Inefficient Markets

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Abstract

We study price formation in laboratory markets with private and common values. Rational expectations, which form the basis for the efficient market hypothesis, predict that the introduction of common values does not affect allocative and informational efficiency. In contrast, a "private" expectations model in which traders' optimal behavior depends on both their private and common-value information predicts that neither allocative nor informational efficiency is possible. We test these competing hypotheses and find that the introduction of common values lowers allocative efficiency by 28%, as predicted by the private expectations model, and that market prices differ substantially from rational expectation levels.

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1. Introduction

The ability of market institutions to aggregate dispersed information and produce correct prices is of central importance to their well functioning. In private-value commodity markets, prices determine traders' opportunity sets and correct prices ensure that the market clears and total gains from trade are maximized. In common-value asset markets, prices play the additional role of informing traders about underlying asset values and correct prices make profitable arbitrage impossible. These desired features have been observed in many laboratory studies that employ the continuous double auction (CDA), the most commonly used trading institution for contemporary financial and commodity markets. Hundreds of experiments have confirmed Vernon Smith's (1962) finding that, in *private value* commodity markets, the CDA converges quickly and reliably to competitive equilibrium outcomes.[1] Furthermore, in *common value* asset market experiments, trade prices in the CDA have been shown to accurately summarize traders' dispersed private information thus providing laboratory evidence for the *efficient market hypothesis* (Fama, 1970, 1991).[2,3] As Cason and Friedman (1996) note "it is folk wisdom, at least among experimenters, that the CDA has remarkable powers to promote price formation."

Real-world markets, however, rarely fit the idealized extreme cases of pure private or pure common values. Private-value commodities may have deficiencies or can be resold, which adds a common-value element. Likewise, in common-value asset markets, private-value differences naturally arise when investors face varying capital gains tax-rates, hold different long/short positions (e.g. Nyborg and Strebulaev, 2004), or different portfolios.

[1] Vernon Smith coined this finding a "scientific mystery" because convergence to competitive equilibrium occurs even when it is not predicted. Static competitive equilibrium theory relies on the assumptions that each trader is a price taker, there is free entry and exit, and there are an infinite number of potential entrants. In experiments, the CDA robustly converges to competitive equilibrium even with few buyers and sellers who act as price makers rather than price takers and who have only private information about values and costs. Furthermore, participants do not need experience or a deep understanding of economics and convergence is robust to changes in subject pools (e.g. students, businessmen, government officials, etc., see Smith, 2010). The competitive equilibrium is reached even when demand (supply) is completely elastic so that the demand (supply) side of the market gets almost no surplus (Smith and Walker, 1993). See Friedman and Rust (1993), Plott and Smith (2005), and Smith (2010) for excellent surveys.

[2] Early experimental evidence was provided by Plott and Sunder (1982), Forsythe, Palfrey, and Plott (1982), and Friedman, Harrison, and Salmon (1984). For a recent study see, e.g., Huber, Angerer, and Kirchler (2011).

[3] The degree to which information gets successfully aggregated depends on certain market features, including the number of informed traders (Camerer and Weigel, 1991), whether the state of nature is revealed ex post (O'Brien, 1990), the complexity of the assets being traded (e.g. single-state versus multi-state assets, Plott and Sunder, 1988; single-period versus multi-period assets, O'Brien and Srivastava, 1991), whether the information technology and distributional assumptions are common knowledge (Forsythe and Lundholm, 1990), and trader experience (Forsythe and Lundholm, 1990). See Sunder (1995) for a thoughtful survey. Our study differs from this prior work in that we consider a setting where allocative and informational inefficiencies may arise even with rational traders.

When both private and common values are present, markets cannot generally achieve full allocative efficiency. This impossibility result was first shown by Maskin (1992) and Dasgupta and Maskin (2000) for one-sided markets (auctions),[4] and generalized to arbitrary mechanisms including two-sided markets by Jehiel and Moldovanu (2001). The reason is that the way in which a trader's information affects her own value from a transaction may differ from the way it impacts the social value. Intuitively, a trader with a high private value should become a net buyer but she may instead sell if her common value information is negative, with adverse consequences for allocative efficiency. In this paper, we link the impossibility of full allocative efficiency to a failure of the efficient market hypothesis. The intuition for this link is simple. If full informational efficiency were possible then the common value would simply shift traders' private values by a *known* constant, which would leave traders' incentives and the total gains from trade unaltered and full allocative efficiency should result.

In a series of laboratory experiments we test price formation in markets with private and common values. Experiments are ideally suited to study market performance in this setting because the various informational conditions, endowments, and preferences can be induced to fit the theoretical models. This allows for a clean measurement of allocative and informational inefficiencies, which would be hard to identify based on econometric analyses of field data. Besides the introduction of private and common-value elements, our experimental design departs from that of previous literature in two important ways. First, market participants do not have preassigned roles of buyers or sellers. Instead, each market participant is a trader who can choose to either buy or sell (or not trade at all) based on their own private information, as is the case in most financial markets. Second, traders receive new private and common value information at the start of each period so that each period represents a new price formation process.[5] The motivation for these two design choices is that it allows for clean theoretical predictions and that fully efficient trade is possible in our setup, i.e. there exists an incentive compatible, individually rational mechanism that delivers all gains from trade (Cramton, Gibbons, and Klemperer, 1987). This would not be possible, for instance, with fixed buyer and seller roles (Myerson and Satterthwaite, 1983).

Rational expectations (e.g. Muth, 1961), which underlie the efficient market hypothesis, predict that the introduction of common values has no adverse consequences for allocative and informational efficiency. In contrast, a "private" expectations model in which traders' optimal

[4]See also Pesendorfer and Swinkels (2000) and Goeree and Offerman (2002, 2003).

[5]Cason and Friedman (1996) and Kagel (2004) also used random values and costs for each trading period in double auction markets where participants had fixed trading roles, either as buyers or as sellers. Importantly, in such a setting with asymmetric property rights, Myerson and Satterthwaite's (1983) impossibility result implies that no incentive compatible, individually rational mechanism can be fully efficient.

behavior depends on both their private and common-value information predicts that neither allocative nor informational efficiency is possible. To test these competing hypotheses, we compare market performance in a treatment with only private values to a treatment with both private and common values. We find that the introduction of common values causes allocative efficiency to drop by 28% on average, as correctly predicted by the private expectations model. In addition, prices are systematically biased away from their rational expectations levels and the observed deviations are increasing in the size of the common value. Also these findings are in line with the predictions of the private expectations model.

