state to another. In other words, canonical measures of mobility and inequality largely quantify similar, rather than distinct, underlying properties of the income distribution. This reinforces the "In Praise of Snapshots" idea (Kanbur, 2019) – suggesting that moving away from equality of outcomes to equality of opportunity is misleading and that the distinction between inequality of outcomes and opportunities is blurred.

Third, this paper derives closed-form expressions for the relationship between canonical measures of intergenerational mobility and economic inequality. These expressions are validated using data. Thus, they could prove useful for simple approximations and back-of-the-envelope calculations of inequality measures in terms of mobility measures, and vice versa.

Finally, from a theoretical perspective, this paper complements the methodological and conceptual debate between Goldberger (1989) and Becker (1989) on the relationship between mobility and inequality. Goldberger (1989) argued that "structural relationships to be found [...] may not rest on utility maximization, but are nonetheless 'behavioral'." Becker (1989) rebutted this argument. His approach remains the standard theoretical approach to intergenerational mobility today. Our findings do not invalidate either of the theoretical approaches. Yet, this paper suggests that such a

[3] See Kanbur (2018) for a review of these works.

validation requires clarifying that any relationship between inequality and mobility measure should be over and above the mechanical links we study.

The paper is structured as follows. Sec. 2 presents the main theoretical results; Sec. 3 discusses empirical evidence, showing its consistence with the theoretical findings. It is followed by a concluding discussion in Sec. 4.

2 The mechanical relationship between intergenerational elasticity and inequality measures

The canonical measure of mobility, the intergenerational earnings elasticity, measures how "sticky" earnings are across generations within the same family or household and is derived from a regression-to-the-mean model. It is defined as the least squares estimate of the coefficient β in the following equation:

$$\log Y_{i,t} = \alpha + \beta \log Y_{i,t-1} + \epsilon_{i,t} \,, \tag{2.1}$$

where Y represents the earnings of an individual or a family indexed by i.

Equation (2.1) can be also thought of as a model for the propagation of Y_i across two generations, indexed $t-1$ and t. The parameter α "captures the trend in average incomes across generations, due, for example, to changes in productivity, international trade, technology, or labor market institutions." (Corak, 2013) ϵ incorporates additional effects on the t generation, which are not correlated with the $t-1$ generation and are not consistent, as otherwise would be considered as a part of the trend α. It can be interpreted as an idiosyncratic income shock. Eq. (2.1) implies that the higher β is, or the higher the IGE is, the more it is possible to deduce on the child's earnings from the parent's earnings and vice versa. Typically, β is within the range 0.1–0.6 (Corak, 2013).

Following this definition, we consider a population consisting of N families (or households), indexed by $i = 1, \ldots, N$, and each assigned an initial income $Y_{i,1}$. We denote by $Y_{i,t}$ the income of the i-th family at generation t, for $t = 1, 2, \ldots$. Denoting $X_{i,t} = \log Y_{i,t}$ and assuming that for a given generation t the income shock, $\epsilon_{i,t}$, follows a random variable with variance σ^2 and zero expectation, we can rewrite Eq. (2.1), so $X_{i,t}$ obey an autoregressive model ($AR\,(1)$):

$$X_{i,t} = \alpha + \beta X_{i,t-1} + \sigma \omega_{i,t} \,, \tag{2.2}$$

where $\omega_{i,t}$ are random variates with zero expectation and unit variance and are independent of $X_{i,t-1}$ (we will also address a case in which this independence assumption is relaxed).

Our aim is to use Eq. (2.2) and find the relationship between β and income inequality measures defined on Y. We address two standard inequality measures – the Gini coefficient and the variance of logarithms. Nonetheless, the same logic would apply to any measure of inequality that is directly derived from the income distribution, such as top income shares (see Appendix C).

2.1 The IGE and the variance of logarithms

We first consider the variance of logarithms VL_t, a common measure of inequality (Cowell, 2011), defined as the variance of the logarithm of incomes. Taking the variance of each side of Eq. (2.2) we obtain

$$VL_t = \beta^2 VL_{t-1} + \sigma^2 \,. \tag{2.3}$$

It follows immediately that the higher β is, or the smaller the mobility is, the higher VL_t is, *i.e.* greater inequality. This simple observation was already discussed by Creedy (1974); Hart (1976). It immediately follows that in a steady state, meaning that the distribution does not change in shape between consecutive generations, the variance of logarithms is

$$VL^* = \frac{\sigma^2}{1-\beta^2} \,. \tag{2.4}$$

VL^* is an increasing function of β for $0 < \beta < 1$. If follows, as expected, that the lower the mobility is, the greater the inequality is. This result is general and does not depend on the distribution of income shocks, $\omega_{i,t}$. In addition, and despite the simplicity of Eq. (2.4), this expression can be practically used as an approximation of the value of VL_t, as we demonstrate in Sec. 3.

Eq. (2.3) illustrates an additional important aspect as to how IGE values should be interpreted. β is usually treated as dimensionless – large β means low mobility, small β means high mobility. Yet, β has units, hence an additional meaning – it determines the speed of convergence of inequality measures:

Proposition 1 *Assuming $0 < \beta < 1$ and income shocks with a fixed standard deviation σ, VL_t converges exponentially to VL^* with convergence time $-\frac{1}{2\log\beta}$.*

The convergence time $-\frac{1}{2\log\beta}$ is measured in the unit of observation – one generation for intergenerational mobility. So small β also means fast convergence, and large β means slow convergence relative to the unit of observation.

For $\beta = 0.45$, similar to that of the United States in the 1980 (Aaronson and Mazumder, 2008), the convergence time is approximately 0.6 generations. For $\beta = 0.2$, similar to that of Canada (Corak, 2016), it is 0.3 generations. In other words, convergence is practically fast in these countries. It takes less time than the observation window length (one generation) for the income distribution to absorb the effect of the parents' distribution. Crucially, the information we extract from the IGE estimation would inform inequality. Put simply, if one knows the values of β and σ in Eq. (2.2), one is able to estimate accurately the variance of logarithms in the children's generation.

β that is as large as, say, 0.8, would correspond to convergence time of more than two generations. This would mean that convergence is slow, and the IGE is not very informative on the level of inequality in the children's generation.

These observations are important for several reasons. First, in order for Eq. (2.4) to be practically

relevant, convergence needs to be faster than the observation window. If the convergence time is several generations, the expression in Eq. (2.4) will not be informative on the observed inequality in the children's generation.

Second, this result provides a scale, otherwise missing from the interpretation of β. It is an objective criterion for telling high mobility from low mobility. Arguing that mobility is high or low is given a tangible meaning this way – mobility is high if convergence is fast relative to the observation window, and low otherwise. In the absence of an objective criterion, mobility can only be said to be low or high relative to arbitrary values, or comparing different countries or the same country over time.

The proximity of the real variance of logarithms to the value predicted by Eq. (2.4) and the short convergence times also validate the steady state assumption. This suggests that despite the evolution of the income distribution over time, changes in inequality between one generation to another can be thought of as changes between one steady state to another, in practice. It follows that for these measures of inequality and intergenerational mobility, it is possible to move from the measurement of mobility to inequality, and the effect of parents on their children's incomes is rapidly absorbed in the children income distribution. Mobility is high enough for this to occur within less than 30 years.

2.2 The IGE and the Gini coefficient

We also use the definition of IGE to derive the Gini coefficient of the income distribution $Y_{i,t}$. In the long time limit ($t \to \infty$), for $AR(1)$ processes with $0 < \beta < 1$, the distribution of X_i (following the notation of Eq. (2.2)), reaches a steady state. Assuming $\omega_{i,t}$ are normally distributed, the steady state log-income distribution would be (Hamilton, 1994)

$$X_i \sim \mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right) . \tag{2.5}$$

It follows that the income distribution is log-normal:

$$Y_i \sim \log\mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right), \tag{2.6}$$

and the Gini coefficient of Y_i is[4]

$$G^* = \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{\sigma^2}{1-\beta^2}}\right), \tag{2.7}$$

where erf is the Gauss error function – $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$. We note that G^* does not depend on α, due to the Gini coefficient invariance to multiplication by a constant.

As in the case of the variance of logarithms, for the Gini coefficient in Eq. (2.7) to be relevant,

[4] The Gini coefficient of $\log\mathcal{N}\left(m, s^2\right)$ is $\operatorname{erf}\left(\frac{s}{2}\right)$ (Crow and Shimizu, 1988).

convergence to a steady state has to be quick. In addition, β and σ are not fixed in time, and the Gini coefficient is constantly changing. Yet, as we will see in Sec. 3, and as explained for the variance of logarithms – it takes a single generation for an observed Gini coefficient to reflect the values of β and σ, and to follow closely Eq. (2.7).

