

# R&D Investment under Currency Depreciation: Should We Beggar-thy-neighbor?

Chun-Che Chi\*

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## Abstract

This paper focuses on the welfare analysis of currency depreciation through endogenous R&D where the economy faces a trade-off between the gain from export and disinvestment of technology. By using country-level data, regressions and panel VAR indicate that undervaluation of the exchange rate and real depreciation are negatively correlated with the R&D activity. The stylized fact can be explained by a model that features endogenous productivity in a small open economy where real depreciation raises the cost of R&D investment. Under real depreciation shock, the economy faces a short-term boom in consumption and output but a long-term bust due to sluggish productivity. Welfare increases slightly following a real depreciation shock when productivity is exogenous. However, when productivity is endogenous, welfare decreases by 0.1% under 1% real depreciation.

*Keywords:* Real depreciation; Welfare; R&D.

## 1 Introduction

Manipulation of the exchange rate in pursuit of the competitiveness of trade and services has been a heated concern since the recent trade war between the US and China. There exist different incentives for governments to intervene in the exchange rate, such as decreasing the burden of the sovereign debt and smoothing the fluctuation of the exchange rate. What lies at the core of the heated debate is the mercantilist motive where countries undervalue their currency to achieve higher competitiveness on export goods. However, depreciation may lead

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\*Department of Economics, Columbia University. E-mail: cc3729@columbia.edu

to welfare loss because countries' long-term productivity may be affected by the exchange rate through firms' investment decisions. Depreciation can result in lower growth of technology when, for example, productivity improvement requires imports of foreign technology.

This paper aims at analyzing the welfare change under real currency depreciation by asking the following questions: Will R&D investment and the technology level affected by the real exchange rate? If so, what is the welfare consequence when a government implements real depreciation? What are the short-term and long-term effects of the exchange rate intervention?

I answer the above questions by first running reduced-form analyses and the panel VAR to show that real depreciation and currency undervaluation lowers R&D investment. By identifying a structural break in the panel data, I show that the effect of the real exchange rate on R&D may explain whether currency undervaluation is expansionary or not. With the empirical evidence in hand, I build up a model that features an endogenous productivity process, which is affected by firms' decisions on R&D investment. Real depreciation discourages R&D investment as it raises the cost of innovation that relies on foreign input. Finally, I calibrate the model and simulate the dynamics under a depreciation shock. Results show that in the conventional model with an exogenous productivity process, the economy will benefit from depreciation as output and consumption increase due to higher foreign demand. However, when endogenous R&D is considered, the economy experiences a short-term boom due to higher demand, but a long-term bust results from a decline in R&D and future technology. Quantitatively, the welfare decreases by 0.1% under 1% real depreciation.

To empirically examine the effect of the exchange rate on R&D and output, I use annual and quarterly country-level panel data from 49 countries. I first regress R&D investment on currency depreciation and find a significantly negative relationship. Next, to understand how R&D investment can potentially affect welfare via changes in output, I regress the output growth on the real exchange rate. Results show that depreciation is expansionary before an identified optimal break of the time series, whereas the effect of the exchange rate is not significant after the break.

I further show that the effect of the exchange rate on R&D investment is the key to explain whether depreciation is expansionary or not. Specifically, I regress R&D investment on depreciation and find a significantly negative coefficient in periods when the exchange rate is not significantly expansionary. On the other hand, when the exchange rate is expansionary, R&D investment is not significantly lowered by depreciation. The effect of depreciation on R&D is also estimated by instrumenting with countries' lagged growth of foreign reserves holdings. The exclusion restriction states that the lagged foreign reserves holding can only

affect current R&D via the change in the real exchange rate. Results with IV are consistent with results without IV.

In addition to the reduced-form analysis, I also run a structural panel VAR with R&D investment, GDP, nominal interest rate, and the real exchange rate, where the last variable is assumed to be the fast variable. The real exchange rate shock, by construction, captures variations that are independent of the nominal interest rate. The result shows that a depreciation shock leads to a significant decline in R&D expenditure for three periods. These findings imply that welfare under depreciation is affected by R&D investment through changes in output.

To quantify the dynamics and the welfare change under real depreciation, I construct a New Keynesian model in a small open economy where aggregate productivity is affected by firms' R&D decisions. Firms choose to invest in R&D when the expected marginal gain in terms of the net profit is larger than the fixed cost of R&D investment. I assume that technology follows an AR(1) process where future technology is increasing in not only current technology, but also current R&D investment. Therefore, real depreciation, which generates higher output due to higher foreign demand, may also lower future productivity and output because R&D innovation that requires foreign input becomes costly. The magnitude of the adverse effect is determined by the persistence of the productivity process.

Next, I estimate the endogenous productivity process by using plant-level data of the manufacturing industry in Taiwan, which is viewed both as a small open economy and a currency manipulator.<sup>1</sup> I then adopt the structural approach introduced by [Olley and Pakes \(1996\)](#), which solves the difficulties arise from the simultaneity problem and unobservable technology. The estimated process shows that current technology is significantly increasing in past R&D. I then generate a depreciation shock by changing the nominal interest rate under the assumptions that the UIP holds, and the price is sticky.

Quantitatively, I show that, under a real depreciation shock, the economy will experience a short-term boom in consumption and output via higher foreign demand, but a long-term bust due to a decline in productivity as the innovation cost of R&D increases. The decrease in R&D investment generates a prolonged effect on output due to the persistence of the technology process. Welfare, which is affected by the long-term bust, drops immediately at the period when the shock hits. Next, I compare the deviation of welfare between models that feature endogenous and exogenous productivity. Results show that welfare increases

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<sup>1</sup> The US Treasury identifies Taiwan as a currency manipulator from 1988 to 1992. [Chen \(2016\)](#) empirically shows that Taiwan conducted asymmetric currency intervention from 1998 to 2012.

slightly following depreciation in the model with exogenous productivity. However, welfare decreases immediately by about 0.1% when we consider endogenous R&D.

This paper is motivated by the fact that currency undervaluation has become more pervasive, both among developing and developed countries.<sup>2</sup> Using annual panel data of 49 countries from 1990 to 2011, Figure 1 plots the percentage of countries that have undervalued their currency. The percentage increases over time except for the sharp drop in 2005.<sup>3</sup> Currency undervaluation, according to Rodrik (2008), is defined as the remaining fluctuation of the real exchange rate, taking into account the dynamics of GDP per capita. The index is constructed by regressing the real exchange rate ( $RER$ ) on the GDP per capita ( $GDPPC$ ), as shown by the following equation:

$$\ln RER_{it} = \alpha_0 + \alpha_1 GDPPC_{it} + f_t + u_{it} \quad (1)$$

where  $f_t$  is the time-fixed effect and the residual  $u_{it}$  is the log of the undervaluation index ( $UVAL$ ).  $UVAL > 1$  implies currency undervaluation. Equation 1 controls the Balassa-Samuelson effect, which suggests that countries with higher productivity growth tend to have higher income growth and higher relative prices of non-tradable goods. The increase in the relative price of non-tradable goods will then generate real appreciation, which implies a negative  $\alpha_1$ .

Figure 2 plots the currency undervaluation among two subgroups of countries: 21 developed countries and 28 developing countries. Several findings emerge from the trends: First, countries that conduct currency undervaluation are mostly developing countries. On average, 70% of the developing countries have undervalued real exchange rate; Second, developed countries feature a more steady growth trend of currency undervaluation. Third, the massive drop in the index during 2005 came from the fluctuation of developing countries. Figure 3 shows the currency undervaluation trend for economies that are on the US exchange rate watch list or are often cited as potential exchange rate manipulators. Key

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<sup>2</sup> There exists no universal definition of currency manipulation, and in particular, currency undervaluation. An often-cited criterion of the currency manipulator is defined by the US Treasury Department in the Trade Facilitation and Trade Enforcement Act of 2015: A country is a currency manipulator if it has (1) bilateral trade surplus larger than \$20 billion. (2) current account surplus larger than 3.0% of the GDP. (3) conducted repeated net purchases of foreign currency that amount to more than 2% of its GDP over the year. Another definition is that a country asymmetrically reacts to appreciation trend but not depreciation pressure.

<sup>3</sup> Although there were five countries that experienced at least one switch of the sign of the  $UVAL$  index (not shown in the paper), the aggregate trend of undervaluation is upward-sloping.

messages behind these facts are that currency undervaluation has become more prevalent, and it is crucial to understand whether a country benefits from the depreciation policy or not.

### *Related Literature*

This paper is closely related to the literature that studies the relationship between the exchange rate and R&D investment. Some papers show that depreciation is negatively correlated with R&D expenditure. For example, [Chen \(2017\)](#) uses country-level panel to show that currency depreciation dampens R&D expenditure. By using sector-level R&D and real exchange rate data, [Tabrizy \(2016\)](#) implements panel VAR and reduced-form analyses to show that a lagged real depreciation will result in lower R&D expenditure. He builds up a partial equilibrium model and proves that currency depreciation increases competitiveness of the export price, and therefore discourages firms to invest in innovation. [Zietz and Fayissa \(1994\)](#) examine US manufacturing data and find that firms initially with high R&D intensity will react to currency appreciation by raising R&D expenditure due to the increase in the within-sector competition. [Bøler et al. \(2015\)](#) use firm-level data from Norway to confirm that the R&D tax credit, which lowers input cost, will increase R&D investment. They propose a general equilibrium model with heterogeneous firms, where imports of intermediate goods and R&D are complementarities. As a result, an increase in the cost of import lowers R&D investment.

On the contrary, other papers show that depreciation is positively correlated with R&D investment. [Becker and Pain \(2008\)](#) focus on panel data of manufacturing industries in UK and show that contemporaneous appreciation is negatively correlated with the R&D activity. They emphasize two possible explanations: first, foreign-owned firms that are located domestically view R&D as a cost of export, which tends to be decreasing in appreciation; Second, firms may lower R&D investment when facing profit loss that results from appreciation. [Funk \(2003\)](#) constructs a firm-specific real exchange rate and shows that currency depreciation leads to higher R&D expenditure. This is especially true for firms that have foreign sales. It is worth noting that the result could be biased due to the biased firm-specific real exchange rate, which is measured by the product of the current industry-specific real exchange rate and the lagged firm-specific sales share. For this to be a correct measure of the firms' exposure to the real exchange rate, sales share should be independent of the industry-specific real exchange rate. This assumption implies a homogeneous markup across all firms in a given industry, which may not hold. I contribute to the literature by providing more empirical evidence on the negative relationship between depreciation and R&D by using reduced-form regressions as well as the structural panel VAR.

This paper is also related to the literature that examines the correlation between real

exchange rate and output growth. [Berg and Miao \(2010\)](#), and [MacDonald and Vieira \(2010\)](#) find that the real currency depreciation or undervaluation is expansionary. [Rodrik \(2008\)](#) uses country-level data that covers 188 countries from 1950 to 2004 and constructs the index of currency undervaluation, taking into account the Balassa-Samuelson effect. He examines the five-year average of GDP and currency undervaluation and shows that they are positively correlated. I contribute to the literature by adding the channel of endogenous R&D, which is affected by the exchange rate. With this additional component, countries may not always benefit from real depreciation, and the resulting long-term effect may be significantly different from the short-term effect.

This paper uses the positive effect of R&D investment on productivity, as highlighted in the literature. Related examples include [Doraszelski and Jaumandreu \(2013\)](#), [Aw et al. \(2011\)](#), [Bøler et al. \(2015\)](#), [Huber \(2018\)](#). [Doraszelski and Jaumandreu \(2013\)](#) use unbalanced panel data of 1,800 firms in the manufacturing industry from Spain and confirm the role of R&D expenditure in explaining the productivity variation across firms. [Bøler et al. \(2015\)](#) focuses on the complementarity between R&D and international sourcing, they adopt a similar approach and obtain the same conclusion regarding R&D and productivity. [Aw et al. \(2011\)](#) use plant-level data of manufacturing sectors from Taiwan and show that both export and the R&D input will promote plant's future productivity, which then could result in more future export, higher R&D expenditure and higher productivity growth. I contribute to the literature by further incorporating the estimated endogenous growth function into a DSGE model in a small open economy and study the welfare implication related to the effect of R&D on productivity.

Finally, this paper provides more evidence regarding the effect of monetary policy on real variables. I show some structural evidence that the monetary policy can affect R&D investment and productivity via the change in the real exchange rate under assumptions of the UIP and price stickiness. Related papers, including [Eichengreen and Charles \(1995\)](#), [Clarida and Gertler \(1997\)](#), [Kim and Roubini \(2000\)](#) have studied this mechanism using VAR/sVAR models.

The remainder of the paper is organized as follows. [Section 2](#) shows the empirical results regarding the relationship between real exchange rate and R&D expenditure through regressions and panel VAR. I also provide some evidence regarding the correlation between output growth and real exchange rate and highlight the potential role of the R&D expenditure. [Section 3](#) lays out the general equilibrium model with endogenous decision on R&D investment. [Section 4](#) calibrates the model parameter and analyzes the welfare consequence of currency undervaluation. [Section 5](#) concludes.

## 2 Empirical Results

There exists a debate over whether real exchange rate depreciation boosts investment and output. On the one hand, literature shows that real depreciation can promote the export sector and foreign demand, and therefore lead to more output growth. For example, [Glüzmann et al. \(2012\)](#) empirically examine the components of GDP and find that undervaluation positively affect saving, investment, and employment. The underlying mechanism is that real depreciation decreases the real labor income, and thus is equivalent to a real transfer from a low-income household to a high-income household, who generally has a higher marginal propensity to save.