We explore how the degree of competition affects market performance.[6] In our experiment, the number of traders varies from two to three to eight. We find that an increase in competition significantly raises allocative efficiency, both with and without common values. However, with common values, allocative efficiency losses remain large ($> 40\%$) even with eight traders. There is little effect of competition on informational efficiency with private values only and price deviations are moderately small. In contrast, with common values, price deviations from rational expectations predictions are substantial and increase with the number of traders.

A final contribution of the paper is to test for equilibrium behavior. As noted by Smith (2010) "the challenge of the CDA empirical results has not yielded game theoretic models that predict convergence to a static competitive equilibrium."[7] We agree that a direct test of equilibrium behavior in this dynamic game of incomplete information where players can move at unspecified times is out of reach. However, for the simple environment employed in the experiment, incentive compatibility makes precise predictions about how traders' equilibrium payoffs should vary with their private information. By comparing predicted and observed payoffs we test for equilibrium and find that observed behavior is in line with predictions of the private expectations model.

1.1. Organization

The reminder of the paper is organized as follows. Section 2 details the experimental design and procedures. Section 3 presents the two theoretical models to be tested. Section 4 reports results on allocative and informational efficiency levels, the determinants of trade, and tests for equilibrium behavior. Section 5 concludes. Instructions, with screen shots of the zTree program (Fischbacher, 2007) used by the subjects, can be found under supplementary material.

[6] In Vernon Smith's (1962) original double auction market experiments with private value commodities, the competitive equilibrium is attained even with a small number of traders. In a common value asset market experiment, Lundholm (1991) finds that an increase in the number of traders does not necessarily lead to better information aggregation.

[7] Friedman (2010) provides a more positive account of game theoretic modeling of behavior in the CDA.

2. Experimental Design and Procedures

The experiments were based on a straightforward 2×3 between-subject design, see Table 1. The treatments included private values (PV) versus private plus common values (PVCV) and variations in group size ($n = 2$, $n = 3$, and $n = 8$). In each of the treatments, the induced private values ranged from 201 to 300 points with each integer number being equally likely. We used the same private values in the PV and associated PVCV treatments (e.g. the same private values in PV_2 as in $PVCV_2$) to ensure that any observed differences in the gains from trade were not due to differences in the random draws. In the PVCV treatments, subjects received an additional common value signal that was either -25 or $+25$, both outcomes being equally likely. The common value was simply equal to the sum of all the common-value signals in a group. A subject's total value for the good was equal to the private value plus the common value.[8] At the start of each period, subjects received new private values and, if applicable, new common-value signals. Private values and common-vale signals were independent across subjects and periods.

Subjects traded in a continuous double auction. At the start of each period, subjects were endowed with one unit of the good and 500 cash. Subjects valued at most two units of the good. Negative holdings of the good or cash were not allowed (i.e. no "short selling"). Only a single unit of the good could be traded by each subject in each period. This design choice follows Cason and Friedman (1996) who note that it allows for sharp theoretical predictions without dubious auxiliary assumptions. In particular, it allows us to predict how traders' equilibrium payoffs vary with their information.

Subjects could submit limit orders (bids and asks) as well as market orders. All orders were executed instantaneously and prioritized according to price in an open bid book. Standing orders and transactions were updated on the traders' screens in real time and the price of the latest transaction was indicated by the "market price." At the end of the period, subjects were shown a results screen, which indicated their information (private value and, if applicable, the common value signal and the common value), their transactions (bought or sold a unit or no trade), and their net earnings. Subjects' earnings were calculated as the difference between their final wealth (value of the items they owned plus final cash position) and their initial wealth (value of one item plus 500 cash). In other words, subjects had to trade to make money.

Subjects were recruited at the University of Zürich and the neighboring ETH. A total of 168 subjects participated in eight sessions with 18-24 people in each session. Each session

[8] The lower bound of 201 for the private values was chosen such that the total value of the good would be positive even if all traders had negative common value signals in $PVCV_8$ treatment.

Treatment	Group Size	Number of Groups	Number of Periods	Private Values	Common Value Signals	Average Earnings
PV_2	2	9	10	U[201, 300]		CHF 38.30
PV_3	3	6	10	U[201, 300]		CHF 36.83
PV_8	8	6	10	U[201, 300]		CHF 46.89
$PVCV_2$	2	9	10	U[201, 300]	U{-25, 25}	CHF 24.60
$PVCV_3$	3	6	10	U[201, 300]	U{-25, 25}	CHF 30.32
$PVCV_8$	8	6	10	U[201, 300]	U{-25, 25}	CHF 34.99

Table 1: The experiments used a 2×3 between-subject design that varied the information/value structure, PV (private values) and PVCV (private and common values), and the group size $n = 2$, $n = 3$, and $n = 8$. The private values are uniformly distributed between 201 and 300 and the common value signals are equally likely to be $+25$ or -25. In the PV treatments, a trader's value is equal to her private value and in the PVCV treatments a trader's value is equal to her private value plus the sum of all common value signals in the group.

consisted of two unpaid practice periods followed by ten paid periods of 120 seconds each. The sessions lasted somewhere between 75 and 90 minutes, including instructions and payment. The exchange rate used in the experiment was 0.2, i.e. five experimental points equaled one Swiss Franc. Average earnings ranged from approximately 25 to 47 Swiss Francs depending on the treatment, see the final column in Table 1.

3. Theoretical Considerations

In Section 3.1 we establish that full allocative efficiency is possible with only private values. In other words, for all possible private value draws, it can be individually rational and incentive compatible for low-value traders to sell to the high-value traders.[9] Section 3.2 considers the case of private plus common-values. One benchmark results by simply imposing full informational efficiency. A second-best benchmark follows by considering what is possible if incentive compatibility and individual rationality are taken into account. In Section 3.3 we discuss the implications of these theoretical predictions for the different treatments.

[9]This possibility result is akin to the efficient dissolution of a partnership when initial property rights are non-extreme, see Cramton, Gibbons, and Klemperer (1987). It contrasts with the impossibility of efficient trade when property rights are extreme, i.e. when buyer and seller roles are fixed, as first shown by Myerson and Sattherwaite (1983).