Eq. (2.7) also shows that G^* increases with β for $0 < \beta < 1$:

$$\frac{\partial G^*}{\partial \beta} = \beta \frac{\sqrt{\frac{\sigma^2}{\pi(1-\beta^2)}}e^{-\frac{\sigma^2}{4(1-\beta^2)}}}{1-\beta^2} > 0\,. \tag{2.8}$$

Thus, not only that the IGE definition implies a certain degree of income inequality, we obtain a built-in positive relationship between the IGE and the income Gini coefficient, or between immobility and inequality.

Instead of using IGE, it is possible to consider a more general case, in which p prior generations affect the income of the current generation (Jæger and Holm, 2007; Hertel and Groh-Samberg, 2014; Braun and Stuhler, 2016; Neidhöfer and Stockhausen, 2018). We consider a set of elasticities, $\{\beta_j\}$, which satisfy the following regression equation:

$$\log Y_{i,t} = \alpha + \sum_{j=1}^{p} (\beta_j \log Y_{i,t-j}) + \epsilon_{i,t}\,, \tag{2.9}$$

where p is the number of generations taken into account for the effect on the current generation's incomes.

The same type of built-in positive relationship between the elasticities, $\{\beta_j\}$, and the income Gini coefficient exists also in this general case. For example, for the case $p = 2$ we obtain (see Appendix D)

$$G^* = \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{\sigma^2}{(1+\beta_2)^2\left[(1-\beta_2)^2-\beta_1^2\right]}}\right), \tag{2.10}$$

for which

$$\frac{\partial G^*}{\partial \beta_1} > 0 \text{ and } \frac{\partial G^*}{\partial \beta_2} > 0\,. \tag{2.11}$$

2.3 Modification – stable idiosyncratic income shock

So far we assumed that the income shock $\epsilon_{i,t}$ is different for every generation t and for each family i and is independent of $X_{i,t-1}$. It is possible that the shock reflects inherited skill and ability, which may consistently increase the family income relative to the overall population or decrease it (see Clark (2014), for example). An important modification of Eq. (2.2) that takes this into

account, is such that the idiosyncratic shock is fixed in time:

$$X_{i,t} = \alpha + \beta X_{i,t-1} + \sigma \omega_i \,, \tag{2.12}$$

assuming that ω_i are normal iid random variables with zero expectation and unit variance.

Following Eq. (2.12) it is possible to show that if $|\beta| < 1$, the distribution of X again converges to a normal distribution:

Proposition 2 *Assuming* $|\beta| < 1$ *and a stable income shock* $\omega_i \sim \mathcal{N}(0,1)$*, the distribution of* X *converges to a normal distribution –* $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \left(\frac{\sigma}{1-\beta}\right)^2\right)$.

Consequently, Y_i follow a log-normal distribution and the steady state Gini coefficient for the income distribution is:

$$\tilde{G}^* = \operatorname{erf}\left(\frac{\sigma}{2(1-\beta)}\right), \tag{2.13}$$

and if $0 < \beta < 1$, $\tilde{G}^*$ increases with β:

$$\frac{\partial \tilde{G}^*}{\partial \beta} = \frac{e^{-\frac{\sigma^2}{4(1-\beta)^2}}\sigma}{\sqrt{\pi}\,(1-\beta)^2} > 0 \,. \tag{2.14}$$

Thus, in both specifications of the mechanical relationship between the Gini coefficient and the IGE – Eq. (2.7) and Eq. (2.13) – income inequality decreases with increasing mobility.

3 Comparison with empirical evidence

3.1 Data

The derivation above demonstrated that the intergenerational elasticity definition may mechanically lead to its positive association with income inequality. In this section we aim to verify that the mechanical relationship holds empirically. Specifically, we will show that the empirical evidence for the relationship between income inequality measures and IGE is almost fully explained by the expressions derived above, suggesting that this relationship is largely mechanical.

For this analysis we use data from several sources:

- Chetty et al. (2017) provide a dataset of income distributions, used for estimating absolute mobility rates in the United States for birth cohorts 1940–1984. These distributions represent the pre-tax family income of 30 year olds in each birth cohort, based on Census and Current Population Survey data. They also provide a 100×100 copula for the United States. It describes the joint rank distribution in the United States of parents and children. Each element in the matrix represents the probability of a child born to parents who were in

income percentile i at age 30, to occupy income percentile j in age 30, with i and j between 1 and 100. This copula is assumed to be fixed in time, motivated by evidence of copula stability since the 1970s (Lee and Solon, 2009; Chetty et al., 2014b, 2017). Combining the copula and the marginal distributions produces joint income distributions of parents and children in the United States, from which β and σ can be estimated, as well as the income inequality measures of the parents' generation and the children's generation. The way in which the copula and the marginal distributions are combined into a joint distribution is detailed in Chetty et al. (2017); Berman (2019).

- The relationship between the IGE and the Gini coefficients is validated against the data used for the Great Gatsby curve (Corak, 2016), complemented by more recent data from the World Bank (Narayan et al., 2018). These data include Gini coefficient and IGE estimates for 36 countries and different birth cohorts (47 Gini and IGE pairs in total). They are detailed in Appendix E.

- The intergenerational elasticity in the United States from 1950 to 2010 estimated by Aaronson and Mazumder (2008) (later extended in Davis and Mazumder (2019)).

3.2 IGE and the variance of logarithms

First, we demonstrate that the steady state expression for the variance of logarithms in Eq. (2.4) is consistent with empirical estimates of the variance of log-incomes. We estimate the historical series of the variance of logarithms in the United States, using income data from Chetty et al. (2017) for the period 1940–2010. We then assume that the copula of the joint parent and child income distribution (defined as the joint distribution of parent and child income ranks) remained stable across all cohorts and use the copula given by Chetty et al. (2017) to estimate β and σ for each cohort.

We follow the methodology of Chetty et al. (2017) and assume that the parents are represented by the income distribution 30 years prior to that of their children. For each year y between 1970 and 2010 we combine the copula and the marginal income distributions for 30 year-olds in years $y-30$ and y. This creates a joint income distribution for years $y-30$ and y, from which we estimate β and σ using an OLS regression (see Eq. (2.1)). We then use β and σ to predict the variance of logarithms according to Eq. (2.4): $VL^* = \frac{\sigma^2}{1-\beta^2}$. The predicted value is compared to the direct estimate of the children's generation variance of logarithms, VL_t.

The results are presented in Fig. 2. They indicate that for the purpose of estimating the variance of logarithms, the steady state assumption is justified and that Eq. (2.4) is a good approximation of the value of VL_t. In other words, even if σ and β are not fixed in time, the distribution changes fast enough so that the steady state expression in Eq. (2.4) is close to the true value of VL_t. Inequality and mobility are both evolving over time and the income distribution is not in a steady state, but it is possible to estimate inequality measures as if it is in a steady state.

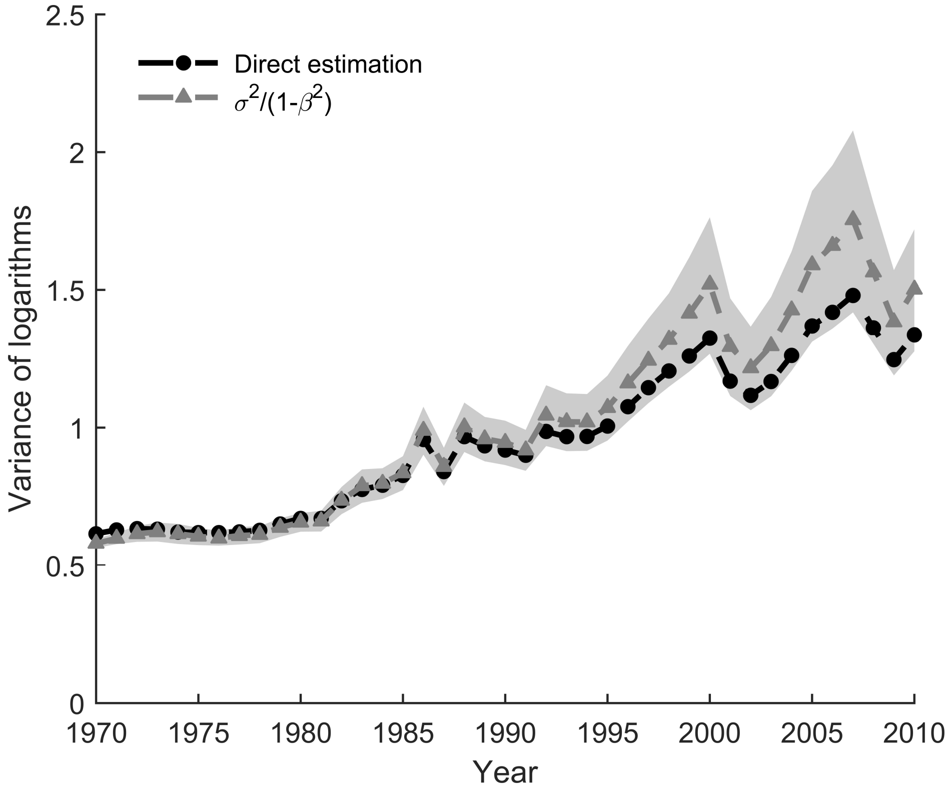

Figure 2: Estimated variance of logarithms for the United States pre-tax incomes (1970–2010). The black circles are the direct estimates of the variance of logarithms based on Chetty et al. (2017). The gray triangles are $\frac{\sigma^2}{1-\beta^2}$ for σ and β estimated for the United States based on a stable copula. The shaded gray area represents 95% confidence intervals for the estimates of $\frac{\sigma^2}{1-\beta^2}$, based on the OLS standard errors. The year on the x-axis refers to the year in which children are assumed to be 30 year-olds.