On the other hand, real depreciation may harm the output growth through a more expensive importing and innovation cost. Depreciation may also limit the room for the monetary policy due to the impossible trinity, leading to an inflationary status that can potentially distort the real production. This section thus aims to test the relationships between the exchange rate and R&D investment as well as the output growth.

### 2.1 Real exchange rate and R&D expenditure

The first step to highlight the effect of the exchange rate on welfare via technology is to examine the relationship between the exchange rate and R&D investment. I use annual unbalanced panel data that covers 48 countries from 1997 to 2014 and proceed with two steps. Following [Chen \(2017\)](#), I first construct the undervaluation index, as shown in equation (1), and subsequently regress R&D investment on the lagged undervaluation index, as shown by the following equation:

$$\Delta \ln RD_{i,t} = \alpha_0 + \alpha_1 \ln(UVAL_{i,t-1}) + \alpha_x X_{i,t} + f_i + \epsilon_{it}, \quad (2)$$

where  $f_i$  is the country fixed effect. The lagged undervaluation index is adopted to avoid the contemporaneous reverse causality problem.  $X_{i,t}$  is a set of characteristics of countries over time and contains five measures, as shown in Table 2. First, the trade openness ( $Openness_{i,t}$ ) is measured by the log of the sum of the export and the import divided by GDP. The coefficient is expected to be positive to capture the idea that trades can promote innovation and knowledge spillovers. Second, the turnover ratio of domestic stocks ( $\Delta Turnover_{i,t}$ ) is measured as the traded value of shares divided by the total market value of the outstanding shares. This is a proxy for financial development and is expected to be positive. The growth in the government spending ( $\Delta Gov_{i,t}$ ) is also considered. The sign of the coefficient can be ambiguous, depending on whether the government spending lowers the cost of R&D (e.g.,

through a tax credit) or serves as additional resource that crowds out private investment from firms. The change of the nominal lending rate ( $\Delta i_{i,t}$ ) is also included to control the constraint of cash in advance when conducting innovation activities. The corresponding sign is expected to be negative as a higher interest rate dampens private borrowing. The change of enrollment in the secondary education ( $\Delta Enrollment_{i,t}$ ) is included to control for the complementarity between technology adoption and human capital. The coefficient of this term is expected to be positive.

Model 1 shows the baseline regression. Consistent with the hypothesis, the coefficient suggests a negative correlation between lagged undervaluation and the log change of the R&D expenditure. Results are robust as more controls are included. The coefficient of the trade openness is positive but slightly significant. The coefficient for the turnover ratio is consistently positive and significant across models. Government expenditure is negative, but the significance is not robust. The change of the interest rate has a significantly negative correlation with the R&D activity, suggesting that the tightness of the nominal rate could slow down the economy through the channel of R&D. Finally, the effect of the enrollment rate of secondary education is insignificant.

As a robustness check, investment on R&D can also be regressed on detrended values of the real effective exchange rate and other controls, as shown in Table 3. The model is given by

$$\log RD_{i,t}^d = \alpha_0 + \alpha_1 e_{i,t-1}^d + \alpha_x X_{i,t}^d + f_i + \epsilon_{it}. \quad (3)$$

Across all model specification, we observe a significantly negative coefficient of the real depreciation. In fact, the results implies an even more direct connection between the exchange rate and the R&D activity since the undervaluation index used in Table 2 mainly considers the deviation of the exchange rate from its equilibrium level. Regarding results of other controls, the effects of the turnover ratio and interest rate remain significant, while other variables do not provide significant effects.

In addition to results of the reduced-form analysis, I also run a panel VAR and plot impulse responses of R&D under a negative shock of the real exchange rate. The reason to adopt the panel VAR is to examine the effect of orthogonal shocks under the limited sample points for each country. The order and the functional form of the VAR are given by

$$\begin{bmatrix} \log RD_{i,t}^d \\ gdp_{i,t}^d \\ i_{i,t} \\ e_{i,t}^d \end{bmatrix} = Y_{i,t} = A_0 + A_T Y_{i,t-1}^T + v_{i,t},$$

where  $x^d$  is the detrended  $x$ . I adopt recursive identification and Cholesky decomposition to derive the orthogonal shocks. The assumption of the order implies that the real exchange rate is the fast variable, that will react to a contemporaneous change in R&D, output and interest rate. On the other hand, the R&D expenditure is the slowest variable that only responds to lagged output, interest rate and real exchange rate. It is important to differentiate the shock of interest rate and real exchange rate because the two variables could be highly correlated due to the uncovered interest rate parity and price stickiness. One can interpret the orthogonal shock of nominal lending rate as the standard monetary shock, which may come from the sudden shift in central bank's preference for monetary policy.

Regarding the interpretation of the real exchange rate shock, it should be variations that are independent of the interest rate but correlated with the real exchange rate. One candidate is a shock to the UIP premium which can be affected by the level of the government-owned foreign reserves. The fact that there exists a UIP premium implies that standard UIP does not hold, and therefore one could affect the nominal exchange rate and real exchange rate without moving the relative interest rate. An alternative explanation can be any factor that affects the relative price of non-tradable goods and tradable goods, such as a shock to the income elasticity of demand. In that case, higher income elasticity of demand for non-tradable goods will drive up the relative price of non-tradable goods given that the price of tradable goods is pinned down internationally. The real exchange rate will then increase whether a country chooses to fix the nominal exchange rate or stabilize the price level by adopting a floating exchange rate regime. Similarly, a shift in fiscal spending to non-tradable goods serves the same purpose.

Figure 4 plots the impulse response of R&D to one standard deviation shock to the real exchange rate. Following [Abrigo et al. \(2016\)](#), I choose the optimal lagged equals three by maximizing the share of variation explained by the panel VAR model. The result shows that there is a significant decline in R&D expenditure for three years (one year if focusing on the two standard deviation bands). A drop in R&D investment following real depreciation is consistent with the findings of the reduced-form analysis. Figure 5 plots the impulse responses to different shocks in the system. Despite the significant effect of the shock to the real exchange rate and R&D, the interest rate does not significantly dampen the R&D. This is in contrast with the evidence in [Table 2](#) and [3](#). One reason might be that the interpretations

of the coefficient between previous regression and the impulse response are different: the latter takes into account the possibility of reverse causality and the simultaneity problem while the former does not. Finally, the significance of the effect of the GDP shock is too weak to draw any conclusion.

Similar approaches have been studied in the literature. [Tabrizy \(2016\)](#) also implements the panel VAR by using sector-level data, and he obtains significant drop following a depreciation shock to the real exchange rate. [Eichanbaum and Charles \(1995\)](#) also run a VAR that includes the nominal interest rate and the real exchange rate in a similar order. It is worth noting that one potential problem of the panel VAR is that it assumes the same lagged coefficients and impulse responses for every country or sector, which can be unrealistic. A more comprehensive data, such as a firm-level panel, is desirable to draw a conclusion. In sum, I show that lagged real depreciation dampens R&D activity by running reduced-form regressions and the panel VAR.

## 2.2 Currency undervaluation, R&D and growth

To understand how depreciation affects welfare, we shall also focus on the effect of currency undervaluation on the change in GDP. Following [Rodrik \(2008\)](#), I run the following regression:

$$growth_{i,t} = \alpha_0 + \alpha_1 \ln(UVAL_{i,t}) + \alpha_2 \log GDP_{i,t-1} + f_i + f_t + \epsilon_{it}$$

where  $growth_{i,t}$  is the growth rate of the GDP per capita and  $\log GDP_{i,t-1}$  is the log of lagged GDP per capita. The coefficient  $\alpha_2$  captures the convergence of the growth rate and is expected to be negative.  $\alpha_1$  is the key estimator that indicates whether undervaluation is expansionary. [Rodrik \(2008\)](#) shows that  $\alpha_1$  is significantly positive by using panel data, which covers 188 countries from 1950 to 2004. I first extend the time coverage by using the unbalanced data from 1951 to 2016 that covers 49 countries. I then separate the samples into two groups: those before and after 2004. For each subgroups, I also run regression (3) to see whether the effect of undervaluation is affected by the correlation between the exchange rate and the R&D investment.

Results are shown in Table 4. Model 1 follows [Rodrik \(2008\)](#) by focusing on the period before 2004. Consistent with the literature, currency undervaluation is expansionary. However, there exists no significant relationship between undervaluation and GDP growth when focusing on the period after 2004, as shown by the result of Model 4. Next, I examine the relationship between R&D investment and the exchange rate separately within the two periods. Model 2 and Model 5 show that the negative relationship between R&D and real

exchange rate only occurs in periods when the real exchange rate is not expansionary.

As a robustness check, I set the lagged growth of government's foreign reserves holding, which is a proxy of foreign exchange rate intervention, as an instrument variable. The exclusion restriction relies on the argument that the only channel through which the lagged foreign reserves holding affects current R&D investment is the dynamic of the real exchange rate. Model 6 shows that the effect of the real exchange rate, when an IV is included, is significantly negative with a sufficiently high joint F-statistic. These results imply that the potential reason why undervaluation is not expansionary after 2004 is that the R&D investment is decreasing in depreciation during that period.

Based on the results in Table 4, it appears that there might exist a structural break which affects the dynamics and correlation among aggregate variables. Since the structural break may not be precisely the year 2004, I use quarterly panel data on GDP to estimate the common breaks, following the method proposed by Bai (2010). I first consider a simple mean-shift model of output and, for each country, compute the mean of GDP before and after an arbitrary break point  $k$ , which are denoted as  $\bar{y}_{i1}$  and  $\bar{y}_{i2}$ . I then calculate the sum of squared residuals for country  $i$ , which is given by

$$s_i(k) = \sum_{t=1}^k (y_{it} - \bar{y}_{i1})^2 + \sum_{t=k+1}^T (y_{it} - \bar{y}_{i2})^2.$$

Next, I aggregate the sums of squared residuals across countries:

$$S(k) = \sum_{i=1}^N s_i(k),$$

where  $N$  is the total number of countries. Finally, I pick the optimal break  $k$  by minimizing  $S(k)$ . The result shows that there exists a significant break at 2001 Q2. Table 5 shows the results of the six models in Table 4 when adopting the optimal break. Results again show a significantly negative relationship between currency undervaluation and the growth of GDP. Moreover, R&D investment is again decreasing in depreciation during the period when the effect of currency undervaluation is not significant. The evidence supports the argument that the R&D activity plays a role in explaining the relationship between the real exchange rate and output. The result is robust when including instrument variable, as shown in Model 6.

As a robustness check, I use quarterly data to check if real depreciation is expansionary only before the optimal break. Table 6 presents the results. When adopting full sample, the key coefficient of undervaluation is not significant. However, when separating the data into two periods, the expansionary depreciation occurs before the break but not after the break,

supporting the previous argument. It is worth noting that the coefficient of the lagged GDP is significantly negative across all models in Table 4, 5, and 6. This supports the standard view that the output growth converges.

### 3 A Model with Endogenous R&D

The primary goal of the model is to explain the stylized fact that R&D investment reacts to the lagged real exchange rate and analyze the welfare under depreciation. I extend the model of [Gali and Monacelli \(2005\)](#) and [Gali \(2015\)](#) by introducing heterogeneous firms' decisions on investment of R&D, which affects the aggregate productivity. The economy features four sectors: the household, a continuum of firms, the central bank, and the government. The household chooses consumption and offers labor, and their utility is strictly increasing in consumption and strictly decreasing in the labor hour. Firms choose production as well as prices subject to Calvo-type price stickiness, and decide whether to invest in technology, depending on the real exchange rate, expected domestic inflation, the foreign output, and the current technology level. The central bank conducts monetary policy and is assumed to be independent of the government. The government offers labor subsidy in a lump sum fashion to guarantee a flexible price equilibrium. The economy is assumed to be a small open economy where the foreign output and price are taken as given.

#### 3.1 Household

The household maximizes the utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t,$$

where  $W_t$  is the nominal wage;  $Q_{t,t+1}$  is the stochastic discount factor;  $D_{t+1}$  is the nominal payoff in period  $t + 1$  of the portfolio in period  $t$ .  $C_t$  is a composite of domestic and foreign good.  $C_{H,t}$  is a composite of all the goods produced in country  $i$ . The functional form of consumption is standard and given by

$$C_t = [(1 - \alpha)^{1/\eta}(C_{H,t})^{(\eta-1)/\eta} + \alpha^{1/\eta}(C_{F,t})^{(\eta-1)/\eta}]^{\eta/(\eta-1)},$$

where

$$C_{F,t} = \left( \int_0^1 C_{i,t}^{\gamma/(\gamma-1)} di \right)^{\gamma/(\gamma-1)},$$

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)},$$

$$C_{i,t} = \left( \int_0^1 C_{i,t}(j)^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)},$$

and  $\epsilon$  is the elasticity of substitution between the variety of domestic goods;  $\eta$  is the elasticity of substitution between home and foreign goods;  $\gamma$  is the elasticity of substitution between goods from different countries;  $\alpha$  measures the openness of the country. The household utility is CRRA and is given by

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\psi}}{1+\psi}$$

where  $\sigma$  is the intertemporal elasticity of substitution. The optimality conditions with first-order Taylor approximation are given by

$$w_t - p_t = \sigma c_t + \psi n_t \tag{4}$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \tag{5}$$

where  $i_t \equiv -\log(E_t(Q_{t,t+1}))$ ,  $\rho = -\log\beta$ ,  $\pi_t = p_t - p_{t-1}$ . Equation (4) is the labor supply which equates the marginal utility of consumption and labor. Equation (5) is the intertemporal optimality condition that equates the marginal utility of consumption and saving.