3.1. Private Values

Recall that in our setup there are no fixed buyers and sellers: each market participant is endowed with one unit of the good and values at most two units. So each market participant can be a "trader" who, depending on the private value, can decide to become a net buyer or a net seller. We normalize traders' private values to lie between 0 and 1 by subtracting 200 from their private value draw and dividing the result by 100. Let $0 \leq v \leq 1$ denote the resulting uniform random variable with distribution $F(v) = v$. Assuming efficient trade, the expected amount bought by a trader with private value v is given by

$$P(v) \;=\; \sum_{k=1}^{n} \mathrm{sign}(2k-n-1)\binom{n-1}{k-1}F(v)^{k-1}(1-F(v))^{n-k} \tag{1}$$

where $P(v) \leq 0$ for $v \leq \frac{1}{2}$ corresponds to minus the probability that a trader with value v sells and $P(v) \geq 0$ for $v \geq \frac{1}{2}$ corresponds to the probability that a trader with value v buys. Below we refer to $P(v)$ as the *trade function.* The binomial terms in the sum on the right side of (1) represent the chance that for a trader with value v there are $n-k$ other traders with higher values and $k-1$ with lower values for $k = 1, \ldots, n$. Each term is weighted with a $+1$ or a -1 depending on whether the trader's value belongs to the top or bottom half of the values respectively,[10] which determines whether the trader should buy or sell.

A simple envelope theorem argument implies that a trader's equilibrium expected payoff satisfies $\pi'(v) = P(v)$, which can be integrated to yield

$$\pi(v) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} P(w)dw,$$

where $\pi(\frac{1}{2})$ is the expected payoff of the trader with the "worst" possible value $v = \frac{1}{2}$. Intuitively, a very low value is beneficial because the trader is likely to sell at a price substantially above her value. Likewise, a very high value is profitable when the trader can buy at a price much lower than her value. With a value of $\frac{1}{2}$ the trader is equally likely to be buy or sell at a price close to her value, resulting in a low payoff.

Efficiency, incentive compatibility, and individual rationality can co-exist if even a trader with the worst value has a non-negative expected payoff. The lowest payoff follows from the condition that the sum of traders' utilities is equal to the total surplus generated from reallocating

[10] And it is weighted with 0 if the trader has the median value in treatments with $n = 3$.

units from low to high-value traders:

$$\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, v_1 \leq \cdots \leq v_n) \;=\; n\int_0^1 \pi(v)dF(v).$$

A direct computation yields that the lowest payoff is positive in all PV treatments.[11]

Let $v_{[k]}$ for $k = 1,\ldots,n$ denote the sequence that results by rearranging the values in increasing order. In other words, $v_{[k]}$ is the k-th order statistic with $v_{[1]} \leq \ldots \leq v_{[n]}$. Furthermore, let $\text{Median}(v_1,\cdots,v_n)$ denote the median value, which is equal to $v_{[(n+1)/2]}$ for n odd and it is equal to $\frac{1}{2}(v_{[n/2]} + v_{[n/2+1]})$ for n even.

Proposition 1. *Incentive compatible, individually rational, fully efficient trade is possible in the private values treatments. The resulting gains from trade and market price are given by*

$$W \;=\; \sum_{k=1}^{n} \text{sign}(2k-n-1)v_{[k]} \tag{2}$$

$$p \;=\; \text{Median}(v_1,\cdots,v_n) \tag{3}$$

The proof of Proposition 1, which is contained in the Appendix, uses standard techniques to verify that, with private values only, there exists a fully efficient, individually rational, and incentive compatible mechanism. Proposition 1 does *not* imply that a particular mechanism, e.g. the continuous double auction, is fully efficient. Rather it provides a tough benchmark to which the performance of the continuous double auction can be compared. We next determine a similar benchmark for the case of private and common values.

3.2. Private plus Common Values

In the private plus common value treatments, trader $i = 1,\cdots,n$ receives an additional signal $\theta_i \in \{-25, 25\}$ about the common value, which is simply equal to the sum of all signals: $\Theta = \sum_{j=1}^{n}\theta_j$. Trader i's total value is thus given by

$$t_i \;=\; v_i + \sum_{j=1}^{n}\theta_j$$

Note that the common value term simply shifts all traders' private values by an equal amount, Θ. First, suppose that we simply *impose* full informational efficiency (FI) and simply assume that traders know the common value.

[11] $\pi(\frac{1}{2})$ is equal to $\frac{1}{12}$, $\frac{1}{12}$, and $\frac{187}{2304}$ in the PV_2, PV_3, and PV_8 treatments respectively.

Proposition 2. *Imposing the assumption of full informational efficiency, the predicted gains from trade and the market price are given by*

$$W_{FI} \;=\; \sum_{k=1}^{n} \text{sign}(2k-n-1)v_{[k]} \tag{4}$$

$$p_{FI} \;=\; \text{Median}(v_1, \cdots, v_n) + \Theta \tag{5}$$

This result follows simply from Proposition 1 since adding a known constant to traders' values does not change the total gains from trade nor traders' incentives. Importantly, Proposition 2 does *not* address whether full informational efficiency is compatible with traders' incentives constraints. We next show it is not and determine the appropriate second-best benchmark that is attainable by an incentive compatible, individually rational mechanism.

First, note that trader i's expected total value depends only on the summary statistic $\xi_i \equiv v_i + \theta_i$ since others' private values and common value signals are independently distributed. Hence, if $\xi_i = \xi_j$ then trader i and j have the same expected total value, even if their private values differ. As a result, bids and asks convey information about traders' summary statistics but not about their private and common value signals separately. It is easy to see how trading based on summary statistics could adversely affect allocative efficiency. Consider, for instance, the case when one trader has a private value of 270 but a negative common value signal while another trader has a private value of 230 with a positive common value signal. Trading based on summary statistics would result in a negative surplus of -40 while trading based on private values yields a positive surplus of $+40$.

Next, we verify that it can be incentive compatible and individually rational for traders with low summary statistics to sell to those with higher summary statistics, which leads to only partial information (PI) revelation. After normalization, the distribution of ξ is given by

$$G(\xi) \;=\; \tfrac{1}{2}F(\xi - \tfrac{1}{4}) + \tfrac{1}{2}F(\xi + \tfrac{1}{4})$$

with support $[-\frac{1}{4}, \frac{5}{4}]$. Incentive compatibility again implies that

$$\pi(\xi) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{\xi} P(\eta)d\eta,$$

where $P(\cdot)$ is defined as in (1) with $F(\cdot)$ replaced by $G(\cdot)$. Individual rationality is ensured if and only if $\pi(\frac{1}{2}) \geq 0$, where $\pi(\frac{1}{2})$ follows from the condition

$$\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, \xi_1 \leq \cdots \leq \xi_n) \;=\; n\int_{-\frac{1}{4}}^{\frac{5}{4}} \pi(\xi)dG(\xi).$$

A direct computation yields that the lowest payoff is positive in all PVCV treatments.[12] Let $v_{\xi[k]}$ denote the sequence that follows by ordering traders' private values according to their summary statistics, e.g. $v_{\xi[1]}$ is the private value of the trader with the lowest summary statistic and $v_{\xi[n]}$ is the private value of the trader with the highest summary statistic.