Given the estimated β values – lower than 0.5 – these results are to be expected. Based on the results in Sec. 2.1 convergence is fast enough. It follows that for IGE and the variance of logarithms, it is possible to move from mobility to inequality, *i.e.* the measurement of mobility is not distinct to the measurement of inequality in practice. The effect of parents on their children's incomes is rapidly absorbed in the children income distribution. We note that for a less mobile economy, *i.e.* had the rank-rank slope been much higher than 0.3, as reported in Chetty et al. (2014b), the convergence might not be as fast. In such a case, the predicted variance of logarithms in Eq. (2.4) will be biased with respect to the real one (see Appendix F).

3.3 IGE and the Gini coefficient

We also test the steady state Gini coefficients in Eq. (2.7) and Eq. (2.13) against data. Using these equations for such a test, we explicitly make the following assumptions:

1. The income shocks ϵ are normally distributed

2. σ is fixed

3. $\alpha = 0$

4. The Gini coefficient reaches its asymptotic value very quickly so that the data is reflected by G^*

These assumptions may seem far from general, especially amidst the increase in income inequality, indicating that the income distribution is not in a steady state. However, the results are robust under a series of tests and changes in the assumptions (see Appendix C).

We can rewrite Eq. (2.7) and Eq. (2.13) as

$$\begin{aligned} \beta &= \sqrt{1 - \frac{\sigma^2}{\left(2\,\mathrm{erf}^{-1}\left(G^*\right)\right)^2}} \\ \tilde{\beta} &= 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}\left(\tilde{G}^*\right)} \end{aligned}, \tag{3.1}$$

where $\tilde{\beta}$ denotes the value of β corresponding to $\tilde{G}^*$.

Using the cross-country data in Corak (2016) and Narayan et al. (2018) we fit Eq. (3.1), and compare predicted IGE values based on the fit to data. The results are presented in Fig. 3, together with the 47 pairs of Gini coefficient and IGE values, on which the fit is based.

The results indicate that the "modified" mechanical relationship – $\beta = 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}(G)}$ – accords well with the empirical evidence. The results also imply that the modified model, in which random income shocks differ between families but not across time, is a better description of the data than the original mechanical model.

To quantify what part of the relationship between the Gini coefficient and the IGE is explained by the mechanical relationship, we estimate the following regressions, based on Eq. (3.1):

$$1 - \beta_i^2 = \alpha + \Gamma \frac{1}{\left(2\,\mathrm{erf}^{-1}\left(G_i\right)\right)^2} + \epsilon_i\,, \tag{3.2}$$

for the non-modified mechanical relationship, and

$$1 - \beta_i = \alpha + \Gamma \frac{1}{2\,\mathrm{erf}^{-1}\left(G_i\right)} + \epsilon_i\,, \tag{3.3}$$

for the modified mechanical relationship.

The main purpose of the regressions is quantifying how well the mechanical relationship describes data. We are also interested in whether the intercept α is significantly different from zero, as expected from Eq. (3.1). For comparison, we also consider the naïve linear relationship:

$$\beta_i = \alpha + \Gamma G_i + \epsilon_i\,. \tag{3.4}$$

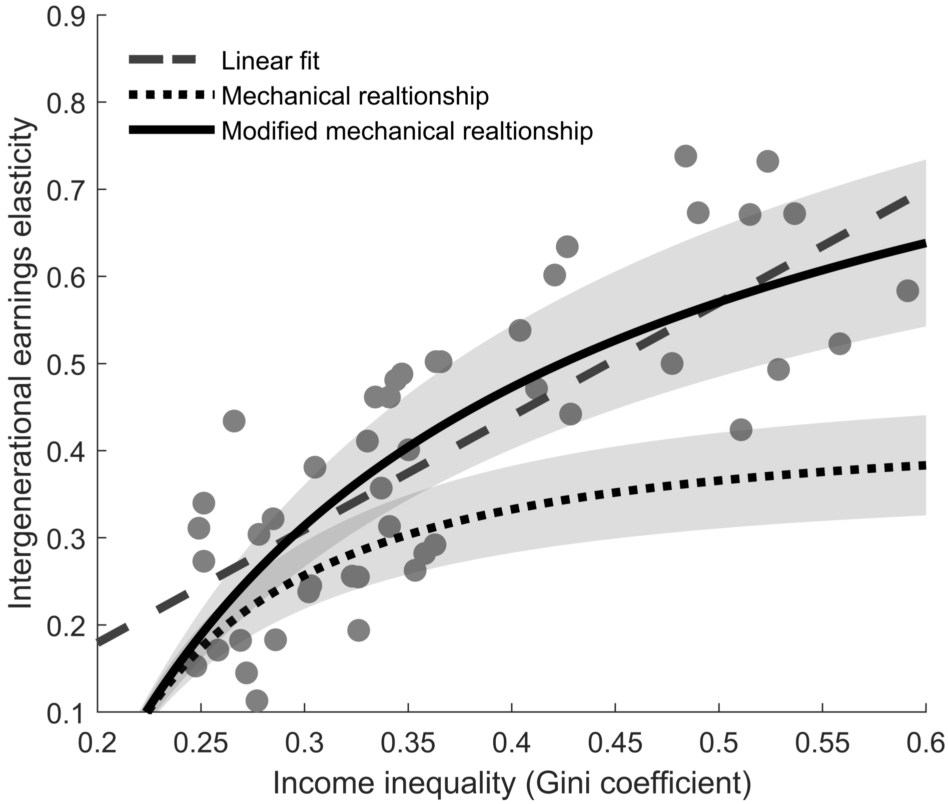

Figure 3: The relationship between IGE and the Gini coefficient: a comparison between theoretical calculations and cross-country data. The black curves represent three fits to data – a linear fit (dashed); $\beta = \sqrt{1 - \frac{\sigma^2}{\left(2\,\mathrm{erf}^{-1}(G)\right)^2}}$ (dotted); $\beta = 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}(G)}$ (solid). The shaded areas indicate $\pm 15\%$ error in β in the fits based on Eq. (3.1) (see Corak (2006)). The gray circles are 47 pairs of Gini coefficient and IGE values on which the fits are based (see Appendix E).

The results are presented in Tab. 1.

Table 1: Regression results for the mechanical relationships between the Gini coefficient and the IGE and for a naïve linear relationship across 36 countries.

	Linear model	Mechanical model	Modified mechanical model
	$\beta_i = \alpha + \Gamma G_i + \epsilon_i$	$1 - \beta_i^2 = \alpha + \Gamma \frac{1}{\left(2\,\mathrm{erf}^{-1}(G_i)\right)^2} + \epsilon_i$	$1 - \beta_i = \alpha + \Gamma \frac{1}{2\,\mathrm{erf}^{-1}(G_i)} + \epsilon_i$
α	-0.08	0.59	0.09
	(0.12)	(0.06)	(0.12)
Γ	1.30	0.09	0.33
	(0.31)	(0.02)	(0.07)
R^2	0.62	0.58	0.64
F-statistic	73.4	62.3	80.5
Observations	47	47	47

As expected, $\tilde{\beta} = 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}(\tilde{G}^*)}$ performs better than the other models. The intercept in this model is not significantly different from zero, which serves as an additional indication for its appropriate description of the data. We thus conclude that the mechanical relationship alone explains about 64% of the variance in mobility across the 36 countries (with 47 data points in total).

The results demonstrate that $\beta = 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}(G)}$ is a good, simple approximation for the IGE, provided the Gini coefficient in a country, based solely on the mechanical relationship. For the data used, these expressions provide IGE estimates that are accurate up to 0.03 (two standard errors). Alternatively, the mechanical relationship serves as an approximation for the Gini coefficient provided the IGE.