### 3.2 Firm

There exists a continuum of monopolistic firms, which supply goods to both domestic and foreign consumers. Firms demand labor and select prices subject to Calvo-type price stickiness. Aggregate development of the technology equals the share of firms that choose to invest in R&D. The investment decision is affected by the marginal benefit of investment,

which is a function of future inflation, the future and current exchange rate, future foreign output, and the current technology level.

### 3.2.1 R&D decision

For tractability, I assume that every firm can fully discover the present technology level which contains all the innovation in the industry. This assumption implies that the innovation contributed by any firm will immediately be revealed to the public (or pirated by other firms), and technology improvement can be used immediately at the next period. The functional form of the log of technology is given by

$$\ln(A_t(j)) = a_t(j) = \exp(\alpha_0 + \alpha_1 a_{t-1} + \alpha_2 d_{t-1}),$$

where  $d_{t-1} = \int d_{t-1}(j) dj$  is the sum of firms' past decision on investment,  $d_{t-1}(j)$ , that equals one if firm  $j$  conducts R&D and zeros otherwise.  $\alpha_1$  is the AR(1) coefficient that controls the persistence of the technology, and  $\alpha_2$  is the key coefficient that measures the effect of the R&D on technology improvement. This setting ensure that the technology process is homogenous across firms. Another reason to assume homogenous technology is to avoid the possibility that firms hit by a low heterogenous innovation cost will choose to invest in R&D, and subsequently has higher marginal benefit to further choose R&D in future periods. This decision process will lead to a permanent and increasing divergence of the individual technology process across firms, which is hard to model.

To provide incentive for firms to put effort into R&D investment, I further assume that firms collectively sign an agreement ex ante to pool their profit together, and a larger portion of the aggregate profit will be allocated to firms that conducted R&D in the last period. The incentive for firms to sign the agreement is that it provides individual firms with an insurance against the uncertainty arises from price stickiness. The profit will thus be even across firms that are allowed to adjust prices and those firms that are not. The complete decision timing of firms is as follows:

1. Time  $t$  begins.
2. Firms fully observe the current aggregate technology,  $A_t$ .
3. Firms sign an agreement regarding ex-post profit distribution.
4. Firms decide whether to invest R&D subject to a heterogeneous R&D cost.
5. Firms adjust the price optimally if allowed.

6. Firms hire labor, produce output, earn and allocate profit according to the agreement signed in period  $t - 1$ .
7. Time  $t + 1$  begins.

The agreement specifies the ex-post allocation of the profit. Let  $\tilde{\Pi}_{t+1}(j)$  be the profit that firm  $j$  receives after the profit from the sales of production at  $t+1$  being pooled and allocated. The profit function is given by

$$\tilde{\Pi}_{t+1}(j) = \begin{cases} \Phi_{t+1} P_{H,t+1}^{1/(1-\alpha_p)} (e^{\alpha_0 + \alpha_1 a_t + \alpha_2})^{1/(1-\alpha_p)} X & \text{if } d_t(j) = 1 \\ \Phi_{t+1} P_{H,t+1}^{1/(1-\alpha_p)} (e^{\alpha_0 + \alpha_1 a_t})^{1/(1-\alpha_p)} X & \text{if } d_t(j) = 0 \end{cases}, \quad (6)$$

where  $\Phi_t \triangleq [(1 - \tau)W_t]^{\alpha_p/(\alpha_p-1)} (\alpha_p^{\alpha_p/(1-\alpha_p)} - \alpha_p^{1/(1-\alpha_p)})$  is a common factor across all firms, and  $X = (\exp(\alpha_2/(1 - \alpha_p))d_t + (1 - d_t))^{-1} \exp(\alpha_2 d_t/(1 - \alpha_p))$  is a normalization factor such that the sum of the before-allocation profit is equal to the sum of the after-allocation profit (i.e.  $\int \tilde{\Pi}_{t+1}(j) dj = \Pi_{t+1}$ ). One can observe that the individual after-allocation profit is proportional to the individual productivity. The only difference between the before-allocation and after-allocation profit is that the uncertainty of price stickiness is eliminated through the aggregation of individual profit.

The firm then decides whether to invest based on the cost and the expected benefit, which equals the increase in the sum of discounted future profit. Let  $\hat{\Pi}_t(j; d_t)$  be firm  $j$ 's discounted sum of the profit stream, the decision is based on the following criterion:

$$d_t(j) = \begin{cases} 1 & \text{if } E_t \hat{\Pi}_t(j; d_t = 1) - E_t \hat{\Pi}_t(j; d_t = 0) \geq c e^{-\mu_t(j)} P_t A_t \\ 0 & \text{otherwise} \end{cases}, \quad (7)$$

where

$$P_t = [(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{1/(1-\eta)},$$

$$\hat{\Pi}_t(j; d_t) = \sum_{s=t+1}^{\infty} \beta_F^s \tilde{\Pi}_s(j; d_t),$$

and  $c$  is the fixed cost of R&D investment.  $\alpha$  is a measure of openness, and  $(1 - \alpha)$  captures the degree of home bias.  $\mu_t(j)$  measures the dispersion of the cost and follows an uniform distribution where  $\mu_t(j) \sim u(\ln \epsilon_d, 0)$ ,  $\epsilon_d \in (0, 1)$ . The reason to include  $A_t$  as a part of

the innovation cost is to capture the “stepping-on-toe” effect that technology becomes more costly as technology increases (See, e.g., [Aghion et al. \(2009\)](#)). It is worth noting that  $P_t$  is a composite of prices of domestic goods and foreign goods, implies that the innovation cost is increasing in input cost. This argument is equivalent to the assumption that the innovation process requires foreign material, and therefore the cost is affected by the relative price of domestic and foreign goods. Proposition 1 shows the functional form of R&D investment,  $d_t$ :

**Proposition 1.** (*R&D investment*) *R&D investment  $d_t$  is endogenously determined by the real exchange rate ( $S_t$ ), the expected domestic inflation ( $E_t\pi_{H,t+1}$ ), the expected foreign output ( $E_t y_{t+1}^*$ ), and the current level of technology ( $a_t$ ). The functional form can be written as:*

$$d_t = \frac{1}{\ln\epsilon_d - z_1} (z_2 + z_3 E_t \pi_{H,t+1} + z_4 E_t S_{t+1} + z_5 S_t + z_6 E_t y_{t+1}^* + z_7 a_t), \quad (8)$$

where

$$\begin{aligned} z_1 &= -1 - \alpha_2 \frac{1 + \psi}{1 - \alpha_p} + e^{\frac{\alpha_2}{1 - \alpha_p}}, \\ z_2 &= f(c, \beta, \alpha_p, \tau, \psi, \alpha_0, \alpha_2), \\ z_3 &= \frac{\alpha_p}{1 - \alpha_p} \left[ 2 \left( \frac{1}{\alpha} + \frac{\psi}{\alpha \sigma \alpha_p} \omega \right) - 1 - \frac{1}{\alpha_p} \right], \\ z_4 &= \alpha \frac{\alpha_p}{1 - \alpha_p} \left( \frac{1}{\alpha} + \frac{\psi}{\alpha \sigma \alpha_p} \omega \right) > 0, \\ z_5 &= \alpha > 0, \\ z_6 &= \frac{\alpha_p}{1 - \alpha_p} \left( \sigma + \frac{\psi}{\alpha_p} \right) > 0, \\ z_7 &= 1 - \frac{\alpha_1}{1 - \alpha_p} (1 + \psi), \\ \omega &= \sigma \gamma + (1 - \alpha)(\sigma \eta - 1), \end{aligned}$$

and  $\frac{\partial d_t}{\partial S_t} < 0$ ,  $\frac{\partial d_t}{\partial E_t S_{t+1}} < 0$ ,  $\frac{\partial d_t}{\partial E_t y_{t+1}^*} < 0$ .

*Proof:* See Appendix 7.1.

The correlation between the aggregate variable highly depends on the sign of  $\ln\epsilon_d - z_1$ , which is determined by the relative share of firms that conducted R&D and the allocated profit. The sign of  $\ln\epsilon_d - z_1$  may be positive when the effect of R&D on technology  $\alpha_2$  is

extremely small. In this scenario, the marginal increases in total output and profit is too tiny such that the after-allocation profit, which is increasing in the share of conducting R&D  $d_t$ , is smaller than the fix cost. Consequently, any factor that decreases the marginal cost of R&D or increases marginal benefit of R&D may, in fact, lowers the total amount of R&D because the after-allocation profit becomes more scarce. Here, we shall assume that  $ln\epsilon_d - z_1$  is positive since the estimated  $\alpha_2$ , as will discussed later, is large enough to ensure that factors that promote the marginal gain or lower the marginal cost of R&D are positively correlated with equilibrium R&D.

Several findings emerge from the proposition. First, real depreciation (or equivalently, deterioration of the terms of trade, which equals the real effective exchange rate divided by  $(1 - \alpha)$  up to the first order approximation.) leads to lower R&D investment as the term  $ln\epsilon_d - z_1$  is negative. The effect of depreciation on R&D investment is increasing in the degree of home bias,  $\alpha$ , since a higher  $\alpha$  implies that the cost of technology becomes more sensitive to the exchange rate.

Second, R&D investment is increasing in current technology level if the AR(1) coefficient  $\alpha_1$  is large enough (i.e.,  $\alpha_1 > (1 - \alpha_p)/(1 + \psi)$ ). The intuition is that if technology is highly dependent on the lagged value, a positive jump of technology will guarantee a higher future technology  $a_{t+1}$ , which leads to a higher marginal benefit of investing R&D. If  $\alpha_1$  is small, the “stepping-on-toe” effect dominates and thus causes the deterioration of R&D.

Third, the expected foreign output is negatively correlated with  $d_t$  as  $z_6$  is positive. The intuition is that a higher future foreign output will lead to higher domestic consumption via improvement of the terms of trade. To provide an incentive to achieve the same amount of labor hour, higher future consumption will raise the future wage due to a lower marginal utility of consumption. Consequently, a higher wage generates a lower marginal benefit of the present R&D.

Fourth, R&D investment is negatively correlated with future terms of trade due to the fact that  $z_4$  is positive. Intuitively, a higher terms of trade leads to higher inflation for any given value of domestic inflation, which will then raise the wage. Therefore, the deterioration of future terms of trade will increase future wage and lower the marginal benefit of investing R&D.

Finally, R&D level is decreasing in future domestic inflation if  $\alpha_p$  is large enough. Future domestic inflation generates two effects; one is the increase in the future profit via a higher  $P_{H,t+1}$ , which increases the marginal benefit of current R&D. The other is the increase in the future wage that lowers the marginal benefit. If  $\alpha_p$  increases, the marginal productivity of labor increases, and therefore the wage channel dominates the profit channel.

### 3.2.2 Labor demand and production

The production function of firm  $j$  is of the Cobb-Douglas type:

$$Y_t(j) = A_t(j)N_t(j)^{\alpha_p},$$

where  $\alpha_p$  is the labor share, and  $A_t(j)$  is the level of the technology. Since the before-allocated profit of an individual firm is proportional to its after-allocated profit, maximizing the former is equivalent to maximizing the latter. Therefore, firms' maximization problem can be presented as follows:

$$\max_{P_{H,t}(j), Y_t(j), N_t(j)} \Pi_t(j) = P_{H,t}(j)Y_t(j) - (1 - \tau)W_tN_t(j),$$

where  $\tau$  is the government subsidy on the labor wage. The optimality condition of labor is given by

$$\alpha_p N_t(j)^{\alpha_p - 1} P_{H,t}(j) A_t(j) = (1 - \tau)W_t. \quad (9)$$

By plugging the optimality condition and the production function, the profit function can also be written as

$$\begin{aligned} \Pi_t(j) &= P_{H,t}(j)A_t(j)\left(\frac{(1 - \tau)W_t}{P_{H,t}(j)\alpha_p A_t(j)}\right)^{\alpha_p/(\alpha_p - 1)} - (1 - \tau)W_t\left(\frac{(1 - \tau)W_t}{P_{H,t}(j)\alpha_p A_t(j)}\right)^{1/(\alpha_p - 1)} \\ &= \Phi_t[P_{H,t}(j)A_t(j)]^{1/(1 - \alpha_p)}. \end{aligned}$$

### 3.2.3 Price setting

Before analyzing the price setting of firms, we shall first observe that the agreement regarding the allocation of profit does not affect firms' pricing decisions. The reason is that the after-allocation profit is increasing in the profit pool, which is increasing in the before-allocation profit. The price-setting problem is thus standard. Throughout the model derivation in this section, I closely follow the work of [Gali and Monacelli \(2005\)](#).