Proposition 3. *Trading based on summary statistics can be incentive compatible, individually rational, in the private plus common values treatment. The resulting gains from trade and the market price are given by*

$$W_{PI} = \sum_{k=1}^{n} \text{sign}(2k-n-1) v_{\xi[k]} \tag{6}$$

$$p_{PI} = \text{Median}(\xi_1, \cdots, \xi_n) \tag{7}$$

The proof of Proposition 3, which is contained in the Appendix, parallels that of Proposition 1. Importantly, the proposition provides a second-best benchmark of what efficiency level is possible in any incentive compatible, individually rational mechanism when both private and common values are present. Like Proposition 1, it does *not* imply that the continuous double auction necessarily attains the predicted efficiency level in (6).

We end this section by noting some features that are the same with two or three traders (or, more generally, with $2n$ and $2n+1$ traders), both with private values only and with private and common values. The reason is that in our setup with endogenous trading positions there will be one trader left out of the market when the number of traders is odd. For example, with three traders, the trader with the highest value buys from the trader with the lowest value and the trader with the middle value is left out. The outcome is the same with two traders since then the high-value trader simply buys from the low-value trader. In other words, the trade function in (1) should be the same with two and three traders.[13] Moreover, the per-capita surplus is the same with two and three traders,[14] and, hence, so are traders' equilibrium payoffs.[15]

Proposition 4. *The trade function $P(v)$ ($P(\xi)$) and the equilibrium payoffs $\pi(v)$ ($\pi(\xi)$) are the same in the private value (private plus common value) treatments with two and three traders.*

We next discuss the implications of these propositions for the experimental results.

[12] $\pi(\frac{1}{2})$ is equal to $\frac{1}{16}$, $\frac{1}{16}$, and $\frac{8237}{131072}$ in the PVCV_2, PVCV_3, and PVCV_8 treatments respectively.

[13] With two traders $P(v) = -(1-F(v)) + F(v) = 2F(v) - 1$ while with three traders $P(v) = -(1-F(v))^2 + F(v)^2 = 2F(v) - 1$. More generally, it is readily verified that (1) is the same with $2n$ and $2n+1$ traders.

[14] With $n = 2$ traders, the expected lowest and highest values are $\frac{1}{3}$ and $\frac{2}{3}$, so the per-capita surplus is $\frac{1}{6}$. With $n = 3$ traders, the expected lowest and highest values are $\frac{1}{4}$ and $\frac{3}{4}$, so the per-capita surplus is also $\frac{1}{6}$. More generally, with uniformly distributed values, the per-capita surplus is the same with $2n$ and $2n+1$ traders.

[15] See also Footnotes 11 and 13, which establish that $\pi(\frac{1}{2})$ is the same with two and three traders.

3.3. Hypotheses

As noted above, Proposition 1 does not necessarily imply that a particular mechanism, such as the continuous double auction, will be fully efficient. However, given its stellar performance in previous private-value experiments, it is natural to conjecture that it is.

Hypothesis PV:

(AE) The continuous double auction results in full allocative efficiency in the PV treatments independent of group size.

(IE) The continuous double auction results in full informational efficiency in the PV treatments independent of group size.

When common values are introduced, market performance is unaffected under the rational expectations (RE) model.

Hypothesis PVCV-RE:

(AE) The continuous double auction results in full allocative efficiency in the PVCV treatments independent of group size.

(IE) The continuous double auction results in full informational efficiency in the PVCV treatments independent of group size.

Under the private expectations model, there will be allocative inefficiencies since buy and sell orders are based on traders' summary statistics not their private values. The predicted fraction of the surplus that is lost is given by

$$\text{allocative efficiency loss} \; = \; \frac{W_{RE} - W_{PE}}{W_{RE}}$$

where W_{RE} and W_{PE} are defined in (4) and (6) respectively. The private expectations model also predicts that observed trade prices will differ from the correct ones, i.e. those predicted by the rational expectations model. Since prices can be too high or too low, we take the absolute value of the difference in predicted prices:

$$\text{informational efficiency loss} \; = \; |p_{RE} - p_{PE}|$$

where p_{RE} and p_{PE} are defined in (5) and (7) respectively.

It is straightforward to compute the predicted allocative and informational losses for the private and common values used in the experiments.[16]

Hypothesis PVCV-PE:

(AE) The introduction of common values causes an allocative efficiency loss of 35.6%, 22.0%, and 28.3% in the PVCV_2, PVCV_3, and PVCV_8 treatments respectively.

(IE) The introduction of common values causes an informational efficiency loss of 12.8, 27.8, and 55.6 in the PVCV_2, PVCV_3, and PVCV_8 treatments respectively.

Finally, Proposition 4 implies some similarities between the outcomes of the PV and PVCV treatments with two and three traders.

Hypothesis 2-3:

(PV) The observed trade and payoff functions are the same in the PV_2 and PV_3 treatments.

(PVCV) The observed trade and payoff functions are the same in the PVCV_2 and PVCV_3 treatments.

The hypothesis is stated in terms of functions, so applies to all private values (summary statistics) in the PV (PVCV) treatments. It does not imply that behavior in the two treatments is necessarily identical, but rather that the average amount bought by a trader with value v (summary statistic ξ) is the same and so is her payoff. The manner in which this comes about might be quite different in the two treatments. Intuitively, the treatments with three traders are more competitive since one trader will be left out, which likely affects behavior.

[16] The ex ante expected allocative loss is given by

$$1 - \frac{\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, \xi_1 \le \cdots \le \xi_n)}{\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, v_1 \le \cdots \le v_n)}$$

which equals 25.0%, 25.0%, and 26.7% for the PVCV_2, PVCV_3, and PVCV_8 treatments respectively. When testing Hypothesis (AE) we use the percentages listed in the main text to avoid rejecting the theory because of the random draws used in the experiment. Importantly, the ex ante expected loss is more or less independent of group size and does not vanish in the limit when n grows large: W_{RE} limits to $\frac{1}{2}n(E(v|v>\frac{1}{2})-E(v|v<\frac{1}{2}))=\frac{1}{4}n$ while W_{PE} limits to $\frac{1}{2}n(E(v|\xi>\frac{1}{2})-E(v|\xi<\frac{1}{2}))=\frac{3}{16}n$, so the expected allocative loss limits to 25%.

4. Results

We first discuss results pertaining to the allocative efficiency losses observed in the experiment (Section 4.1) and then discuss the informational efficiency losses (Section 4.2). In Section 4.3 we study the determinants of trade and Section 4.4 tests for equilibrium behavior.