In practice, in studies such as Corak (2006); Narayan et al. (2018) estimating mobility was not done using the same income distribution from which income inequality measures are estimated. The latter are based on the entire working force during the relevant year, or on large scale household surveys. The former are based on limited datasets with parent-offspring couples, and a variety of adaptations for fitting the raw data to the regression model (Corak, 2006). Therefore, one should not expect the theoretical results to precisely match the data, which are not standardized between countries and over time, the same way the data used for testing the prediction for the variance of logarithms in the United States. Our finding, that the mechanical relationship explains 64% of the variance in mobility, is potentially understated.

Therefore, we also use the data in Chetty et al. (2017) in a similar way to the cross-country data. We fit Eq. (3.1) using the regressions above, based on US Gini coefficient estimates from Chetty et al. (2017) and the IGE data in Davis and Mazumder (2019). Fig. 4 presents these results. For both non-modified and modified mechanical models we get $R^2 = 0.94$. Unlike the results in Fig. 2, here we assume that σ is fixed in time. Still, we find that the IGE is well-approximated by the simple formulas in Eq. (3.1). It follows that it is also possible to move from inequality to mobility, *i.e.* the measurement of inequality is not distinct to the measurement of mobility in practice.

4 Conclusion

We study the mechanical relationship between intergenerational mobility and income inequality. We compare this relationship with empirical evidence and find that the mechanical relationship alone explains 64% of the variance in mobility across 36 countries. We also find that income inequality measures converge rapidly in comparison to changes in the intergenerational dynamics. This seemingly technical point is fundamental. It suggests that despite the evolution of the income distribution over time, changes in inequality between one generation to another can be thought of as changes between one steady state to another, in practice. It follows that for these measures of inequality and intergenerational mobility, one could move from mobility to income distribution, *i.e.* the measurement of inequality is not distinct to the measurement of mobility in practice, and vice versa.

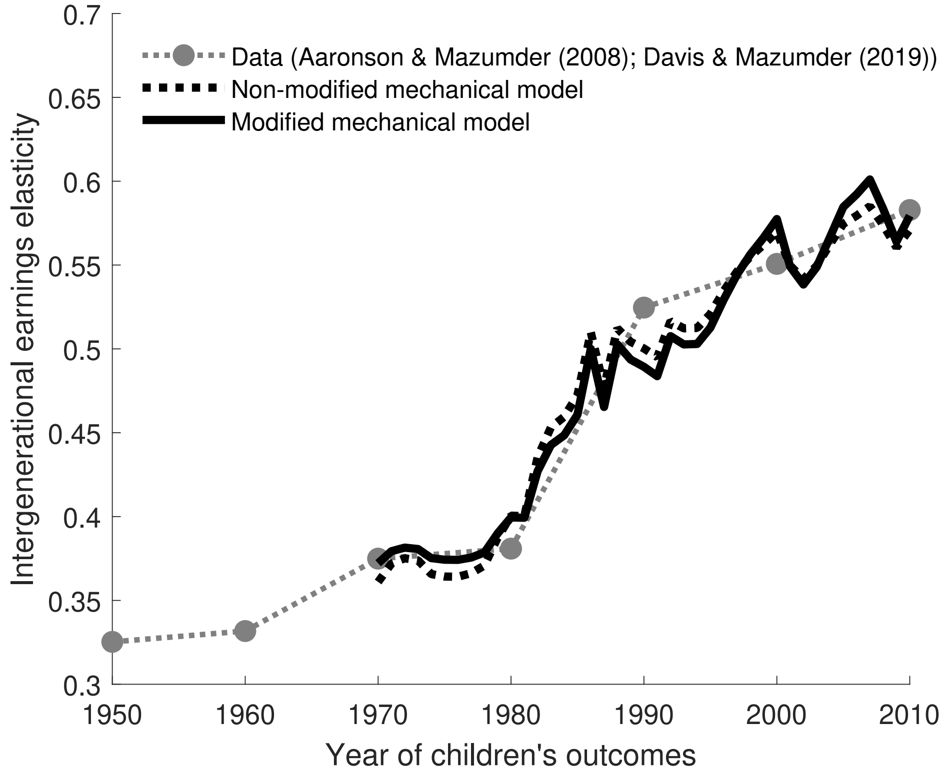

Figure 4: The intergenerational elasticity in the United States, 1950 to 2010. The gray circles are the IGE estimates from of Aaronson and Mazumder (2008); Davis and Mazumder (2019). The black curves represent two fits to data – $\beta = \sqrt{1 - \frac{\sigma^2}{\left(2\,\mathrm{erf}^{-1}(G)\right)^2}}$ (dotted); $\beta = 1 - \frac{\sigma}{2\,\mathrm{erf}^{-1}(G)}$ (solid).

Thus, our results indicate that a large part of the effect documented in the Great Gatsby curve is mechanical rather than causal. Therefore, our findings suggest that in order to draw conclusions about the causal links between mobility and inequality, other mobility measures, which are not mechanically linked ti inequality measures, would have to be used. Any empirical attempt to demonstrate causal links between mobility and inequality when measured using the IGE and the Gini coefficient (or similar inequality measures) will have to be over and above the mechanical relationship discussed in this paper.

A potential alternative measure is the rank correlation (Spearman's ρ or the rank-rank slope), which only depends on the copula between marginal distributions and is more robust than IGE (Jäntti and Jenkins, 2013; Chetty et al., 2014a). However, near the steady state, Spearman's ρ and β are approximately linearly related (see Appendix B and Trivedi and Zimmer (2007)), and the mechanical relationship derived will still apply. The empirical observation that in practice the IGE and the rank correlation are close, externally re-validates the steady state assumption of our derivation. Furthermore, recent empirical evidence suggests that indeed, the rank correlation has increased together with the IGE in the United States after 1980 (Davis and Mazumder, 2019). New approaches to measuring intergenerational mobility have also appeared recently (see, for example, Blundell and

Risa (2018)), however these approaches will need to be consistent with the theoretical literature in order to enable the identification of the causal links suggested by the latter.

Our results also illustrate an additional important aspect as to how IGE values should be interpreted. β is usually treated as dimensionless – large β means low mobility, small β means high mobility. Yet, β has units and it determines the speed of convergence of inequality measures: $-\frac{1}{2\log\beta}$ in the unit of observation, one generation for intergenerational mobility. This observation is important as it helps validating the steady state assumptions made in this paper. However, it provides a scale, otherwise missing from the interpretation of β. It is an objective criterion for telling high mobility from low mobility. Arguing that mobility is high or low is given a tangible meaning this way – mobility is high if convergence is fast relative to the observation window, and low otherwise. In the absence of an objective criterion, mobility can only be said to be low or high relative to arbitrary values, or comparing different countries or the same country over time.

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A Proofs

A.1 Proof of proposition 1

Assuming $0 < \beta < 1$ and income shocks with fixed standard deviation σ, VL_t converges exponentially to VL^* with convergence time $-\frac{1}{2\log\beta}$.

Proof: We begin with the standard linear regression used for the estimation of the IGE, where X_t are the log-incomes at generation t and ϵ_t is the noise term at generation t (leading from $t-1$ to t):

$$X_t = \alpha + \beta X_{t-1} + \epsilon_t \,. \tag{A.1}$$

Taking the variance of each side of the equation we obtain

$$\begin{aligned} VL_t \equiv \mathrm{Var}\left(X_t\right) &= \beta^2 VL_{t-1} + \sigma^2 \\ &= \beta^2\left(\beta^2 VL_{t-2} + \sigma^2\right) + \sigma^2 \\ &= \beta^{2t} VL_0 + \sigma^2\left(1 + \beta^2 + \cdots + \beta^{2(t-1)}\right) \\ &= \beta^{2t} VL_0 + \sigma^2 \frac{1-\beta^{2t}}{1-\beta^2} \\ &= \beta^{2t} VL_0 + VL^*\left(1-\beta^{2t}\right) \\ &= VL^* + \left(VL_0 - VL^*\right)\beta^{2t} \end{aligned} \,, \tag{A.2}$$

where σ is the standard deviation of the noise term ϵ_t for any t and $VL^* = \frac{\sigma^2}{1-\beta^2}$ (see Eq. (2.4)). VL_0 is the initial value of the variance of logarithms.