Firms adjust prices by maximizing the following sum of the discounted stream of profit:

$$\max_{P_{H,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} (P_{H,t}(j)Y_{t+k|t}(j) - \mathcal{C}_{t+k}(Y_{t+k|t}(j))) \},$$

where  $\mathcal{C}$  is the cost function and  $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$  is the standard stochastic discount factor.  $\theta$  is the probability that firm is constrained by price stickiness.  $Y_{t+k|t}(j)$  denotes firm

$j$ 's output in period  $t+k$  when period  $t$  is the last period of price adjustment. The first-order condition is given by

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{H,t+k|t} \left( \frac{P_{H,t}^*(j)}{P_{H,t-1}(j)} - \mathcal{M} \tilde{M}C_{t+k|t}(j) \frac{P_{H,t+k}(j)}{P_{H,t-1}(j)} \right) \right\} = 0, \quad (10)$$

where  $\mathcal{M}$  is the steady-state mark-up, and  $\tilde{M}C_{t+k|t}(j)$  is the real marginal cost in period  $t+k$  when the price of firm  $j$ 's production is last set in period  $t$ . I then log-linearize equation (10) around the zero-inflation (on every goods) steady state where  $Q_{t,t+k}^{s.s.} = \beta^k$  and  $\tilde{M}C = 1/\mathcal{M}$  ( $\tilde{m}c = \ln(1/\mathcal{M})$ ). Note that under homogenous technology, the marginal cost and the price setting are the same across firms. One can show that the optimal price setting is characterized by the following equation:

$$p_{H,t}^* - p_{H,t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [\hat{m}c_{t+k|t} + (p_{H,t+k} - p_{H,t-1})], \quad (11)$$

where  $\hat{m}c_{t+k|t} \triangleq \tilde{m}c_{t+k|t} - \tilde{m}c$ . Rearranging (11), we obtain the following equation of the price setting:

$$p_{H,t}^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [\tilde{m}c_{t+k|t} + p_{H,t+k}]$$

where  $\mu = -\tilde{m}c = \log(\mathcal{M})$  is the log of markup.

### 3.3 Competitive Equilibrium

The competitive equilibrium, under a given path of the interest rate  $i_t$  and the wage subsidy  $\tau_t$ , is characterized by firms' optimality conditions (9), (10) as well as households' optimality conditions (4), and (5). The equilibrium R&D investment is then determined by equation (8). The equilibrium dynamic can be characterized by the following standard representation:

**Proposition 2.** (*Equilibrium dynamics*) *The equilibrium dynamics in terms of the output gap and domestic inflation is given by*

$$\begin{aligned} \tilde{y}_t^n &= E_t \tilde{y}_{t+1}^n - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - r_t^n), \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + K \tilde{y}_t, \end{aligned}$$

where

$$K = \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \left( \frac{\psi}{\alpha_p} - \frac{\alpha_p - 1}{\alpha_p} \right),$$

$$r_t^n = \rho + \frac{\zeta\sigma_\alpha\psi}{\sigma_\alpha + \psi} E_t(\Delta y_{t+1}^*) - \sigma_\alpha \Gamma_a (-\alpha_0 + (1 - \alpha_1)a_t - \alpha_2 d_t)$$

is the natural rate of interest, and  $\tilde{y}_t^n = y_t - y_t^n$  is the output gap. The natural level of output is given by

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*$$

$$\Gamma_0 = \frac{\mu - \ln(1 - \tau) + \ln\alpha_p}{\left(\frac{\psi}{\alpha_p} - \frac{\alpha_p - 1}{\alpha_p} + \sigma_\alpha\right)}$$

$$\Gamma_a = \frac{1 + \psi}{1 + \psi - \alpha_p(1 - \sigma_\alpha)}$$

$$\Gamma_* = (\sigma_\alpha - \sigma) \frac{1}{\left(\frac{\psi}{\alpha_p} - \frac{\alpha_p - 1}{\alpha_p} + \sigma_\alpha\right)}$$

$$\sigma_\alpha = \sigma / [1 + \alpha(\sigma\gamma + (1 - \alpha)(\sigma\eta - 1) - 1)]$$

*Proof:* See Appendix 7.2.

R&D investment  $d_t$  increase the natural rate of interest as higher investment leads to a higher equilibrium output and consumption in the next period. The fact that households consumes more in the future can only be accomated with a higher natural rate of interest. Although R&D investment does not affect the natural rate of output in the current period, it changes the output gap jointly through the natural rate of interest, future domestic inflation, and the future output gap. Section 4 provides an example of the dynamics under the real depreciation shock, which dampens R&D investment.

## 4 Numerical Results

This section first estimates the monetary policy and the technology process, which is the key through which endogenous R&D investment affects the dynamics, using firm-level data from Taiwan. I then calibrate the labor supply elasticity and the intertemporal elasticity of substitution to match the relative second moments of the labor to output as well as consumption to output. Finally, I analyze the dynamics and welfare under a real depreciation

shock, which is generated by a shock to the nominal interest rate according to the uncovered interest parity.

## 4.1 Estimation of the Technology Process

The critical parameter that determines the relevance of endogenous R&D investment is the effect of lagged R&D on current technology; that is, the coefficient  $\alpha_2$  in the following technology process:

$$a_t = \alpha_0 + \alpha_1 a_{t-1} + \alpha_2 d_{t-1} + \epsilon_t^a \quad (12)$$

I follow [Aw et al. \(2011\)](#) and use plant-level data of manufacturing industry in Taiwan from 2000 to 2004. The dataset is collected by the Taiwanese MOEA Factory Operation Census and covers 1,237 plants. The data includes firms' revenue, the capital input, and the amount of R&D investment. In each year, there are roughly 20% of plants that choose to conduct R&D. Productivity is estimated by first regressing firms' revenue on the aggregate effect of capital and technology, where the capital level is assumed to contain certain information of the productivity level. Given the assumed relationship between capital and technology, I then regress the fitted value of the aggregate effect on capital level, the past aggregate effect and past R&D investment to obtain the coefficients of the technology process.

This approach has several advantages.<sup>4</sup> First, it can solve the difficulty that the technology level is unobservable. Second, it avoids the simultaneity problem arises from the jointly determined input choice and technology. Third, it does not lead to selection bias due to the jointly determined relationship between firms' entry as well as exit and technology level. Specifically, the estimation is proceeded with the following steps:

First, I assume that production is of standard Cobb-Douglas type where  $Y_{it} = A_{it} K_{it}^{\alpha_k}$ , and  $K_{it}$  is the capital stock. With standard Dixit-Stiglitz demand and the technology process (12), the log of the firm revenue  $\ln r_{it}$  can be given by

$$\ln r_{it} = (1 + \epsilon) \ln \left( \frac{\epsilon}{1 + \epsilon} \right) + \Phi_t + (1 + \epsilon) (\beta_0 + \beta_k \ln K_{it} + \beta_p \ln P_{Kt} - a_{it}), \quad (13)$$

where  $\epsilon$  is the elasticity of substitution between varieties,  $P_{Kt}$  is the factor price of of input  $K$ , and  $\Phi_t$  is a demand shifter. However, equation (13) can not be estimated because technology

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<sup>4</sup> See, e.g., [Olley and Pakes \(1996\)](#), [Aw et al. \(2011\)](#), [Doraszelski and Jaumandreu \(2013\)](#), [Bøler et al. \(2015\)](#).

is unobservable. One way to solve this problem is to approximate technology  $a_{it}$  by using all available information regarding inputs. The underlying assumption is that the inputs of firms contain information on the level of productivity. Following [Aw et al. \(2011\)](#), I assume that  $h(\cdot)$  captures the aggregate effect of capital and technology on firms' revenue and is given by

$$h(K_{it}, a_{it}) \triangleq (1 + \epsilon)(\beta_K \ln K_{it} - a_{it}).$$

Next, I run two regressions. The first step regresses firm's revenue on the aggregate effect of inputs capital and technology:

$$\ln r_{it} = \gamma_0 + \sum_{t=1}^T \gamma_t D_t + h(K_{it}, M_{it}) + u_{it},$$

where  $D_t$  is the time-fixed effect. The fitted value of  $\hat{h}$  can then be inverted as an estimation of technology:

$$a_{it} = -\frac{1}{1 - \epsilon} \hat{h} + \beta_K \ln K_{it},$$

which can be plugged into the process [\(12\)](#). Finally, coefficients  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  can be estimated by running the second regression, which is given by

$$\hat{h}_{it} = (1 - \alpha_1) \beta_K^* \ln K_{it} - \alpha_0^* + \alpha_1 \hat{h}_{i,t-1} - \alpha_2^* d_{i,t-1} + \epsilon_{it}^*,$$

where  $X^* = (1 + \epsilon)X$  for any variable  $X$ . The estimated result with endogenous R&D investment is as follows:

$$a_t = -0.002 + 0.825a_{t-1} + 0.044d_{t-1} + \epsilon_t^a, \tag{14}$$

(0.013) (0.013) (0.006)

where the standard deviation of technology  $\sigma_a = 0.5924$ . As a comparison with the endogenous productivity model, the standard AR(1) process of technology is also estimated and given by

$$a_t = -0.040 + 0.846a_{t-1} + \epsilon_t^a.$$

(0.011) (0.013)

Compared with the standard exogenous technology process, the persistence of the lagged productivity in the endogenous productivity process is larger, and the standard deviation is significantly higher. The key coefficient of R&D dummy is significant and equals 0.044. Although the coefficient of past R&D investment is not large, a shock to R&D decision can generate a long-lasting effect as the technology process is persistent.

## 4.2 Parameters and Calibration

Before calibration, I first close the model by adopting an empirical targeting rule estimated by [Wu and Wu \(2014\)](#) and is given by

$$i_t = 0.88i_{t-1} + (1 - 0.88)(2.21 + 1.46E\pi_{t+1} + 0.92\tilde{y}_t - 0.57e_t)$$

To simulate the result around an inefficient steady state, I assume that the wage subsidy  $\tau$  equals zero such that constant markup is not entirely eliminated by the government. The markup is assumed to be 20%, which is characterized by a standard value of the elasticity of substitution across variety  $\epsilon$  that equals 6. The degree of openness is approximated by the import to GDP ratio over the period 2001 to 2016. The fixed cost of R&D investment is calibrated such that the steady state of the percentage of firms that spend in R&D investment equals the empirical mean. The AR(1) foreign output process is estimated using the time series of US GDP provided by U.S. Bureau of Economic Analysis and is given by

$$y_t^* = 0.99y_{t-1}^* + \epsilon_t^*,$$

where the standard deviation equals 0.004. For targeting parameters, the elasticity of labor supply and the intertemporal elasticity of substitution is calibrated to match the empirical second moments  $\sigma_N/\sigma_Y$  and  $\sigma_C/\sigma_Y$  using data from Taiwan. The empirical moments are calculated using Taiwan data from 1981 to 2016, and all the price-related variables are calculated based on the constant price at 2011. The data is detrended using Hodrick-Prescott (HP) filter. [Table 7](#) lists the parameter values, [Table 1](#) presents the data source.

Finally, I compare the performance of the moments between the model with and without R&D, as shown in [Table 8](#). The first two rows present the targeting moments. The model with endogenous R&D performs better than the model without endogenous R&D in most of the ratios and second moments. However, the model fails to match the correlation between output and employment, as well as the autocorrelation of employment. One possible explanation is that the shock of technology is much stronger than the shock of the foreign output. Given the large standard deviation of technology, a significant part of the variation

in output will be affected by the technology fluctuations, resulting in less cyclical employment. However, if we consider the foreign output shock being the only shock in the model and therefore not dominated by the shock of productivity, we can generate (not included in the paper) a high correlation between output and employment and a high autocorrelation of employment. Adding more shocks that feature strongly cyclical employment, such as a demand shock or a markup shock, can be a solution.

### 4.3 Real Exchange Rate Depreciation

With the calibrated model in hand, this section then simulates the dynamic response to a real depreciation shock. The goal here is to examine whether it is welfare-improving for a country to depreciate its own currency when considering endogenous R&D investment. I will also compare the results of the model with and without the endogenous R&D to highlight the importance of endogenous R&D.

There are two ways to generate a real depreciation shock: First, the real exchange rate can be affected by changing the nominal interest rate via the assumption of UIP under certain price stickiness. Second, [Benes et al. \(2015\)](#) shows that when UIP does not hold, the real exchange rate can be influenced by accumulation of foreign reserves, which is assumed to be a determinant of the UIP premium. In that framework, the government can alter the exchange rate without moving the nominal interest rate.

Here, I adopt the first approach and then leave the second approach for future extensions. To derive a simple closed-form relationship between the real exchange rate and the monetary policy, I first assume that the central bank implements a simple Taylor-type monetary policy, which only reacts to domestic inflation and the output gap. [Proposition 3](#) describes how a monetary shock to interest rate leads to a shock to the real exchange rate.

**Proposition 3.** (*Real exchange rate shock*) For a simple Taylor-type interest rate policy  $i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + v_t$  where the shock  $v_t = \rho_v v_{t-1} + \epsilon_t^v$  is assumed to be an  $AR(1)$  process. The expected changes of the terms of trade and the real exchange rate are given by

$$E_t(\Delta s_{t+1}) = (\rho_v - 1) \left[ \frac{(G_3 + G_4 \rho_v) G_5}{1 - \beta \rho_v} - (G_1 - G_2 \rho_v) \right]^{-1} v_t,$$

and

$$E_t(\Delta q_{t+1}) = (1 - \alpha) E_t(\Delta s_{t+1})$$

up to a first-order approximation where

$$G_1 = 1 + \frac{\phi_y}{\sigma_\alpha} - \frac{1}{\sigma_\alpha} \alpha_2 \xi_2 B_3$$

$$G_2 = 1 + \frac{B_2}{\sigma_\alpha}$$

$$G_3 = \frac{\phi_\pi + \alpha_2 \xi_2 B_4}{\sigma_\alpha}$$

$$G_4 = \frac{1 + \alpha_2 \xi_2 B_1}{\sigma_\alpha}$$

$$\rho = -\log \beta$$

*Proof:* See Appendix 7.3.