4.1. Allocative Efficiency

The loss measures introduced in the previous section were constructed under the assumption that either the rational expectations model or the private expectations model applies. Of course, in the experiments neither one of them may be 100% correct. To measure deviations from either model without assuming that if one fails the other applies, we introduce the following loss measures:

$$\text{allocative efficiency loss RE} = \frac{1}{GT}\sum_{g=1}^{G}\sum_{t=1}^{T}\frac{W_{RE}^{g,t}-W_{obs}^{g,t}}{W_{RE}^{g,t}}$$

$$\text{allocative efficiency loss PE} = \frac{1}{GT}\sum_{g=1}^{G}\sum_{t=1}^{T}\frac{W_{PE}^{g,t}-W_{obs}^{g,t}}{W_{RE}^{g,t}}$$

with G the number of different groups per treatment (see Table 1), $T = 10$ the number of periods, and the t and g superscripts indicate that observed and predicted surpluses are determined for each group and each period separately.

The results are shown in the top panel of Figure 1. The top-left panel pertains to the PV treatments and the top-right panel to the PVCV treatments. Recall that in the PV treatments, there is no difference between the private and rational expectations models – both models predict full allocative efficiency. The non-negligible losses indicated by the bars in the top-left panel of Figure 1 suggest that this prediction is not borne out by the data. We test this and other hypotheses formally by running an OLS regression where the independent variable is the percentage efficiency loss for a group in a given period and the regressors are group size dummies (Two Traders, Three Traders, Eight Traders). The results are shown in the first two columns of Table 2. The top panel of Table 2 labeled "PV" shows that Hypothesis PV(AE) can be rejected: the allocative efficiency losses are 32.2%, 22.4%, and 12.8% for groups of size two, three, and eight respectively.

> **Result 1:** In the private values treatments, the continuous double auction results in significant allocative efficiency losses that are decreasing with group size.[17]

[17] The test results of Table 2 are corroborated by non-parametric tests. For example, a Kruskal-Wallis test rejects the hypothesis that allocative efficiency loss is independent of group size in the PV treatments ($p = 0.07$).

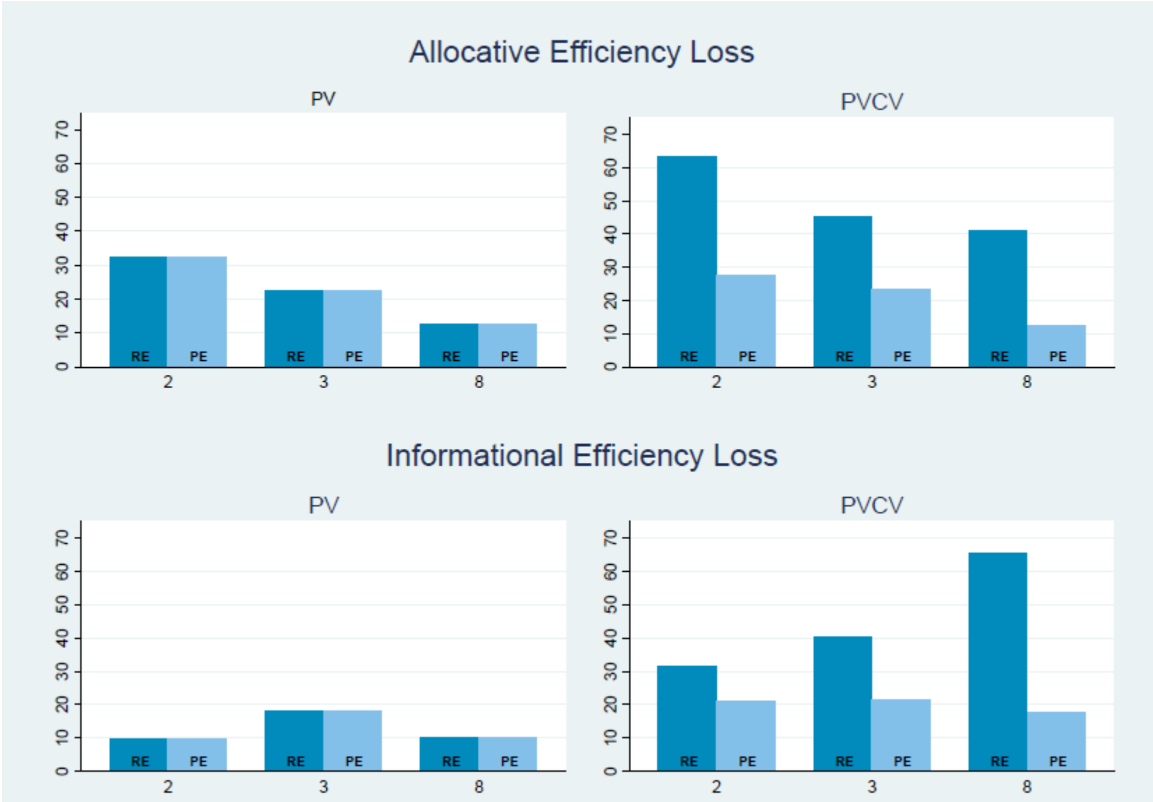

Figure 1: Allocative efficiency losses (top panels) and informational efficiency losses (bottom panels) in the different treatments. The two left panels pertain to the PV treatments and the two right panels to the PVCV treatments. The bars show the allocative and informational losses with respect to the private expectations model (light) or the rational expectations model (dark).

To understand why the CDA does not always yield efficient allocations in the PV treatments suppose the private value draws are such that many traders have low values. Then mostly sell orders will be submitted and a fully efficient outcome with some low-value traders buying may not materialize, especially when the total gains from trade are small.

The top-right panel of Figure 1 shows allocative efficiency losses when common values are introduced. The dark bars indicate that observed allocative efficiency losses are significant and substantial: 63.3%, 45.4%, and 40.9% for groups of size two, three, and eight respectively. Hypotheses PVCV-RE(AE) is also rejected.

> **Result 2:** In the private plus common values treatments, the continuous double auction results in significant allocative efficiency losses that are decreasing with group size.[18] However, losses remain substantial ($> 40\%$) even with eight traders.

[18] A Kruskal-Wallis test rejects the hypothesis that allocative efficiency loss is independent of group size in the PVCV treatments ($p = 0.016$).