Assuming $|\beta| < 1$, it is clear that the variance of logarithms converges exponentially to VL^*. We can then write

$$VL^* + \left(VL_0 - VL^*\right)\beta^{2t} = VL^* + \left(VL_0 - VL^*\right)e^{-\frac{t}{\tau}} \,, \tag{A.3}$$

where τ is the convergence time, which represents how fast the convergence to VL^* occurs. We then get immediately

$$\beta^{2t} = e^{-\frac{t}{\tau}} \,, \tag{A.4}$$

so

$$\tau = -\frac{1}{2\log\beta} \,. \tag{A.5}$$

■

A.2 Proof of proposition 2

Assuming $|\beta| < 1$ *and fixed income shocks* $\omega_i \sim \mathcal{N}(0,1)$*, the distribution of* X *converges to a normal distribution –* $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \left(\frac{\sigma}{1-\beta}\right)^2\right)$.

Proof: Following Eq. (2.12), and assuming $|\beta| < 1$, we can write for $t > 0$:

$$\begin{aligned} X_{i,t+1} &= (\alpha + \sigma\omega_i) + \beta X_{i,t} \\ &= (\alpha + \sigma\omega_i) + \beta\left((\alpha + \sigma\omega_i) + \beta X_{i,t-1}\right) \\ &= (\alpha + \sigma\omega_i) + \beta\left((\alpha + \sigma\omega_i) + \beta\left((\alpha + \sigma\omega_i) + \beta X_{i,t-2}\right)\right) \\ &= (\alpha + \sigma\omega_i)\left[1 + \beta + \beta^2 + \cdots + \beta^t X_{i,1}\right] \end{aligned}. \tag{A.6}$$

Without loss of generality we assume $X_{i,1} = 1$, and obtain that in the limit $t \to \infty$, which is equivalent to assuming fast convergence (see Sec. 3), the log-income of family i is

$$X_i^* = \frac{(\alpha + \sigma\omega_i)}{1-\beta}\,. \tag{A.7}$$

Hence X_i follow a normal distribution – $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \left(\frac{\sigma}{1-\beta}\right)^2\right)$.

■

B Intergenerational earnings elasticity and rank correlation

The mechanical relationship between the IGE and measures of inequality we studied in this paper is not a new insight in and of itself. It goes back to the works of Gibrat (1931); Kalecki (1945); Champernowne (1953); Creedy (1974); Hart (1976). We wish to test if this relationship is also valid when measuring intergenerational mobility with Spearman's rank correlation, the Rank-Rank Slope (RRS) (Dahl and DeLeire, 2008).[5]

Chetty et al. (2014a) state that "the correlation of log incomes ρ_{XY} [IGE] and the correlation of ranks ρ_{PR} [RRS] are closely related scale-invariant measures of the degree to which child income depends on parent income."

Not only that the correlation of log-incomes and the correlation of ranks are closely related empirically, it can be shown that they are strongly dependent theoretically. The same way the dynamics implied by IGE relate to income inequality, they also relate to the rank correlation. Specifically, we obtain that the steady state rank correlation is roughly 4.5% lower than the IGE (see also Trivedi and Zimmer (2007)):

Proposition 3 *Considering Eq. (2.2), assuming $\beta < 1$, the rank correlation, ρ_{PR}, converges approximately to $\frac{3\beta}{\pi}$ in the limit $t \to \infty$.*

Proof: We start from Eq. (2.2):

$$X_{i,t} = \alpha + \beta X_{i,t-1} + \sigma\omega_{i,t}\,, \tag{B.1}$$

where $\omega_{i,t}$ are random variates of a normal distribution with zero expectation and unity variance. We assume $\beta < 1$, and look at the converged distribution which follows $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right)$ (see Sec. 2). The percentile rank of an individual i, with log-income X_i, would therefore be

$$P_i = \frac{1}{2}\left(1 + \text{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right)\right)\,. \tag{B.2}$$

Assuming a converged distribution, the percentile rank of the next generation R_i would be

$$R_i = \frac{1}{2}\left(1 + \text{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left((\alpha + \beta X_i + \sigma\omega_i) - \frac{\alpha}{1-\beta}\right)\right)\right)\,, \tag{B.3}$$

[5] We follow the definition of Chetty et al. (2014a):

> "Let R_i denote child i's percentile rank in the income distribution of children and P_i denote parent i's percentile rank in the income distribution of parents. Regressing the child's rank R_i on his parents' rank P_i yields a regression coefficient $\rho_{PR} = Corr\,(P_i, R_i)$, which we call the rank-rank slope. The rank-rank slope ρ_{PR} measures the association between a child's position in the income distribution and his parents' position in the distribution."

for the same X_i as in P_i, and where ω_i are random variates of a normal distribution with zero expectation and unity variance and erf is the error function.

We wish to calculate $\rho_{PR} = Corr\left(P_i, R_i\right)$ in this case:

$$\rho_{PR} = \frac{E\left[P_i R_i\right] - E\left[P_i\right] E\left[R_i\right]}{\sqrt{\left(E\left[P_i^2\right] - E\left[P_i\right]^2\right)\left(E\left[R_i^2\right] - E\left[R_i\right]^2\right)}}. \tag{B.4}$$

By definition $E\left[P_i\right] = E\left[R_i\right] = 1/2$ and similarly $E\left[P_i^2\right] = E\left[R_i^2\right] = 1/3$ and therefore $\rho_{PR} = 12E\left[P_i R_i\right] - 3$. We will explicitly calculate $E\left[P_i R_i\right]$ using Eq. (B.2) and Eq. (B.3):

$$\begin{aligned} E\left[P_i R_i\right] = \frac{1}{4} + \frac{1}{4}E&\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right)\right] + \\ &\frac{1}{4}E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right)\right] + \\ &\frac{1}{4}E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right) \times \operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right)\right] \end{aligned}. \tag{B.5}$$

$\frac{1}{4}E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right)\right]$ and $\frac{1}{4}E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right)\right]$ are 0 by definition and therefore

$$\begin{aligned} &\rho_{PR} = 12E\left[P_i R_i\right] - 3 = \\ &\quad 3E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right) \times \operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right)\right] \end{aligned}. \tag{B.6}$$

We denote $E\left[\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right) \times \operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right)\right]$ by K. In order to calculate K, we expand the error functions:

$$\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i - \frac{\alpha}{1-\beta}\right)\right), \tag{B.7}$$

and

$$\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(\left(\alpha + \beta X_i + \sigma\omega_i\right) - \frac{\alpha}{1-\beta}\right)\right) \tag{B.8}$$

as a second order power series around $\frac{\alpha}{1-\beta}$:

$$\begin{aligned}
&\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left(X_i-\frac{\alpha}{1-\beta}\right)\right)\times\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2\sigma^2}}\left((\alpha+\beta X_i+\sigma\omega_i)-\frac{\alpha}{1-\beta}\right)\right)\approx \\
&\sqrt{\frac{2\left(1-\beta^2\right)}{\pi\sigma^2}}\operatorname{erf}\left(\sqrt{\frac{1-\beta^2}{2}}\omega_i\right)\left(X_i-\frac{\alpha}{1-\beta}\right)+\frac{2\beta\left(1-\beta^2\right)}{\pi\sigma^2}e^{-\frac{1-\beta^2}{2}\omega_i}\left(X_i-\frac{\alpha}{1-\beta}\right)^2
\end{aligned} . \tag{B.9}$$

Averaging over $X_i \sim \mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right)$ and $\omega_i \sim \mathcal{N}(0,1)$, we obtain $K \approx \beta/\pi$, hence

$$\rho_{PR} = 3K \approx \frac{3\beta}{\pi} . \tag{B.10}$$

■

We also note that the steady state rank correlation is independent of σ, the standard deviation of the income shocks in Eq. (2.2) and it is reached fast enough that we should expect the measured rank correlation to be very close to the measured IGE, the same way we find it for income inequality measures in Fig. 2. This is illustrated in Fig. 5. We use the value of the IGE to simulate the income dynamics of $N = 10^7$ families and numerically calculate the resulting rank correlation. Convergence is fast – reached within 2–3 generations (see also Appendix C).

Chetty et al. (2014a) report an IGE estimate of 0.344 and a rank correlation estimate of 0.341. Based on the data reported by Boserup, Kopczuk and Kreiner (2013); Corak and Heisz (1999), they also estimate the rank correlation in Denmark as 0.18 and in Canada as 0.174, while Corak (2013) estimate their IGEs as 0.15 and 0.19, respectively. We note, however, that the periods and cohorts used for estimation in Corak (2013) are different from those used in Boserup, Kopczuk and Kreiner (2013); Corak and Heisz (1999). Nevertheless, these empirical observations are not inconsistent with the theoretical finding, as the empirical error in measuring both the rank correlation and IGE is larger than the theoretical 4.5% discrepancy between them (Corak, 2006; Chetty et al., 2014a). These observations imply that the relationship found between the IGE and the Gini coefficient is also expected between the rank correlation and the Gini coefficient.