Before analyzing the impulse response to the depreciation shock, we should first determine the coefficients of the monetary policy. Following the standard value, I set  $\phi_\pi$  to be 1.5 and then calibrate  $\phi_y$  equals 1.95 by matching the empirical correlation between output and employment. Figure 6 and 7 plot the impulse responses to 1% of real depreciation, which is generated by the monetary shock. The shock is assumed to be temporary (i.e.,  $\rho_v = 0$ ).

Several findings emerge from the impulse responses. First, from the top two panels of Figure 6, we observe a short-term boom on both output and consumption. The driving force of the boom is the increase in foreign demand due to depreciation, which supports the conventional argument of the mercantilist motive. Second, when incorporating the endogenous R&D channel, the fact that the exchange rate depreciates, immediately increases the cost of innovation and thus drives down the R&D level, as shown in Figure 7. The temporary drop of R&D in period 0 leads to a drop in technology in period 1, which will then sluggishly return to the steady state according to the AR(1) process. Consequently, the decline in technology causes a long-term bust, which features a persistent decrease in consumption and output. Third, the expansionary interest rate that leads to jumps in output gap and domestic inflation. Fourth, the impulse response of employment is similar (two impulse responses do not coincide, even during the flat part after period 1) because the R&D is extremely sensitive to the terms of trade shock such that the change in output is mostly explained by the change in the technology.

Note that the magnitudes of the interest rate shocks that are used to generate the same size of depreciation shocks in the two models are, in fact, not identical. The reason is that

the equilibrium interest rate is partly determined by the output gap, which is affected by the future output gap according to aggregate demand. Since the future output gap is subject to R&D investment and the persistence  $\alpha_2$ , the fact that  $\alpha_2$  is zero in the model of exogenous productivity results in the difference between the required monetary shocks.<sup>5</sup>

Next, we compare the welfare change of the two models under 1% real depreciation. as shown in Figure 8. The fact that two models feature similar responses of employment but significantly different responses of consumption suggests that the primary determinant of welfare gap is the difference in consumption. The welfare deviation of the model without endogenous R&D is positive, which is consistent with the argument of the mercantilist motive where countries can benefit from real depreciation.

However, the welfare decreases in the model with endogenous R&D. The result shows that welfare drops immediately in period 0 because it measures the sum of future discounted utility, and therefore takes into account the long-term bust. Subsequently, welfare drops even more in period one as the initial boom of output and consumption ends. Finally, welfare slowly converges back to the steady state as the shock dies out. Comparing the scales of the welfare deviation, we can observe that for 1% real depreciation, welfare in endogenous R&D model decreases by about 0.1%. On the other hand, the model of exogenous productivity, although brings a welfare gain from foreign demand, does not show a large increase in welfare due to disutility of labor hours during booms. The difference between welfare under the two models is about 0.1% under 1% real depreciation, and the aggregate decline in welfare across time is 4.5%.

In sum, I show that real depreciation generates a short-term boom and a long-term bust. More importantly, welfare decreases when R&D investment is endogenously determined and only increase slightly when the productivity process is exogenous.

## 5 Conclusion

This paper focuses on the welfare analysis of real depreciation through the endogenous productivity process, which is affected by firms' R&D decisions. I first present some empirical evidence on the relationship of the exchange rate and R&D investment by using data from

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<sup>5</sup>To be more clear, the negative shock  $v_t$  will drive down the nominal interest rate, which then depreciates the real exchange rate and decreases the present R&D. Endogenous R&D will change both the future output level and the future natural rate of output, but in different degrees. Specifically,  $\partial y_{t+1}/\partial d_t = \alpha_2$  and  $\partial y_{t+1}^n/\partial d_t = \Gamma_a \alpha_2$  where  $\Gamma_a > 1$  under the parametrization in Table 7. Lowering current R&D then leads to an increase in the future output gap and therefore boosts the current output gap, implying a drop in the nominal rate. The fact that the equilibrium partly offsets the nominal shock implies that a stronger negative deviation is required to achieve the same amount of depreciation in the model with endogenous productivity.

Taiwan. Results of the reduced-form regressions show that currency undervaluation as well as real depreciation, are negatively correlated with R&D investment. By running panel VAR using country-level data, I show that R&D investment significantly drops when a country is hit by an orthogonal shock of real depreciation. The reduction in R&D activity lasts for three periods within the band of one standard deviation.

To highlight the role of the R&D on the output growth, which is the key factor that determines welfare, I regress the output growth on undervaluation index, taking into account a common break of the times series among countries. Results suggest that currency undervaluation is expansionary during the period before the break, yet it fails to be expansionary after the break. I argue that the key reason that explains this difference is the effect of the exchange rate on R&D investment. Specifically, when regressing R&D investment on the real exchange rate separately for the two periods, depreciation dampens R&D investment when undervaluation is not expansionary. On the contrary, depreciation does not significantly lower R&D investment when undervaluation is not expansionary.

In addition to the empirical findings, I contribute to the theoretical modeling of a New Keynesian model in a small open economy where aggregate productivity can be affected by firms' decisions on R&D investment. I show that equilibrium R&D investment is decreasing in real depreciation, expected foreign output, and the expected terms of trade. Consistent with the negative correlation between the exchange rate and R&D investment in data, the model predicts a decrease in R&D when facing a depreciation shock.

To study the welfare consequence and the impulse responses under depreciation, I first estimate the endogenous productivity process by using plant-level data of manufacturing industry in Taiwan and adopting the structural approach introduced by [Olley and Pakes \(1996\)](#), which solves the difficulties arise from the simultaneity problem and unobservable technology. A depreciation shock is then generated by the changes in the nominal interest rate through the UIP and price stickiness.

Quantitatively, I show that, under a real depreciation shock, the economy will experience a short-term boom in consumption and output via higher foreign demand, but a long-term bust due to a decline in productivity as the innovation cost of R&D increases. Welfare, which is affected by the long-term bust, drops immediately at the period when the shock hits. Comparing the deviation of welfare between models that feature endogenous and exogenous productivity, I find that welfare increases slightly in the model with exogenous productivity, but decreases immediately by 0.1% when we consider endogenous R&D.

In sum, this paper incorporates endogenous productivity to evaluate the welfare under real depreciation. Real depreciation is not only subject to the conventional criticism of “Beggars-thy-neighbor”, but also leads to welfare loss of the country that actually implements

depreciation. Depreciation policy that only aims to take advantage of the export competitiveness can, therefore, be welfare-deteriorating.

# 6 Figures and Tables

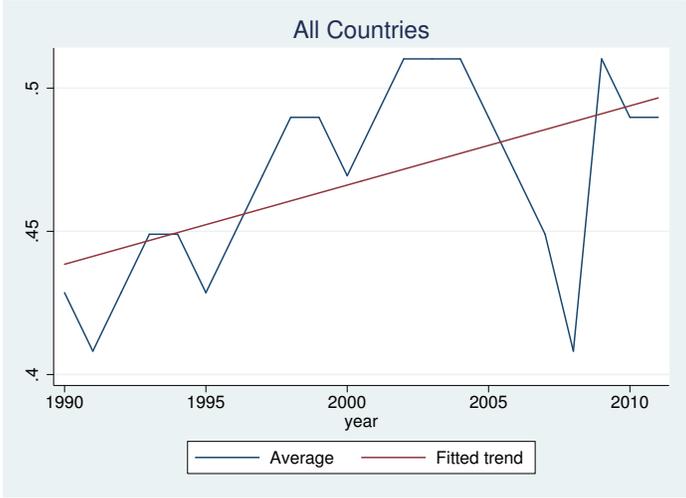


Figure 1: Percentage of countries conducts currency undervaluation

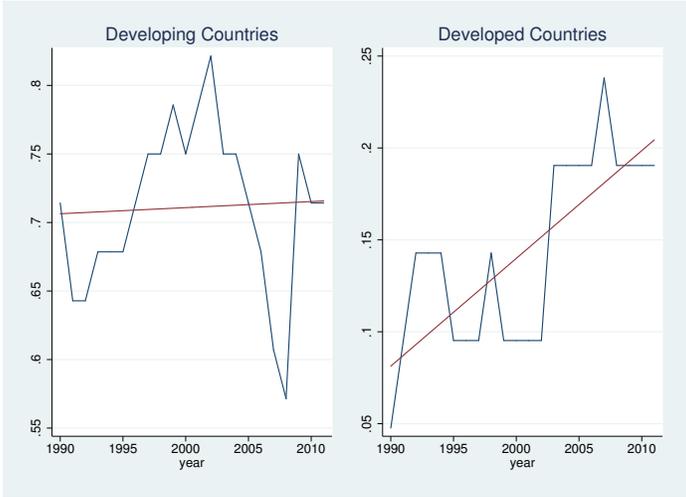


Figure 2: Percentage of countries conducts currency undervaluation: by development

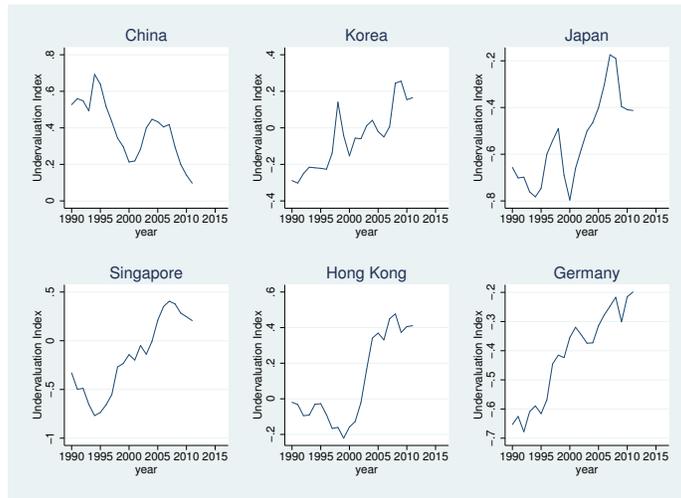


Figure 3: Currency undervaluation index

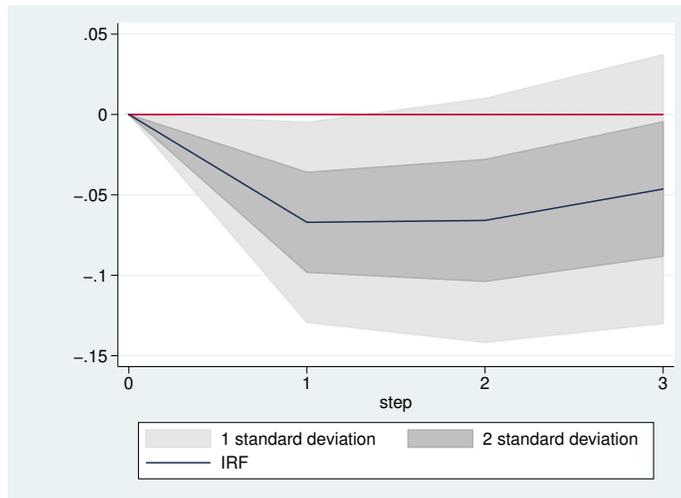


Figure 4: Impulse response of R&D to real exchange rate shock

Notes: The shaded areas represent the bands of one and two standard deviations.

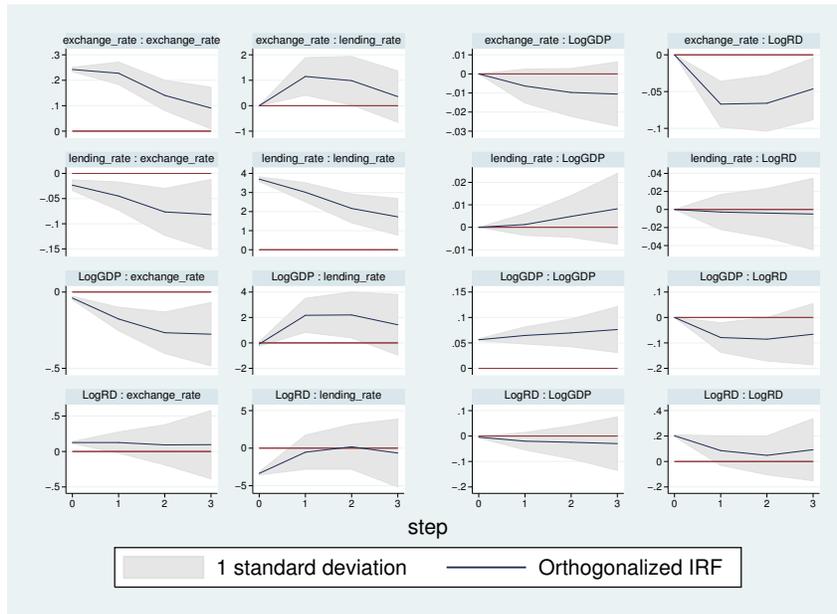


Figure 5: Impulse responses to structural shocks

*Notes:* In each title, the variable before the colon represents the corresponding structural shock and the variable after the colon is the objective hit by the shock. The last column shows the impulse responses of R&D expenditure to four different shocks. The one-standard-deviation bands are shaded.