OLS Regression: Efficiency Losses				
Model: Rational Expectations (RE) or Private Expectations (PE)				
	Allocative Loss		Informational Loss	
	RE	PE	RE	PE
PV				
Two Traders	32.22**	32.22**	9.68**	9.68**
	(5.49)	(5.49)	(1.10)	(1.10)
Three Traders	22.39**	22.39**	18.18**	18.18**
	(4.46)	(4.46)	(3.39)	(3.39)
Eight Traders	12.79**	12.79**	10.14**	10.14**
	(1.34)	(1.34)	(0.68)	(0.68)
PVCV - PV				
Two Traders	31.11**	-4.44	21.94**	11.30**
	(6.75)	(6.31)	(3.64)	(2.95)
Three Traders	23.01**	0.96	21.95**	3.08
	(7.21)	(8.48)	(6.99)	(4.84)
Eight Traders	28.09**	-0.23	55.43**	7.44*
	(6.03)	(4.40)	(4.35)	(3.22)
Observations	420	420	610	610
Log Likelihood	-2237	-2295	-2872	-2571
Number of Clusters	42	42	42	42
Competition Effects (F test)				
PV	7.59**	7.59**	2.88	2.88
PVCV	6.47**	4.25*	19.17**	0.43

Robust standard errors in parentheses, clustered by groups
** p<0.01, * p<0.05

Table 2: OLS regressions of allocative and informational efficiency losses on group size and common value dummies. The "PV" panel shows identical losses under the RE and PE models. The "PVCV−PV" panel shows the *additional* losses when common values are introduced. The "Competition Effects" in the bottom panel test whether efficiency losses are independent of group size.

Interestingly, comparing the light bars in the top-left and top-right panels of Figure 1 shows that the differences between observed allocative efficiencies and those predicted by the private expectations model are very similar for the PV and PVCV treatments. The PE column in the middle panel of Table 2 labeled "PVCV−PV" confirms this: none of the group size dummies that measure the difference between the PVCV and PV treatments are significant. This suggests that the PE model correctly describes how traders incorporate the additional common value information. To test this formally we compare the numbers in the RE column of the "PVCV−PV" panel of Table 2 to those in Hypothesis PVCV-PE(AE). The result is that Hypothesis PVCV-PE(AE) cannot be rejected.[19]

[19]An F-test whether the three predicted numbers (35.6%, 22.0%, 28.3%) are the same as the observed ones (31.1%, 23.0%, 28.1%) yields an insignificant test statistic $F(3, 41) = 0.15$, where 41 is the residual degrees of freedom given that we had 42 independent groups. These results can be corroborated by a non-parametric sign test based on 21 observed and predicted differences (between PV and PVCV).

Result 3: The increase in allocative efficiency losses when common values are introduced are correctly predicted by the private expectations model.

To summarize, the CDA results in allocative efficiency losses even with private values only, which is not predicted by either the private or the rational expectations model. Losses diminish with competition and are roughly 13% in large groups of eight traders, which is in line with results of previous studies.[20] When common values are introduced, losses are significantly higher and remain substantial ($> 40\%$) even with large groups. The *increase* in allocative efficiency losses are correctly predicted by the private expectations model. Averaging over all treatments, the observed increase in allocative efficiency loss is 27.9%, which is very close and not significantly different from the 28.5% increase predicted by the PE model.

4.2. Informational Efficiency

The informational efficiency loss measures for the RE and PE models are also defined at the group/period level and then averaged over all groups and periods:

$$\text{informational efficiency loss RE} = \frac{1}{GTJ}\sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{j=1}^{J}|p_{obs}^{g,t,j} - p_{RE}^{g,t}|$$

$$\text{informational efficiency loss PE} = \frac{1}{GTJ}\sum_{g=1}^{G}\sum_{t=1}^{T}\sum_{j=1}^{J}|p_{obs}^{g,t,j} - p_{PE}^{g,t}|$$

where J is the observed number of trades in a period.[21]

The results are shown in the bottom panel of Figure 1. As before, there is no difference between the predictions of the RE and PE models with private values only – both predict full informational efficiency. The bars in the left-bottom panel suggest otherwise. To test this formally consider the OLS regression results in the final two columns of Table 2. The top panel labeled "PV" shows that Hypothesis PV(IE) can be rejected: the informational efficiency losses are 9.7, 18.2, and 10.1 for groups of size two, three, and eight respectively.

Result 4: In the private values treatments, the continuous double auction results in significant informational efficiency losses for all group sizes.

[20] For instance, Cason and Friedman (1995) find in a setting with random values/costs and fixed buyer/seller roles that efficiency in the CDA averages 86% – 94% when there are eight to ten traders.

[21] Since subjects were endowed with one unit and valued at most two units, J is at most one when $n = 2, 3$ and J is at most four when $n = 8$. Periods for which the observed number of trades is zero are discarded when computing the informational efficiency losses.

While the observed deviations are statistically significant they seem small, especially taking into account that we used point predictions for the RE (or PE) model. In treatments with an even number of traders there typically is a range of possible equilibrium prices and we simply used the midpoint of that range.

The bottom-right panel of Figure 1 shows the informational efficiency losses when common values are introduced. The dark bars show that informational losses are now quite large: 31.6, 40.1, and 65.6 for groups of size two, three, and eight respectively. Also Hypothesis PVCV-RE(IE) is rejected.

> **Result 5:** In the private plus common values treatments, the continuous double auction results in significant informational efficiency losses for all group sizes. Losses are especially large with eight traders.

The light bars in the bottom-right panel in Figure 1 are higher than those in the bottom-left panel, indicating that the introduction of common values results in larger deviations from the private expectations model (see also the significant dummies in the PE column of the "PVCV−PV" panel in Table 2). This is somewhat intuitive in that the introduction of additional common value signals results in a more noisy environment with more volatile prices. We next test Hypothesis PVCV-PE(IE).[22]

> **Result 6:** The increase in informational efficiency losses when common values are introduced are correctly predicted by the private expectations model.

Observed prices differ from rational expectations predictions especially in large groups, see Result 5. To understand why this is the case, recall that the predicted price under the rational expectations model is the median private value plus the sum of all common value signals. Under the private expectations model the predicted price is equal to the median summary statistic, which consists of a private value and a *single* common value signal. As a result, the difference between the predictions of the rational and private expectations models grows (roughly) linearly with the size of the common value.

The dashed line in Figure 2 shows this difference in predictions based on the draws used in the experiment. The "V" shape confirms the above argument that the difference grows

[22] An F-test whether the three predicted numbers (12.8, 27.8, 55.6) are the same as the observed ones (21.9, 22.0, 55.4), see the RE column of the "PVCV−PV" panel in Table 2, yields an insignificant test statistic $F(3, 41) = 2.34$ with a p-value of 0.09 (the difference is mainly driven by the $n = 2$ treatment). A non-parametric sign test based on 21 observed and predicted differences (between PV and PVCV) also does not yield a significant difference (p-value of 0.66).