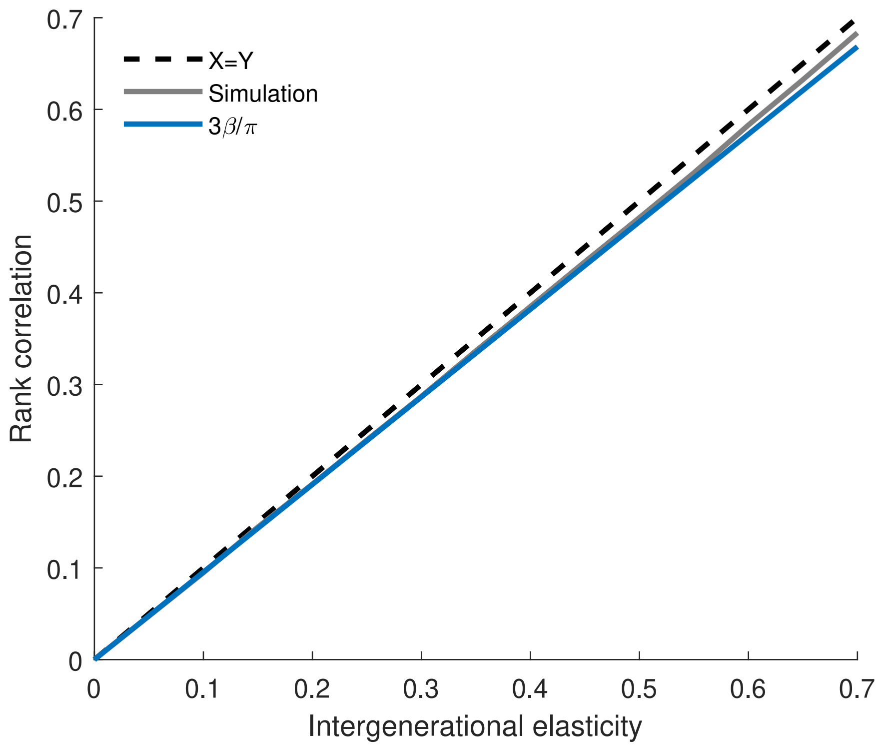

Figure 5: The relationship between the intergenerational elasticity and the rank correlation in steady state. For each value of β we use Eq. (2.1) and numerically simulate the income dynamics of a population of $N = 10^7$ families over 10 generations. We obtain that within 2–3 generations the rank correlation converges to its asymptotic value, which is very similar to the intergenerational elasticity (about 2.5% lower). The small discrepancy observed at higher β values between $3\beta/\pi$ and the simulated rank correlation is due to higher order terms in the approximations Eq. (B.7) and Eq. (B.8) (see proof above). The numerical calculations were done for $\sigma = 0.4$. As explained, the steady state value of the rank correlation is independent on the value of σ, only the speed of convergence.

C Robustness of results

The validity of the results rely on their robustness with respect to the simplifying assumptions made. Below we review the assumptions and discuss the robustness of the results:

1. The income shocks ϵ are normally distributed – this assumption leads to a log-normal income distribution in the long run, consistent with a simple Gibrat's law (Gibrat, 1931) (see also Salem and Mount (1974); Bénabou (2000); Pinkovskiy and Sala-i-Martin (2009)). The results obtained for the variance of logarithms are independent of this assumption. For the Gini coefficient, removing this assumption and letting the income shocks take any form will quantitatively change the results. Qualitatively, it will not have an effect.

2. σ is fixed within generations – the linear regression residuals when estimating β is somewhat different for the various countries analyzed. However, realistically, the σ values are relatively similar, whereas the values of β and the Gini coefficient of the different countries differ substantially (Grawe, 2004; Corak, 2006; Neidhöfer and Stockhausen, 2018). Using Eq. (2.13) it is also possible to calculate σ for each country reported by Corak (2016), according to its value of β and Gini coefficient. These values of σ lie within a narrow band with an average of 0.39 and standard error of 0.07.

3. $\alpha = 0$ – α has no effect on the Gini coefficient since it is invariant under multiplication by a constant.

4. The Gini coefficient reaches its asymptotic value very quickly so that the data is reflected by G^* – using a Monte-Carlo simulation we were able to estimate the number of generations required for reaching the steady state values of G^* and $\tilde{G}^*$ starting from a perfectly equal economy (meaning that every family has exactly the same income) and assuming fixed β and σ. For the parameters considered, meaning $\sigma \in [0.2,\ 0.6]$ and $\beta \in [0.1,\ 0.6]$, convergence, defined as being as close as 0.1% to G^*, was reached within 2–3 generations. From this we can assume that in practice, when the initial distribution is realistic and not perfectly equal, steady state is reached within a single generation (see Appendix C.1). The robustness of the results to this assumption is also demonstrated in Fig. 2 – when comparing empirical observations of inequality to the predicted inequality measures, the differences remain relatively small. Following the argument illustrated in Fig. 2, even if the income distribution and β change over time, the changes in the distribution are fast compared to the changes in mobility and therefore in practice, the steady state expressions for the inequality measures are empirically valid (see Appendix C.2).

C.1 Fast convergence of the Gini coefficient

In the discussion above, we assumed that in the case of $|\beta| < 1$, the convergence of the income distribution to its asymptotic shape, and hence the convergence of the income Gini coefficient to

its asymptotic value, is very fast. In order to test this assumption we use the dynamics implied by the IGE definition and simulate the incomes of $N = 10^7$ families for different β values. We assume that the initial income of each family is the same ($X = 1$), meaning a perfectly equal distribution. Convergence is reached within 2–3 generations when starting from a perfectly equal distribution. Without assuming an initially perfectly equal distribution, convergence is reached within a single generation. Therefore, we conclude that the convergence is indeed fast enough to treat the Gini coefficient in its asymptotic value. We note, however, that if β increases, the convergence is slower. Therefore, the fast convergence assumption when β is about 0.8 or higher, is less valid – in such a case convergence will require 5–6 generations.

Figure 6 demonstrates one of the simulations done, for $\sigma = 0.4$, and assuming $\beta = 0.4$, which changes to 0.1 after reaching a steady state.

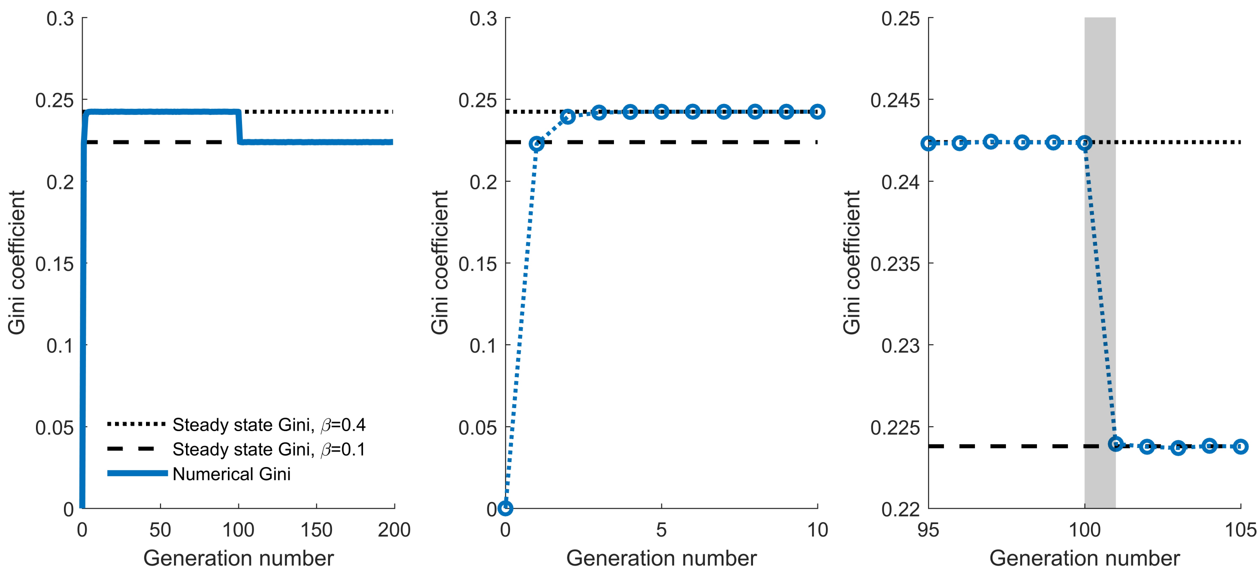

Figure 6: The convergence of the Gini coefficient. We simulate $N = 10^7$ families for 200 generations, assuming σ is 0.4 with $\beta = 0.4$ for the first 100 generations and then $\beta = 0.1$ for the next 100 generations. The black lines are the theoretical asymptotic Gini coefficients for $\beta = 0.4$ (dotted) and $\beta = 0.1$ (dashed). The blue curve is the simulated Gini coefficient. The middle and right panels are blow-ups of the left panel around different generations.