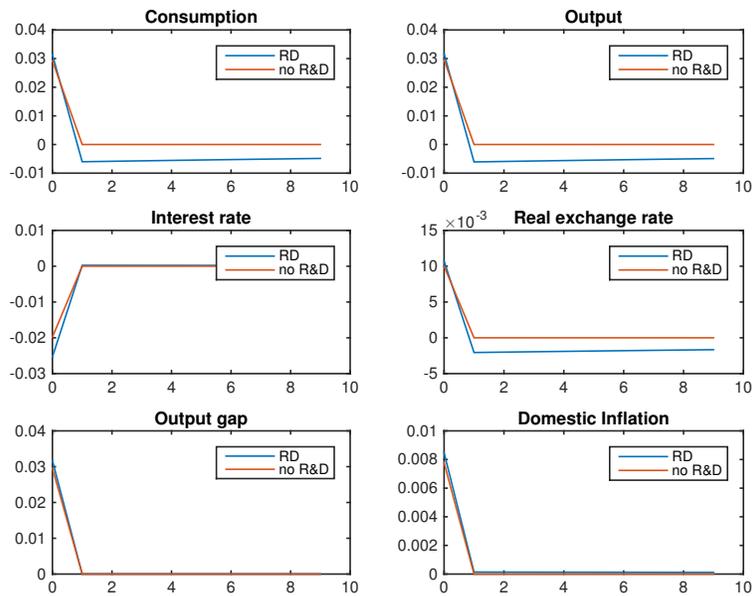


Figure 6: Impulse responses under 1% real depreciation

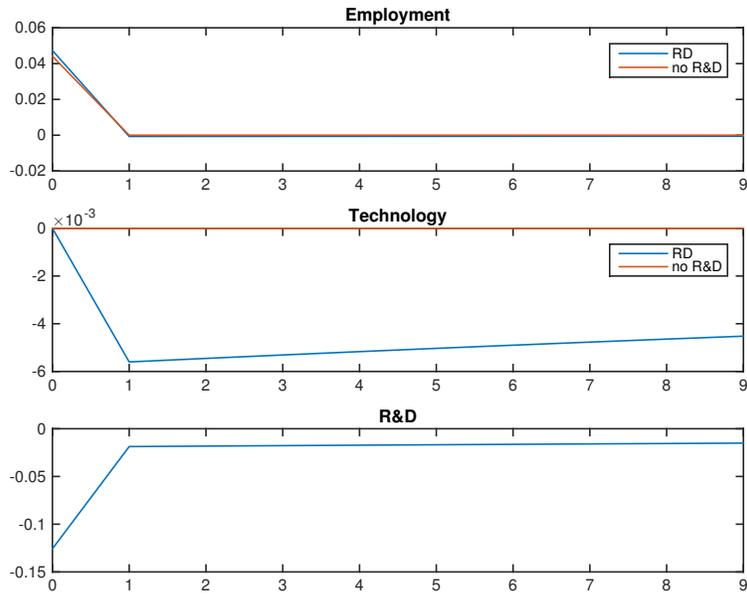


Figure 7: Impulse responses under 1% real depreciation

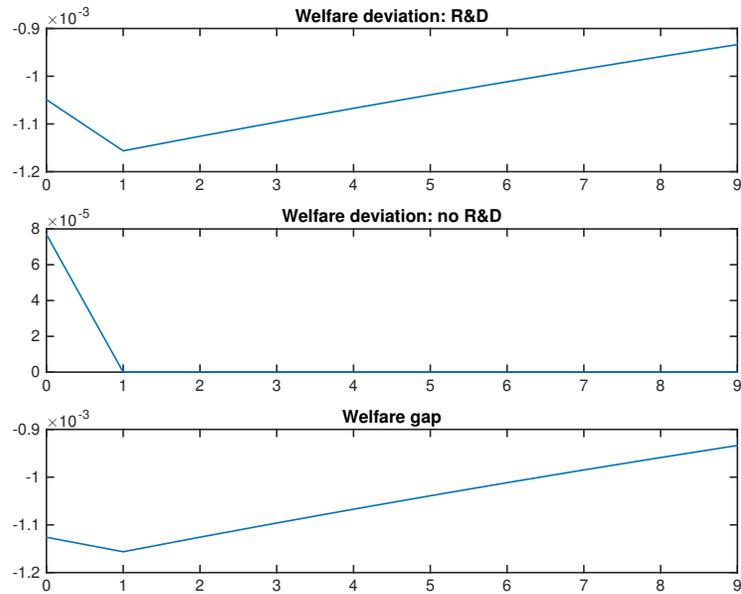


Figure 8: Changes in welfare under 1% real depreciation

Table 1: Sources of Data and Description

	Sources	Description
Figure 1, 2 and 3		
Nominal GDP	Penn World Tables 8.1	rgdpo, the output-side GDP at constant 2005 US dollar.
Population	Penn World Tables 8.1	Annual data measured in millions.
Real exchange rate	Penn World Tables 8.1	pl_gdpo, defined as conversion factors over exchange rate.
Table 2, 3, 4, 5 and Figure 4 & 5		
R&D	UNESCO Institute for Statistics	Gross domestic expenditure on R&D per capita (in current PPP)
Trade openness	Penn World Tables 8.1	Import plus export trade share. (i.e. $(v\_x + v\_m)/v\_gdp$ )
Turnover ratio	The World Bank data	Value of shares over total value of the outstanding shares.
Government Spending		
Lending rate	The World Bank data	In percentage.
Education Enrollment	The World Bank data	The enrollment in secondary education, includes both sexes.
Foreign Reserves	Global Economic Monitor	Measured in months import cover.
FXI index	Penn World Tables 8.1 & Global Economic Monitor	Measured as import value times foreign reserves in months Import cover. $(v\_m \times ForeignReserves)$ .
Table 6		
Nominal GDP	Global Economic Monitor	Constant local currency unit (2010) in millions.
Population	The World Bank data	We assume that population is fixed within a year.
Real effective exchange rate	Global Economic Monitor	Calculate the quarterly average from monthly data.
Calibration		
GDP	Taiwan National Statistic	Constant 2011 price (in millions NTD)
Employment	Taiwan National Statistic	Include both number (in thousands) and rate
Consumption	Taiwan National Statistic	Constant 2011 price (in millions NTD)
Productivity	Taiwan National Statistic	Base year 2011. Defined as output/total labor hour
Import	Taiwan National Statistic	in millions NTD
Real effective exchange rate	Penn World Tables 8.1	2005 US price level

Table 2: R&amp;D Activity and Currency Undervaluation

VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
$\ln(UVAL_{i,t-1})$	-0.0794*	-0.0856**	-0.125***	-0.114***	-0.1000**	-0.128*
	(-0.0409)	(-0.0365)	(-0.031)	(-0.0405)	(-0.0409)	(-0.0718)
$Openness_{i,t}$		0.0906*	0.103	0.103	0.0991*	0.114
		(-0.0496)	(-0.0692)	(-0.0666)	(-0.0578)	(-0.0948)
$\Delta Turnover_{i,t}$			0.0002*	0.0002**	0.0002***	0.0002**
			(0.0000)	(0.0000)	(0.0000)	(0.0000)
$\Delta Gov_{i,t}$				-0.00823	-0.0137	-0.0243**
				(-0.0111)	(-0.0255)	(-0.0111)
$\Delta i_{i,t}$					-0.0069***	-0.0076***
					(-0.00223)	(-0.0022)
$\Delta Enrollment_{i,t}$						0.0686
						(-0.102)
Constant	0.0678***	0.0816***	0.0710***	0.0754***	0.0686***	0.0713***
	(-0.0080)	(-0.0124)	(-0.0158)	(-0.0155)	(-0.0108)	(-0.0166)
Country fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	731	690	474	454	356	283
$R^2$	0.007	0.014	0.075	0.076	0.099	0.127
Number of Country	48	48	42	40	37	33

\*Significant at the 10% level; \*\*Significant at the 5% level; \*\*\*Significant at the 1% level.

The entries in brackets are the standard errors.

Table 3: R&amp;D Activity and Currency Depreciation

VARIABLES	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
$e_{it-1}^d$	-0.104*** (-0.0176)	-0.0936*** (-0.0183)	-0.0502*** (-0.0186)	-0.0586*** (-0.0188)	-0.0934*** (-0.0205)	-0.117*** (-0.0219)
$Openness_t^d$		0.013 (-0.0911)	0.162** (-0.0798)	0.0744 (-0.0766)	0.0969 (-0.0757)	0.112 (-0.0781)
$Turnover_{i,t}^d$			0.0003*** (0.0000)	0.0004*** (0.0000)	0.0004*** (0.0000)	0.0004*** (0.0000)
$Gov_{i,t}^d$				-0.0067 (-0.0158)	0.0198 (-0.0173)	0.00866 (-0.0178)
$i_{i,t}$					-0.0006** (-0.0003)	-0.0005** (-0.0002)
$Enrollment_{i,t}^d$						0.00367 (-0.0916)
Constant	-2.287*** (-0.0608)	-2.308*** (-0.0735)	-2.208*** (-0.0628)	-2.213*** (-0.06)	-0.275*** (-0.0682)	-0.241* (-0.128)
Country fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	731	690	494	474	375	323
$R^2$	0.054	0.045	0.075	0.082	0.172	0.211
Number of Country	48	48	42	40	37	36

\*Significant at the 10% level; \*\*Significant at the 5% level; \*\*\*Significant at the 1% level.

The entries in brackets are the standard errors.

Table 4: Break at 2004 (annual data)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Period	$\leq break$	$\leq break$	$\leq break$	$> break$	$> break$	$> break$
Dependent Var	$GDP\ growth_{i,t}$	$logR\&D_{it}^d$	$logR\&D_{it}^d$	$GDP\ growth_{i,t}$	$logR\&D_{it}^d$	$logR\&D_{it}^d$
IV			$FXI_{i,t-1}$			$FXI_{i,t-1}$
$logGDP_{i,t-1}$	-0.0946*** (-0.0298)			-0.364*** (-0.037)		
$ln(UVAL_{i,t})$	0.0485*** (-0.0131)			-0.0204 (-0.0331)		
$e_{it-1}^d$		-0.0286 (-0.0215)	0.821 (-1.913)		-0.0855** (-0.0388)	-0.801*** (-0.288)
Constant	-0.515*** (-0.189)	-2.671*** (-0.0937)	-0.737 (-1.107)	-1.557*** (-0.168)	-0.397*** (-0.037)	0.418* (-0.232)
Joint F			1.01258			29.3218
Country fixed effect	Yes	Yes	No	Yes	Yes	No
Time fixed effect	Yes	No	No	Yes	No	No
Observations	1,861	404	157	343	377	147
R-squared	0.193	0.99		0.725	0.99	0.373

\*Significant at the 10% level; \*\*Significant at the 5% level; \*\*\*Significant at the 1% level.

The entries in brackets are the standard errors.

Table 5: Optimal break (annual data)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Period	$\leq break$	$\leq break$	$\leq break$	$> break$	$> break$	$> break$
Dependent Var	$GDP\ growth_{i,t}$	$logR\&D_{it}^d$	$logR\&D_{it}^d$	$GDP\ growth_{i,t}$	$logR\&D_{it}^d$	$logR\&D_{it}^d$
IV			$FXI_{i,t-1}$			$FXI_{i,t-1}$
$logGDP_{i,t-1}$	-0.0938*** (-0.0338)			-0.135*** (-0.0326)		
$ln(UVAL_{i,t})$	0.0559*** (-0.0146)			0.0239 (-0.0265)		
$e_{it-1}^d$		-0.0438 (-0.0354)	7.498 (-69.15)		-0.150*** (-0.0259)	-0.579** (-0.282)
Constant	-0.500** (-0.213)	-2.741*** (-0.126)	-3.448 (-29)	-0.616*** (-0.156)	-0.336*** (-0.0247)	0.253 (-0.226)
Joint F			.014634			19.8937
Country fixed effect	Yes	Yes	No	Yes	Yes	No
Time fixed effect	Yes	No	No	Yes	No	No
Observations	1,714	260	84	490	521	220
$R^2$	0.21	0.993		0.555	0.985	0.336

\*Significant at the 10% level; \*\*Significant at the 5% level; \*\*\*Significant at the 1% level.

The entries in brackets are the standard errors.

Table 6: Optimal break (quarterly data)

	Model 1	Model 2	Model 3
Dependent var	$GDP\ growth_{i,t}$	$GDP\ growth_{i,t}$	$GDP\ growth_{i,t}$
Period	All	$\leq break$	$> break$
$logGDP_{i,t-1}$	-0.00686** (-0.0033)	-0.0752*** (-0.0135)	-0.00674* (-0.0038)
$ln(UVAL_{i,t-1})$	0.00137 (-0.0034)	0.0200** (-0.0088)	0.000113 (-0.0025)
Constant	0.000277 (-0.0033)	-0.0461*** (-0.0096)	-0.00219 (-0.0029)
Country fixed effect	Yes	Yes	Yes
Time fixed effect	Yes	Yes	Yes
Observations	2,606	1,040	1,566
R-squared	0.343	0.308	0.471

\*Significant at the 10% level; \*\*Significant at the 5% level; \*\*\*Significant at the 1% level.

The entries in brackets are the standard errors.