Figure 2: The dashed line shows the difference between rational expectations and private expectations predictions. The thick red line shows observed deviations from rational expectations predictions and the thin blue line shows observed deviations from private expectations predictions.

linearly with the magnitude of the common value. Figure 2 also shows the difference between observed prices and predictions of the private expectations model (thin blue line) and the rational expectations model (thick red line). The thin blue line is more or less flat at a height of 12.5, indicating there are small deviations from the private expectations model (see also Table 2) but these deviations are independent of the common value. In contrast, the thick red line shows that deviations from the rational expectations model grow with the size of the common value. These findings complement Result 6.

> **Result 7:** Price deviations from rational expectations predictions are increasing in the size of the common value as predicted by the private expectations model.

4.3. Determinants of Trade

The individual trade data allow us to estimate the relative weight that subjects place on their common value signal vis-à-vis their private signal. The private expectations model predicts this weight to be 1 while full allocative efficiency requires this weight to be 0. Specifically, we run the ordered Pobit regression

$$Y_j = \sum_{n=2,3,8} (v_j - \tfrac{1}{2})\beta_n^{PV} d_n^{PV} + \sum_{n=2,3,8} (v_j + \alpha_n\theta_j - \tfrac{1}{2})\beta_n^{PVCV} d_n^{PVCV} + \varepsilon_j$$

Ordered Probit Regression: Trade (Y = -1, 0, 1)	(a)	(b)
PV		
Two Traders	2.80**	
	(0.34)	
Three Traders	2.52**	
	(0.32)	
Two & Three Traders		2.66**
		(0.23)
Eight Traders	4.36**	4.36**
	(0.26)	(0.26)
PVCV		
Two Traders	1.43**	
	(0.31)	
Three Traders	1.91**	
	(0.31)	
Two & Three Traders		1.65**
		(0.22)
Eight Traders	2.55**	2.57**
	(0.21)	(0.21)
Relative Weight of CV Signal		
Two Traders	1.31**	
	(0.34)	
Three Traders	0.88*	
	(0.35)	
Eight Traders	0.87**	
	(0.22)	
Two & Three & Eight Traders		0.97**
		(0.16)
Observations	1,680	1,680
Log Likelihood	-1472	-1474
Cut points	(-0.47, 0.47)	(-0.47, 0.47)
Model (a) versus Model (b)		
Likelihood-ratio test		3.18

** p<0.01, * p<0.05

Table 3. Ordered Probit regression with trade ($Y = -1$ for sell, $Y = 0$ for no trade, and $Y = 1$ for buy) as the dependent variable and treatment dummies as regressors. The test in the bottom panel shows that trade is the same in treatments with two and three traders and that the weight on the common value signal is the same for all group sizes. The weight is not significantly different from 1.

where Y_j is -1, 0, or $+1$ when the trader sold, did not trade, or bought respectively, the d's are dummy variables that are 1 for the relevant treatment and 0 otherwise, and v_j and θ_j are the trader's private information. Finally, the α_n measure the relative weights placed on the common value signal in each of the three PVCV treatments.

The results are shown in the column labeled "(a)" in Table 3. The column labeled "(b)" shows a reduced model in which the dummies for the treatments with two and three traders are forced to be the same and the weight placed on the common value signal is forced to be the same for all group sizes. This model fits equally well, see the likelihood-ratio test in the bottom panel.

Result 8: The trade function is the same in treatments with group size two or three but is more responsive to the private value/summary statistic in treatments with a group size of eight.[23]

That the trade functions are the same with two or three traders does not imply that behavior in these treatments is the same. Figure 3 shows the evolution of realized surplus in the different treatments by blocks of 15 seconds (there are eight such blocks since the period lasted for two minutes). Obviously, there is more of a "hold out" problem in the treatment with two traders where most of the surplus is realized in the final 30 seconds. In contrast, in the private values treatment with three traders almost all surplus is realized in the first 45 seconds. Note that for all group sizes the introduction of common values shifts trades towards the second half of the period as traders become more cautious.

Result 9: The hold out problem is more severe with two traders and is exacerbated by the introduction of common values.

The extent to which there was a hold out problem in the different treatments can also be measured by comparing the two possible sources of inefficiencies: missing trades or suboptimal trades. The forgone surplus in the PV_2 treatment is mainly due to missing trades (92.6%) and rarely due to wrong trades (7.4%) that occur when a high-value trader sells to a low-value trader. In contrast, there are no missing trades in the PV_3 treatment where the entire loss in surplus is due to suboptimal trades, e.g. the high-value trader buying from trader with the medium value. When common values are introduced, the loss due to wrong trades more than doubles to 18.8% in $PVCV_2$ while in $PVCV_3$ the loss from missing trades jumps to 23.3%.[24] Despite the different sources of inefficiencies, the average amount bought or sold by a trader of a certain type is the same with two and three traders (Result 8).

Importantly, the estimation results in Table 3 show that the relative weight placed on the common value signal is independent of group size and not significantly different from one, providing additional support for the private expectations model.

Result 10: The relative weight placed on the common value signal is not significantly different from 1.

[23] A χ^2-test whether $\beta_{2,3} = \beta_8$ is rejected at a p-value less than 0.0001 for the PV treatments and it is rejected at a p-value of 0.002 for the PVCV treatments.

[24] In the PV_8 treatment, 19.7% of the loss is because of missing trades and 80.3% due to suboptimal trades. In the $PVCV_8$ treatment, the loss due to missing trades is 30.8% and due to suboptimal trades is 69.2%.