C.2 Robustness of the predicted variance of logarithms

Section 3.2 showed that it is possible to use the results of the IGE regression to produce an estimates of the children's generation the variance of logarithms, *i.e.* to use the estimates of mobility measures to estimate inequality, as if the distribution is in a steady state. Fig. 2 demonstrated the high similarity between the actual variance of logarithms and the predicted variance of logarithms based on the IGE estimates. This similarity indicates that for the purpose of estimating the variance of logarithms, the steady state assumption is justified and that Eq. (2.4) provides a good approximation of the value of VL_t. Inequality and mobility are both evolving over time and the

income distribution is not in a steady state, but it is possible to estimate inequality measures as if it is in a steady state. Thus, it follows that for IGE and the variance of logarithms, there is no real benefit from looking at "movies" over "snapshots".

The prediction of the variance of logarithms in Fig. 2 was under the assumption that the rank-rank slope is stable and equal to 0.3. This is motivated by evidence of copula stability since the 1970s (Lee and Solon, 2009; Chetty et al., 2014b, 2017). In order to test the robustness of the results to this assumption, we repeated the analysis for rank-rank slopes going from 0.1 to 0.5, *i.e.* much more mobile and much less mobile than recorded for the United States. Fig. 7 presents the results of this test. Based on these limits, the confidence intervals of the predicted variance of logarithms estimates are higher than in Fig. 2, but only marginally, indicating the low sensitivity of the results to the stable rank-rank slope assumption.

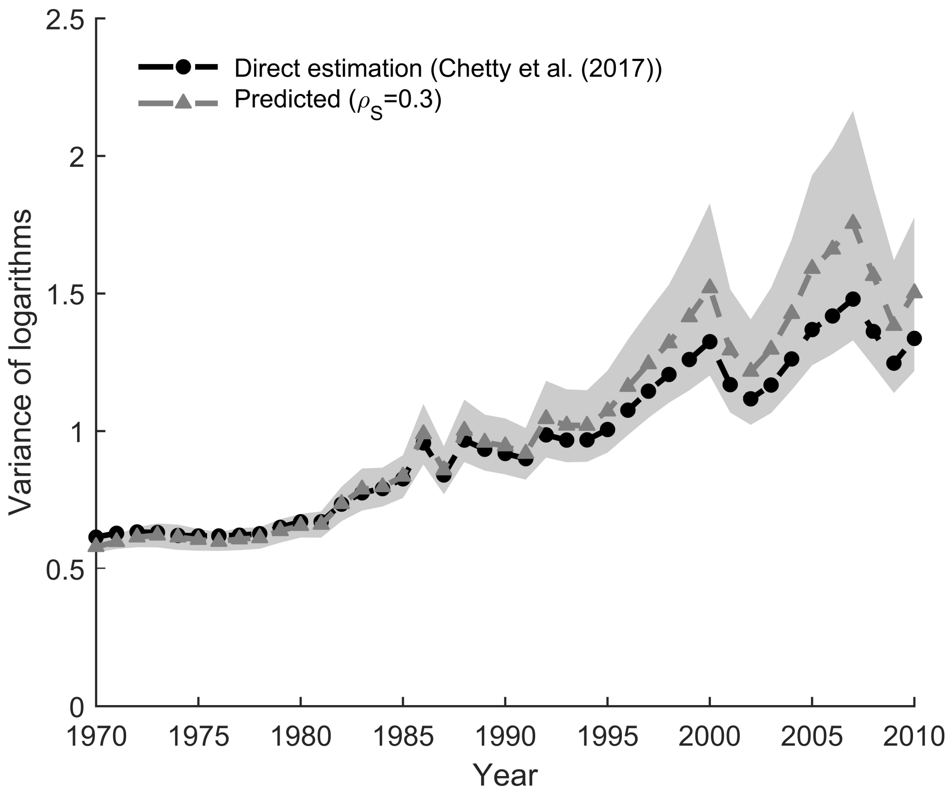

Figure 7: The variance of logarithms for the United States pre-tax incomes (1970–2010). The black circles are the direct estimates of the variance of logarithms based on Chetty et al. (2017). The gray triangles are $\frac{\sigma^2}{1-\beta^2}$ for σ and β estimated for the United States based on a stable copula with rank-rank slope of 0.3. The shaded gray area covers represents 95% confidence intervals for the estimates of $\frac{\sigma^2}{1-\beta^2}$ produced by bootstrapping using a range of rank-rank slopes going from 0.1 to 0.5. The year on the x-axis refers to the year in which children are assumed to be 30 year-olds – the variance of logarithms derived in Eq. (2.4) refers to this distribution.

D Mobility and inequality in a multigenerational mobility framework

Instead of using IGE, it is possible to consider a more general case, in which p prior generations affect the income of the current generation. Considering a set of elasticities, $\{\beta_j\}$, we can rewrite Eq. (2.9) as

$$X_{i,t} = \alpha + \sum_{j=1}^{p} \left(\beta_j X_{i,t-j}\right) + \sigma\omega_{i,t} \,, \tag{D.1}$$

similarly to Eq. (2.1) and Eq. (2.2), so that $X_{i,t}$ now follows an $AR\,(p)$ process. Therefore, assuming quick convergence, as we did for the IGE (or the $p = 1$ case), the distribution of Y_i is log-normal. If $p = 2$, meaning that we consider the effect of grandparents' and parents' incomes on children's incomes, we obtain some steady state Gini coefficient, similar to that in Eq. (2.7):

Proposition 4 *Considering Eq. (D.1) for $p = 2$, and assuming $\beta_1 + \beta_2 < 1$ and $\beta_2 - \beta_1 < 1$, the income distribution converges asymptotically, and the asymptotic income Gini coefficient is*

$$G^* = \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{\sigma^2}{\left(1+\beta_2\right)^2\left[\left(1-\beta_2\right)^2 - \beta_1^2\right]}}\right). \tag{D.2}$$

Proof: We start from Eq. (D.1):

$$X_{i,t} = \alpha + \sum_{j=1}^{2} \left(\beta_j X_{i,t-j}\right) + \sigma\omega_{i,t} \,, \tag{D.3}$$

where α is common income growth trend between consecutive generations, and σ is the standard deviation of the income shocks. β_1 and β_2 are the intergenerational elasticities – β_1 (β_2) measures the effect of the parents' (grandparents') generation on the children's generation.

Convergence is obtained if $\beta_1 + \beta_2 < 1$ and $\beta_2 - \beta_1 < 1$. Since parents' effect is larger than the grandparents' effect, $\beta_1 > \beta_2$. For $p = 1$, most of the β values obtained were less than 0.5, and β_1 is expected to be ever smaller. Therefore, both criteria are most likely to be met and the distribution of X_i reaches a steady state and follows (Hamilton, 1994):

$$\mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{\left(1+\beta_2\right)^2\left[\left(1-\beta_2\right)^2 - \beta_1^2\right]}\right). \tag{D.4}$$

Therefore, assuming the convergence is quick, as was illustrated for IGE, the distribution of Y_i is log-normal. According to Crow and Shimizu (1988), the Gini coefficient of $\log\mathcal{N}\left(m, s^2\right)$ is $\operatorname{erf}\left(\frac{s}{2}\right)$.

It follows that there exists a steady state Gini coefficient for the income distribution of Y_i:

$$G^* = \operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{\sigma^2}{(1+\beta_2)^2\left[(1-\beta_2)^2-\beta_1^2\right]}}\right). \tag{D.5}$$

■

G^* is an increasing function in both β_1 and β_2 . Once again, even if additional generations are considered, it follows that more mobility directly implies less inequality. This result is illustrated in Fig. 8.