Table 7: Value of Parameters

Parameter	Value	Description	Source
$\eta$	0.89	Substitutability btw domestic and foreign goods	Feenstra et al. (2014)
$\gamma$	1.06	Substitutability btw goods from different countries	Feenstra et al. (2014)
$\epsilon$	6	Elasticity of substitution between varieties	Standard value
$\psi$	1.7215	Labor supply elasticity = $1/\psi$	Match $\sigma_N/\sigma_Y$
$\sigma$	0.3416	Intertemporal elasticity of substitution	Match $\sigma_C/\sigma_Y$
$\alpha_p$	0.67	Output elasticity of labor	Standard value
$\beta$	0.99	Discount rate	Standard value
$\alpha$	0.495	Degree of openness	Empirical Import/GDP ratio
$\theta$	0.75	Price Stickiness	Gali and Monacelli (2005)
$\tau$	0	Wage subsidy	Assume 20% mark-up
$\epsilon_d$	0.9729	Dispersion of R&D cost	Estimated from data
$c$	1.3703	Fixed R&D cost	Match the empirical $d_t$

Table 8: Empirical and Simulated Unconditional Moments

Moment	Data	Endogenous productivity	Exogenous productivity
$\sigma_N/\sigma_Y$	0.2428	0.2429	0.1752
$\sigma_C/\sigma_Y$	0.9910	0.9902	0.9897
$\sigma_Q/\sigma_Y$	0.4588	0.3220	0.3396
$\sigma_A/\sigma_Y$	1.1476	0.9778	0.9852
$Corr(Y, N)$	0.9860	0.2162	0.1838
$Corr(Y, C)$	0.9880	1.0000	1.0000
$Corr(Y, A)$	0.9695	0.9867	0.9931
$auto(Y)$	0.9863	0.9516	0.8954
$auto(C)$	0.9999	0.9517	0.8473
$auto(Q)$	0.9742	0.9465	0.8963
$auto(N)$	0.9998	-0.0613	0.5211
$auto(A)$	0.9966	0.8909	0.8473

$Y$ : GDP,  $C$ : Consumption,  $Q$ : Real exchange rate,  $A$ : Productivity,  $N$ : Employment.

$Corr$ : correlation,  $auto$ : autocorrelation.

## 7 Appendix

### 7.1 Proof of Proposition 1

First note that the marginal benefit of R&D investment described in the decision rule 7 equals the marginal benefit at  $t + 1$  due to the assumption of the homogeneous technology process. That is,

$$E_t \hat{\Pi}_t(j; d_t = 1) - E_t \hat{\Pi}_t(j; d_t = 0) = E_t(\tilde{\Pi}_{t+1}(j; d_t(j) = 1) - \tilde{\Pi}_{t+1}(j; d_t(j) = 0))$$

The reason to focus only the period  $t + 1$  is that the increase of current technology only generate more income at  $t + 1$ . This can be observed via the effect of R&D on the revenue in period  $t + 1$ :

$$\begin{aligned} \frac{\partial \tilde{\Pi}_{t+2}(j)}{\partial d_t(j)} &= \frac{\partial \tilde{\Pi}_{t+2}(j)}{\partial d_{t+1}(j)} \times \frac{\partial d_{t+1}(j)}{\partial d_t} \times \frac{\partial d_t}{\partial d_t(j)}, \\ \frac{\partial d_t}{\partial d_t(j)} &= \frac{\partial \int_0^1 d_t(i \neq j) di}{\partial d_t(j)} = 0. \end{aligned}$$

Similarly, for any period onward, the effect of R&D on future revenue is zero:

$$\frac{\partial \tilde{\Pi}_{t+i}(j)}{\partial d_t(j)} = 0 \quad \forall i > 1.$$

Therefore, the marginal gain of R&D investment is given by

$$E_t \hat{\Pi}_t(j; d_t = 1) - E_t \hat{\Pi}_t(j; d_t = 0) = E_t[\Phi_{t+1} P_{H,t+1}^{1/(1-\alpha_p)} X(e^{\alpha_0 + \alpha_1 a_t})^{1/(1-\alpha_p)} (e^{\alpha_2/(1-\alpha_p)} - 1)].$$

The R&D decision is then given by

$$d_t(j) = \begin{cases} 1 & \text{if } \ln[\beta E_t(\tilde{\Pi}_{t+1}(j; d_t(j) = 1) - \tilde{\Pi}_{t+1}(j; d_t(j) = 0))] \geq \ln c - \mu_t(j) + p_t + a_t, \\ 0 & \text{otherwise} \end{cases},$$

where  $c$  is the fixed cost of innovation. For tractability, I hereafter use the approximation that  $E \ln(G) \simeq \ln(E(G))$  when an arbitrary variable  $G$  is small. The marginal benefit ( $MB$ ) is given by

$$\begin{aligned}
MB &= \ln\beta + E[\ln(\Phi_{t+1} P_{H,t+1}^{1/(1-\alpha_p)} X (e^{\alpha_0 + \alpha_1 a_t})^{1/(1-\alpha_p)} (e^{\alpha_2/(1-\alpha_p)} - 1))] \\
&= \ln\beta + E[\phi_{t+1} + (\frac{1}{1-\alpha_p}) p_{H,t+1} + \ln X + \frac{1}{1-\alpha_p} (\alpha_0 + \alpha_1 a_t + \ln(e^{\frac{\alpha_2}{1-\alpha_p}} - 1))] \\
&= \ln\beta + \frac{\alpha_p}{\alpha_p - 1} [\ln(1 - \tau) + E_t w_{t+1}] + \ln(\frac{\alpha_p}{1-\alpha_p} - \alpha_p^{\frac{1}{1-\alpha_p}}) + \frac{1}{1-\alpha_p} E_t p_{H,t+1} \\
&\quad + \frac{1}{1-\alpha_p} (\alpha_0 + \alpha_1 a_t) + \ln(e^{\frac{\alpha_2}{1-\alpha_p}} - 1) + (1 + \frac{\alpha_2}{1-\alpha_p} - e^{\frac{\alpha_2}{1-\alpha_p}}) d_t,
\end{aligned}$$

where

$$X = \frac{e^{\alpha_2 d_t / (1-\alpha_p)}}{e^{\alpha_2 / (1-\alpha_p)} d_t + (1 - d_t)},$$

which is derived from the contract regarding the allocation of profits:

$$\Pi_t = \int \tilde{\Pi}_t(j) dj = d_{t-1} \tilde{\Pi}_t(j; d_{t-1} = 1) + (1 - d_{t-1}) \tilde{\Pi}_t(j; d_{t-1} = 0).$$

It follows that

$$(e^{\alpha_0 + \alpha_1 a_{t-1} + \alpha_2 d_{t-1}})^{1/(1-\alpha_p)} = d_{t-1} (e^{\alpha_0 + \alpha_1 a_{t-1} + \alpha_2})^{1/(1-\alpha_p)} X + (1 - d_{t-1}) (e^{\alpha_0 + \alpha_1 a_{t-1}})^{1/(1-\alpha_p)} X,$$

and therefore,

$$\begin{aligned}
\ln X &= \ln\left(\frac{e^{\alpha_2 d_t / (1-\alpha_p)}}{e^{\alpha_2 / (1-\alpha_p)} d_t + (1 - d_t)}\right) \\
&= \frac{\alpha_2}{1-\alpha_p} d_t - \ln(1 + (e^{\frac{\alpha_2}{1-\alpha_p}} - 1) d_t) \\
&\simeq \frac{\alpha_2}{1-\alpha_p} d_t - (e^{\frac{\alpha_2}{1-\alpha_p}} - 1) d_t \\
&= (1 + \frac{\alpha_2}{1-\alpha_p} - e^{\frac{\alpha_2}{1-\alpha_p}}) d_t.
\end{aligned}$$

Plugging the condition (23), (28) and (20) into equation (4), we can write the future wage  $w_{t+1}$  as

$$\begin{aligned}
w_{t+1} &= \left(1 + \frac{1}{\alpha} \frac{\sigma + \psi/\alpha_p}{\sigma_\alpha} - \omega\right) \pi_{t+1} + p_t + \left(\sigma + \frac{\psi}{\alpha_p}\right) y_{t+1}^* \\
&\quad - \frac{1}{\alpha} \left(\frac{\sigma + \psi/\alpha_p}{\sigma_\alpha} - \alpha\omega\right) \pi_{H,t+1} + \left(\frac{\sigma + \psi/\alpha_p}{\sigma_\alpha} - \alpha\omega\right) - \frac{\psi}{\alpha_p} (\alpha_0 + \alpha_1 a_t + \alpha_2 d_t).
\end{aligned}$$

Firms will choose to invest in R&D if

$$\begin{aligned}
MB &\geq lnc - \mu_t(j) + p_t + a_t \\
\Rightarrow \mu_t(j) &\geq lnc + p_t + a_t - MB \triangleq f(E_t \pi_{t+1}, p_t, E_t y_{t+1}^*, E_t \pi_{H,t+1}, s_t, a_t, p_{H,t}, d_t).
\end{aligned}$$

The aggregate R&D must satisfies  $d_t = \int_0^1 d_t(j) dj = 1 - (f - lnc)/(-lnc)$  in the equilibrium; that is,

$$d_t = \frac{1}{lnc - c_{11}} [c_3 + c_4 E_t \pi_{t+1} + c_5 p_t + c_6 E_t y_{t+1}^* + c_7 E_t \pi_{H,t+1} + c_8 s_t + c_9 a_t + c_{10} p_{H,t}], \quad (15)$$

where

$$c_1 = \frac{1}{\alpha} + \frac{\psi}{\alpha \sigma \alpha_p} [\sigma \gamma + (1 - \alpha)(\sigma \eta - 1)]$$

$$c_2 = (c_1 - 1)\alpha$$

$$c_3 = lnc - lnc - \frac{\alpha_p}{\alpha_{p-1}} \left[ \ln(1 - \tau) - \frac{\psi \alpha_0}{\alpha_p} \right] - \ln\left(\frac{\alpha_p}{\alpha_{p-1}} - \alpha_p^{\frac{1}{1-\alpha_p}}\right) - \frac{1}{1 - \alpha_p} \alpha_0 - \ln\left(e^{\frac{\alpha_2}{1-\alpha_p}} - 1\right) + 1$$

$$c_4 = \frac{\alpha_p}{1 - \alpha_p} c_1$$

$$c_5 = 1 + \frac{\alpha_p}{1 - \alpha_p}$$

$$c_6 = \frac{\alpha_p}{1 - \alpha_p} \left(\sigma + \frac{\psi}{\alpha_p}\right)$$

$$c_7 = \frac{\alpha_p}{1 - \alpha_p} \frac{c_2}{\alpha} - \frac{1}{1 - \alpha_p}$$

$$c_8 = \frac{\alpha_p}{1 - \alpha_p} c_2$$

$$c_9 = 1 - \frac{\alpha_1}{1 - \alpha_p} (1 + \psi)$$

$$c_{10} = -\frac{1}{1 - \alpha_p}$$

$$c_{11} = -1 - \alpha_2 \frac{1 + \psi}{1 - \alpha_p} + e^{\frac{\alpha_2}{1 - \alpha_p}}$$

Rearranging the equation, we obtain the functional form of R&D presented in Proposition 1 (One can compare the notations here with those in Proposition 1, it follows that  $z_1 = c_{11}$ ,  $z_2 = c_3$ ,  $z_3 = c_4 + c_7$ ,  $z_4 = \alpha c_4$ ,  $z_6 = c_6$ , and  $z_7 = c_9$ ).

*QED*

## 7.2 Proof of Proposition 2

Domestic production is affected by foreign production via the exchange rate that affects foreign demand and the endogenous productivity process. We shall first focus on the relationship between open market ratio (e.g., the terms of trade, nominal exchange rate, and the real exchange rate). The terms of trade is defined as the relative price of foreign goods and domestic goods:

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \frac{(\int_0^1 P_{i,t}^{1-\gamma} di)^{1/(1-\gamma)}}{P_{H,t}} = (\int_0^1 S_{i,t}^{1-\gamma} di)^{1/(1-\gamma)},$$

where  $S_{i,t}$  equals  $\frac{P_{i,t}}{P_{H,t}}$  is the terms of trade of country  $i$ . After log-linearizing around the symmetric steady state where  $S_{i,t} = S^* = 1$ , we obtain the effective terms of trade:

$$s_t \triangleq \ln S_t = \int_0^1 \ln S_{it} di = \int_0^1 s_{it} di$$

Assume that the price of goods is linked via the law of one price such that  $P_{i,t}(j) = \epsilon_{i,t} P_{i,t}^i(j)$  where  $\epsilon$  is the nominal exchange rate and  $P_{i,t}^i(j)$  is the domestic price of good  $j$  in country  $i$ , denominated in its local currency.  $P_{i,t}^i = (\int_0^1 P_{i,t}^i(j)^{1-\epsilon} dj)^{1/(1-\epsilon)}$ ,  $p_{i,t}^i(j)$  is the price of country  $i$ 's good  $j$  expressed in terms of  $i$ 's currency. After log-linearizing the price index around the symmetric steady state, the terms of trade and the nominal exchange rate can be linked:

$$s_t = p_{F,t} - p_{H,t} = e_t + p_t^* - p_{H,t} \tag{16}$$

where  $e_t = \int e_{i,t} di$  is the effective nominal exchange rate, and  $p_t^* = \int_0^1 p_{i,t}^i di$  is the world price index. The real exchange rate between the domestic country and country  $i$  is denoted as  $Q_{i,t}$ , which is defined as the nominal exchange rate times the relative price level; that is,

$Q_{i,t} = \epsilon_{i,t} P_t^i / P_t$ . The log of the real exchange rate is given by

$$q_{i,t} = e_{i,t} - p_t^i - p_t,$$

where the real effective exchange rate is given by

$$q_t \triangleq \int_0^1 q_{i,t} di = \int_0^1 \ln Q_{i,t} di. \quad (17)$$

By plugging in equation (16), The real effective exchange can also be written as a function of the terms of trade:

$$q_t = s_t + p_{H,t} - p_t. \quad (18)$$

Next, I log-linearize the price index  $P_t = [(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{1/(1-\eta)}$  around the steady state where  $S_{i,t} = S^* = 1$ . The linear approximation is given by

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t}, \quad (19)$$

$$\Rightarrow \pi_t = \pi_{H,t} + \alpha \Delta s_t. \quad (20)$$

Plugging equation (19) into equation (18), we obtain the relationship between real exchange rate and the terms of trade:

$$q_t = (1 - \alpha)s_t. \quad (21)$$

I assume that the market is complete and that securities can be traded freely across countries. In the symmetric steady state, the relationship between consumption of two countries is linked via the real exchange rate and is given by

$$C_t = C_t^i Q_{i,t}^i. \quad (22)$$

Taking log on both sides of equation (22) and plugging it into equation (21), we obtain the relationship between foreign consumption, domestic consumption and terms of trade.

$$c_t = c_t^* + \frac{1 - \alpha}{\sigma} s_t \quad (23)$$

With the functional form of consumption in hand, we move onto the labor market where

the aggregate labor demand is given by

$$\begin{aligned}
N_t &= \int_0^1 N_t(j) dj \\
&= \int_0^1 \left( \frac{Y_t(j)}{A_t(j)} \right)^{1/\alpha_p} dj \\
&= Y_t^{1/\alpha_p} \int_0^1 \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon/\alpha_p} \left( \frac{1}{A_t(j)} \right)^{1/\alpha_p} dj, \\
\Rightarrow \alpha_p n_t &= y_t - a_t - G_t \\
G_t &= \alpha_p \log \left( \int_0^1 (P_{H,t}(i)/P_{H,t})^{-\epsilon/\alpha_p} di \right),
\end{aligned} \tag{24}$$

where equation (24) holds under the assumption of homogeneous technology. According to Galí (2015),  $G$  is equal to zero up to a first-order approximation. Consequently, the equilibrium output is given by

$$y_t = a_t + \alpha_p n_t. \tag{25}$$

The goods market clears when the supply equals demand from both the domestic market and foreign markets:

$$\begin{aligned}
Y_t(j) &= C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) di \\
&= \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\epsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right].
\end{aligned} \tag{26}$$

The log-linearization of equation (26) around the symmetric steady state yields

$$y_t = c_t + \frac{\alpha[\sigma\gamma + (1 - \alpha)(\sigma\eta - 1)]}{\sigma} s_t. \tag{27}$$

By plugging the market clearing condition (27) and the global market clearing condition,  $y_t^* = c_t^*$ , into equation (23) we obtain

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t, \tag{28}$$

where  $\sigma_\alpha = \sigma/[1 + \alpha(\sigma\gamma + (1 - \alpha)(\sigma\eta - 1) - 1)]$ . Equation (28) states that the domestic output is affected by foreign demand through the foreign output and the terms of trade. Next, we move onto the dynamics of price and how it is affected by price stickiness. The log of the marginal cost is

$$\begin{aligned}\tilde{m}c_t &= mc_t^n - p_{H,t} \\ &= \ln(1 - \tau) + w_t - a_t - \ln\alpha_p - (\alpha_p - 1)n_t - p_{H,t} \\ &= \ln(1 - \tau) + w_t - a_t - \ln\alpha_p - \frac{\alpha_p - 1}{\alpha_p}(y_t - a_t) - p_{H,t}\end{aligned}\quad (29)$$

$$= \ln(1 - \tau) - \ln\alpha_p + \sigma y_t^* + s_t + \left(\frac{\psi}{\alpha_p} - \frac{\alpha_{p-1}}{\alpha_p}\right)y_t - \left(\frac{1 + \psi}{\alpha_p}\right)a_t. \quad (30)$$

Equation (30) is derived by plugging in equation (23), (5) and the condition where  $y_t^* = c_t^*$ . Substitute the terms of trade with equation (28), we obtain

$$\hat{m}c_t = \left(\frac{\psi}{\alpha_p} - \frac{\alpha_p - 1}{\alpha_p}\right)\tilde{y}_t, \quad (31)$$

where  $\tilde{y}_t$  is the output gap. Similar to equation (30), the log of marginal cost in period  $t + k$  when period  $t$  is the last period of price adjustment is given by

$$\tilde{m}c_{t+k|t} = \ln(1 - \tau) - \ln\alpha_p + w_{t+k} - p_{H,t+k} - \frac{1}{\alpha_p}(a_t - (1 - \alpha_p)y_{t+k|t}). \quad (32)$$

The corresponding output is given by

$$Y_{t+k|t} = \left(\frac{P_{H,t}^*(j)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k},$$

where the log of output is given

$$y_{t+k|t} = -\epsilon(p_{H,t}^* - p_{H,t+k}) + y_{t+k}.$$

By comparing equation (29) and (32), the corresponding marginal cost is given by

$$\tilde{m}c_{t+k|t} = \tilde{m}c_{t+k} - \frac{(1 - \alpha_p)\epsilon}{\alpha_p}(p_{H,t}^* - p_{H,t+k}) \quad (33)$$

According to Galí (2015), the aggregate price dynamic can be written as

$$\pi_{H,t} = (1 - \theta)(p_{H,t}^* - p_{H,t-1}) \quad (34)$$

After plugging equation (33), (34), and (31) into the optimal price setting (11), the

Phillips curve is given by

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + K \tilde{y}_t \quad (35)$$

where  $K = \frac{(1-\beta\theta)(1-\theta)}{\theta} (\frac{\psi}{\alpha_p} - \frac{\alpha_p-1}{\alpha_p})$ . Next, we rearrange the aggregate demand (5), which can be written in terms of the natural rate of interest and output gap. Plugging in equation (28) into equation (30), the marginal cost can be written as

$$\tilde{m}c_t = \ln(1-\tau) - \ln\alpha_p + (\sigma - \sigma_\alpha)y_t^* + \left(\frac{\psi}{\alpha_p} - \frac{\alpha_p-1}{\alpha_p} + \sigma_\alpha\right)y_t - \left(\frac{1+\psi}{\alpha_p}\right)a_t$$

where the natural rate of output can be derived from replacing the markup with the markup in the flexible price equilibrium:

$$-\mu = \ln(1-\tau) - \ln\alpha_p + (\sigma - \sigma_\alpha)y_t^* + \left(\frac{\psi}{\alpha_p} - \frac{\alpha_p-1}{\alpha_p} + \sigma_\alpha\right)y_t^n - \left(\frac{1+\psi}{\alpha_p}\right)a_t. \quad (36)$$

After rearranging equation (36), the natural rate of output is given by

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*, \quad (37)$$

where

$$\begin{aligned} \Gamma_0 &= \frac{\mu - \ln(1-\tau) + \ln\alpha_p}{\left(\frac{\psi}{\alpha_p} - \frac{\alpha_p-1}{\alpha_p} + \sigma_\alpha\right)}, \\ \Gamma_a &= \frac{1+\psi}{1+\psi - \alpha_p(1-\sigma_\alpha)}, \\ \Gamma_* &= (\sigma_\alpha - \sigma) \frac{1}{\left(\frac{\psi}{\alpha_p} - \frac{\alpha_p-1}{\alpha_p} + \sigma_\alpha\right)}. \end{aligned}$$

Next, by plugging equation (27) and (28) into the aggregate demand (5), the IS curve can be written as

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - \rho) + \zeta E_t (\Delta y_{t+1}^*), \quad (38)$$

where  $\zeta = \alpha(\sigma\gamma + (1-\alpha)(\sigma\eta - 1) - 1)$ . Finally, equation (38) and (37), the aggregate demand

in terms of output gap,  $\tilde{y}_t^n = y_t - y_t^n$ , is given by

$$\tilde{y}_t^n = E_t \tilde{y}_{t+1}^n - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - r_t^n),$$

where the natural rate of interest is given by

$$r_t^n = \rho + \frac{\zeta \sigma_\alpha \psi}{\sigma_\alpha + \psi} E_t (\Delta y_{t+1}^*) - \sigma_\alpha \Gamma_a (-\alpha_0 + (1 - \alpha_1) a_t - \alpha_2 d_t).$$

*QED*

### 7.3 Proof of Proposition 3

The equilibrium is characterized by the following equations:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + K \tilde{y}_t^n, \quad (39)$$

$$\tilde{y}_t^n = E_t \tilde{y}_{t+1}^n - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - r_t^n), \quad (40)$$

where

$$r_t^n = \rho + \xi_1 E_t (\Delta y_{t+1}^*) - \xi_2 (-\alpha_0 + (1 - \alpha_1) a_t - \alpha_2 d_t), \quad (41)$$

$$i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t^n + v_t,$$

$$\xi_1 = \frac{\zeta \sigma_\alpha \psi}{\sigma_\alpha + \psi}, \quad (42)$$

$$\xi_2 = \sigma_\alpha \Gamma_\alpha, \quad (43)$$

$$K = \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \left( \frac{\psi}{\alpha_p} - \frac{\alpha_p - 1}{\alpha_p} \right). \quad (44)$$

From Proposition 7.1, by plugging equation (20) and (28) into equation (15), R&D investment can be given by

$$d_t = \frac{1}{ln\epsilon_d - c_{11}} [c_3 + c_4[E_t\pi_{H,t+1} + \sigma_\alpha E_t y_{t+1} - \sigma_\alpha y_t - \sigma_\alpha E_t \Delta y_{t+1}^*] + c_5(\pi_t + p_{t-1}) + c_6 \rho_y y_t^* + c_7 E_t \pi_{H,t+1} + c_8 \sigma_\alpha (y_t - y_t^*) + c_9 a_t + c_{10}(\pi_{H,t} + p_{H,t-1})]. \quad (45)$$

Note that the natural rate of output is

$$\begin{aligned} y_t^n &= \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*, \\ \Rightarrow E_t y_{t+1} &= E_t \tilde{y}_{t+1}^n + g_1(y_t^*, a_t) + \Gamma_a \alpha_2 d_t, \end{aligned}$$

where  $\tilde{y}_{t+1}^n = y_{t+1} - y_{t+1}^n$ . Denote  $den \equiv ln\epsilon_d - c_{11}$ , R&D investment can now be written as

$$\begin{aligned} d_t &= g_2(y_t^*, a_t, X_{t-1}) + \frac{1}{G \times den} [(c_4 + c_7)E_t \pi_{H,t+1} + c_4 \sigma_\alpha E_t \tilde{y}_{t+1}^n + \sigma_\alpha (-c_4 + c_5 + c_8) \tilde{y}_t] \\ &= g_2(y_t^*, a_t, X_{t-1}) + B_1 E_t \pi_{H,t+1} + B_2 E_t \tilde{y}_{t+1}^n + B_3 \tilde{y}_t, \end{aligned} \quad (46)$$

where  $X_{t-1}$  is the information set that contains all the predetermined variables, and

$$\begin{aligned} G &= \frac{den}{den - (\sigma_\alpha \Gamma_a \alpha_2 c_4)}, \\ B_1 &= \frac{1}{G} \frac{c_4 + c_7}{den}, \\ B_2 &= \frac{1}{G} \frac{c_4 \sigma_\alpha}{den}, \\ B_3 &= \frac{1}{G} \frac{\sigma_\alpha (-c_4 + c_5 + c_8)}{den}. \end{aligned}$$

Plugging equation (46) first into equation (41) and then into equation (40), the dynamics of the output gap can be given by

$$G_1 \tilde{y}_t^n = G_2 E_t \tilde{y}_{t+1}^n + G_3 \pi_{H,t} + G_4 E_t \pi_{H,t+1} + \frac{1}{\sigma_\alpha} (\hat{r}_t^n - v_t), \quad (47)$$

where

$$\hat{r}_t^n = r_t^n - (B_1 E_t \pi_{H,t+1} + B_2 E_t \tilde{y}_{t+1}^n + B_3 \tilde{y}_t) - \rho,$$

$$\begin{aligned}
G_1 &= 1 + \frac{\phi_y}{\sigma_\alpha} - \frac{1}{\sigma_\alpha} \alpha_2 \xi_2 B_3, \\
G_2 &= 1 + \frac{B_2}{\sigma_\alpha}, \\
G_3 &= \frac{\phi_\pi + \alpha_2 \xi_2 B_4}{\sigma_\alpha}, \\
G_4 &= \frac{1 + \alpha_2 \xi_2 B_1}{\sigma_\alpha}.
\end{aligned}$$

Since the natural rate of interest is not affected by the monetary shock,  $\hat{r}_t^n$  should be zero. By conjecturing  $\tilde{y}_t^n = \psi_{yv} v_t$  as well  $\pi_{H,t} = \psi_{\pi v} v_t$  and using equation (39) and (47), the terms of trade (28) can be written as

$$\begin{aligned}
s_t &= \sigma_\alpha (y_t - y_t^*) \\
&= \left[ \frac{(G_3 + G_4 \rho_v) G_5}{1 - \beta \rho_v} - (G_1 - G_2 \rho_v) \right]^{-1} v_t - \sigma_\alpha y_t^*, \\
s_{t+1} &= \sigma_\alpha \psi_{yv} (\rho_v v_t + \epsilon_{t+1}^v).
\end{aligned}$$

Since we perturb the economy using only the monetary shock,  $y_t^*$  is always the steady state value and therefore can be normalized to zero. The change in the terms of trade, therefore, is given by

$$E_t(\Delta s_{t+1}) = (\rho_v - 1) \left[ \frac{(G_3 + G_4 \rho_v) G_5}{1 - \beta \rho_v} - (G_1 - G_2 \rho_v) \right]^{-1} v_t.$$

Finally, the change in real depreciation results from the monetary shock can then be derived based on equation (21).

*QED*

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