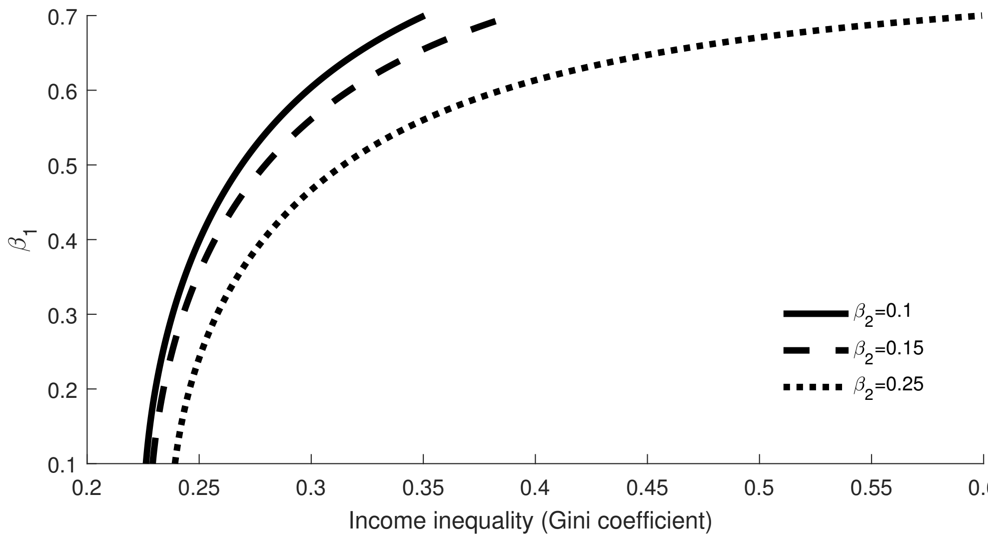

Figure 8: The theoretical relationship between the Gini coefficient and β_1, in an $AR(2)$ mobility model. The different curves depict an additional dependence on β_2. The value of σ used was 0.4. The results are independent of α (see Eq. (D.2)).

E Gini coefficient and IGE data

Table 2 presents the Gini coefficient and IGE data used in Sec. 3.3. In the data taken from Corak (2016), the Gini coefficient data are based on disposable household income for 1985 as provided by the OECD. The IGE values are based on "data on a cohort of children born, roughly speaking, during the early to mid 1960s and measuring their adult outcomes in the mid to late 1990s." (Corak, 2013). In the data taken from Narayan et al. (2018), the Gini coefficients and IGE data are for early 2010s (in most countries sons' outcomes were measured in 2010, but not in all. See Narayan et al. (2018) for full documentation).

Table 2: Gini coefficient and IGE data used in Sec. 3.3

Country	Gini coefficient	Intergenerational elasticity	Source
Argentina	0.53	0.49	Corak (2016)
Australia	0.35	0.26	Corak (2016)
Austria	0.30	0.25	Narayan et al. (2018)
Belgium	0.29	0.18	Narayan et al. (2018)
Brazil	0.59	0.58	Corak (2016)
Brazil	0.52	0.67	Narayan et al. (2018)
Canada	0.33	0.19	Corak (2016)
Chile	0.56	0.52	Corak (2016)
Chile	0.49	0.67	Narayan et al. (2018)
China	0.42	0.60	Corak (2016)
Czech Republic	0.27	0.43	Narayan et al. (2018)
Denmark	0.25	0.15	Corak (2016)
Denmark	0.27	0.15	Narayan et al. (2018)
Ecuador	0.48	0.74	Narayan et al. (2018)
Finland	0.27	0.18	Corak (2016)
Finland	0.28	0.11	Narayan et al. (2018)
France	0.33	0.41	Corak (2016)
France	0.34	0.36	Narayan et al. (2018)
Germany	0.29	0.32	Corak (2016)
Germany	0.30	0.24	Narayan et al. (2018)
Ghana	0.43	0.63	Narayan et al. (2018)
Greece	0.34	0.31	Narayan et al. (2018)
Ireland	0.32	0.26	Narayan et al. (2018)
Italy	0.37	0.50	Corak (2016)
Italy	0.35	0.49	Narayan et al. (2018)
Japan	0.25	0.34	Corak (2016)
Luxembourg	0.31	0.38	Narayan et al. (2018)
Mexico	0.48	0.50	Narayan et al. (2018)

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Country	Gini coefficient	Intergenerational elasticity	Source
Netherlands	0.28	0.30	Narayan et al. (2018)
New Zealand	0.36	0.29	Corak (2016)
Norway	0.26	0.17	Corak (2016)
Pakistan	0.33	0.46	Corak (2016)
Panama	0.52	0.73	Narayan et al. (2018)
Peru	0.54	0.67	Corak (2016)
Portugal	0.36	0.28	Narayan et al. (2018)
Singapore	0.43	0.44	Corak (2016)
Slovak Republic	0.27	0.60	Narayan et al. (2018)
Slovenia	0.25	0.31	Narayan et al. (2018)
South Africa	0.63	0.62	Narayan et al. (2018)
Spain	0.35	0.40	Corak (2016)
Spain	0.51	0.42	Narayan et al. (2018)
Sweden	0.25	0.27	Corak (2016)
Switzerland	0.34	0.46	Corak (2016)
Switzerland	0.33	0.26	Narayan et al. (2018)
United Kingdom	0.36	0.50	Corak (2016)
United Kingdom	0.34	0.48	Narayan et al. (2018)
United States	0.41	0.47	Corak (2016)
United States	0.40	0.54	Narayan et al. (2018)

F Intergenerational mobility and top income shares

The aim of the derivation is to highlight a pitfall in the interpretation of intergenerational mobility measures and their relationship to income inequality. Since most of the papers studying this relationship are focusing on the Gini coefficient, we follow this convention (see Eq. (2.7) and Eq. (2.13)). However, many studies of income inequality study top income shares (Piketty and Saez, 2003; Piketty, Saez and Zucman, 2017).

Following from the derivation of the steady state Gini coefficient, the steady state top income shares can also be derived:

- In the original version of the model, X_i follow $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right)$ in the steady state. In this case, the steady state top q income share (for example $q = 0.1$ for the top 10%) is (following Crow and Shimizu (1988)):

$$sh_q = 1 - \Phi\left(\Phi^{-1}\left(1-q\right) - \sqrt{\frac{\sigma^2}{1-\beta^2}}\right). \tag{F.1}$$

- In the modified model, X_i follow $\mathcal{N}\left(\frac{\alpha}{1-\beta}, \left(\frac{\sigma}{1-\beta}\right)^2\right)$ in the steady state. Similarly, the steady state top q income share is (following Crow and Shimizu (1988)):

$$\tilde{sh}_q = 1 - \Phi\left(\Phi^{-1}\left(1-q\right) - \frac{\sigma}{1-\beta}\right). \tag{F.2}$$

Similarly to what was found for the Gini coefficient and the IGE, the top income shares are also increasing functions of β, as demonstrated in Fig. 9.

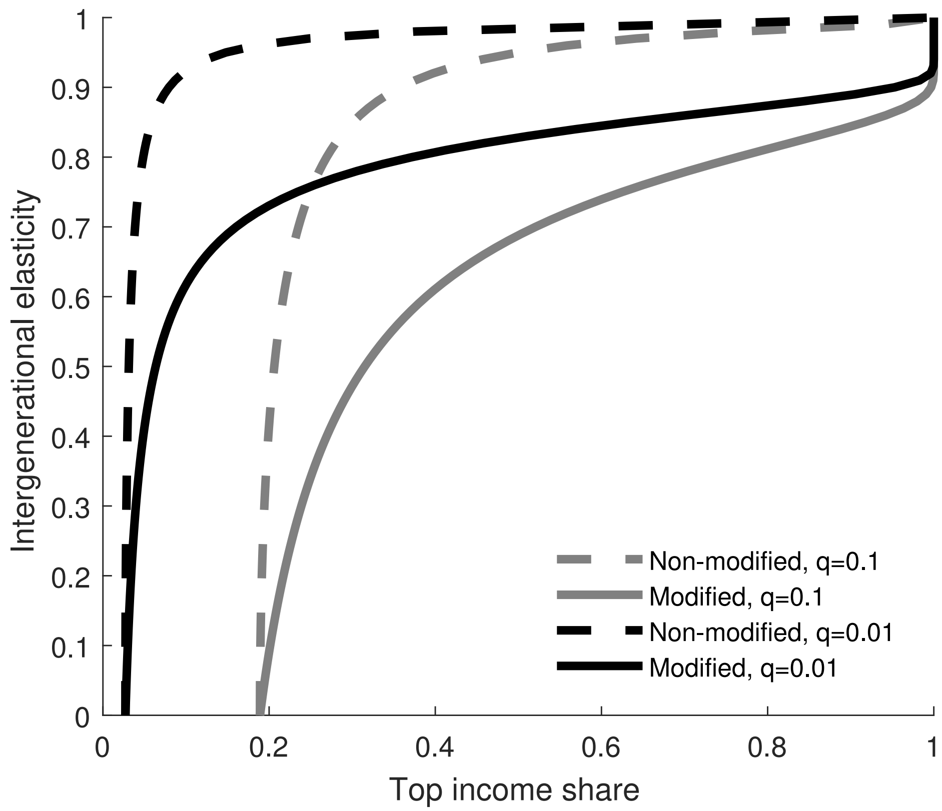

Figure 9: The theoretical relationship between the top income shares and the intergenerational elasticity. The different curves depict the dependence of the IGE (β) on sh_q (dashed) and $\tilde{sh}_q$ (solid) for $q = 0.1$ (gray) and $q = 0.01$ (black).