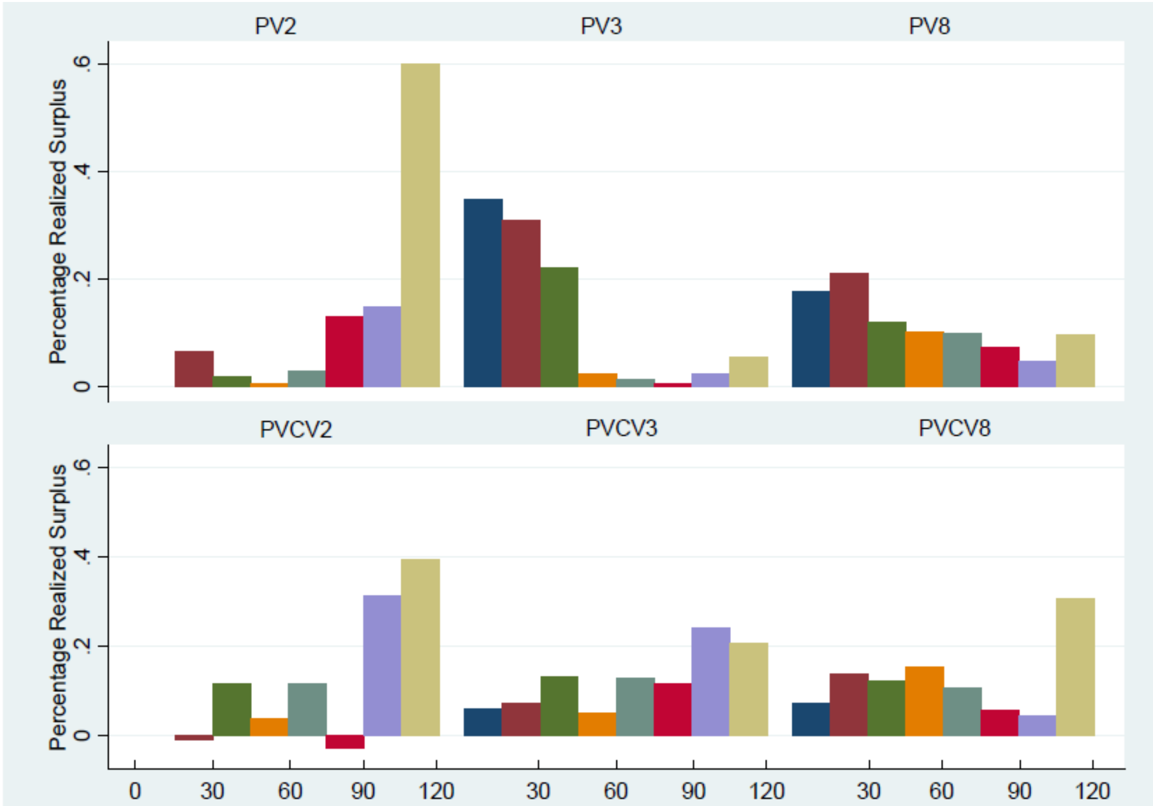

Figure 3: Evolution of surplus by blocks of 15 seconds in each of the treatments. The bars indicate the fraction of the total surplus that was realized in each time block.

The estimation results in the final column of Table 3 can be used to construct the empirical analogues of (1), i.e. the expected amount bought by a trader with value v in the PV treatments or summary statistic ξ in the PVCV treatments. Importantly, the two cut-points produced by the ordered Probit regressions are located symmetrically around zero: the first cut-point is at $-c = -0.47$ and the second one is at $c = 0.47$, see Table 3. This implies that the empirical analogues of (1) are *anti*-symmetric around $v = \frac{1}{2}$ ($\xi = \frac{1}{2}$) for the PV (PVCV) treatments:[25]

$$P_{obs}(v) = \Phi(\beta(v - \tfrac{1}{2}) - c) + \Phi(\beta(v - \tfrac{1}{2}) + c) - 1$$

$$P_{obs}(\xi) = \Phi(\beta(\xi - \tfrac{1}{2}) - c) + \Phi(\beta(\xi - \tfrac{1}{2}) + c) - 1$$

The empirical trade functions are shown by the orange lines in Figure 4. The top panels pertain to the PV treatments with $n = 2, 3$ pooled on the left and $n = 8$ on the right. The bottom panels pertain to the PVCV treatments. Figure 4 also displays the observed average amount bought (plus or minus one standard deviation) according to private values or summary

[25]In the PV treatments $\beta = 2.66$ when $n = 2, 3$ and $\beta = 4.36$ when $n = 8$, and in the PVCV treatments, $\beta = 1.65$ when $n = 2, 3$ and $\beta = 2.57$ when $n = 8$, see Table 3. It is readily verified that $P_{obs}(v) = -P_{obs}(1 - v)$ and $P_{obs}(\xi) = -P_{obs}(1 - \xi)$ for all v, ξ.

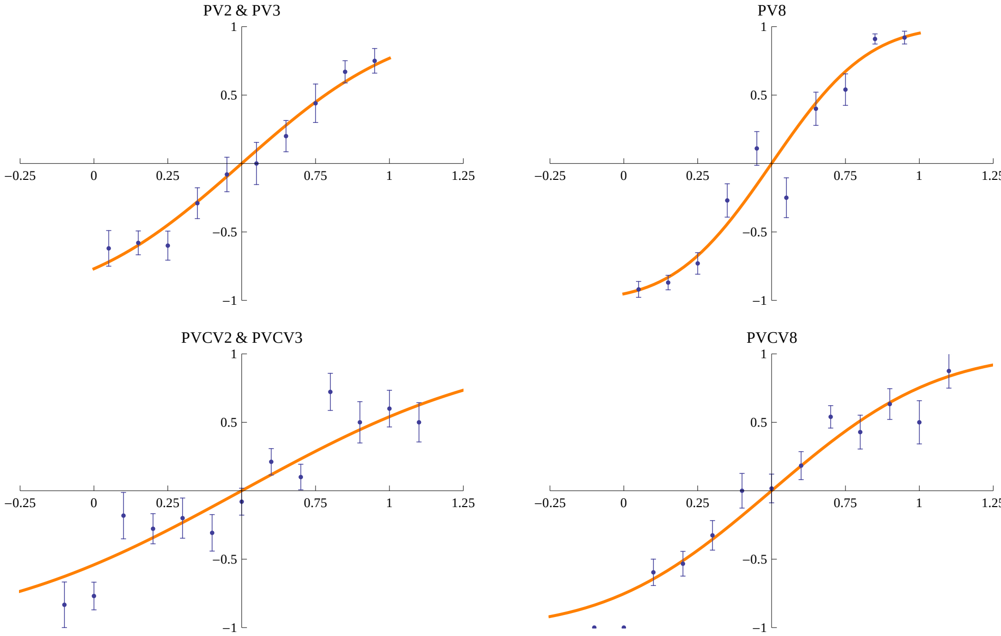

Figure 4: The orange lines show the estimated trade function for a trader with value v in the PV treatments (top panels) or a trader with summary statistic ξ in the PVCV treatments (bottom panels). The estimated lines are based on the ordered Probit regressions reported in Table 3. The data points with error bars indicate the average observed amount bought (plus or minus one standard deviation) for private values (top) or summary statistics (bottom) that are categorized by bins of size 10.

statistics, which are grouped in bins of size 10. While there are some discrepancies between the observed and estimated amounts bought (in particular for the PVCV treatments), the ordered Probit regressions of Table 3 result in a good fit of the observed trade functions.

4.4. Testing for Equilibrium Behavior

There does not exist a complete description of equilibrium behavior for the dynamic continuous-time double auction where players have private information and can move at unspecified times. However, an indirect test follows from the observation that incentive compatibility, or equilibrium behavior, implies that $\pi'(v) = P(v)$. Using the empirical trade functions derived above we can test for equilibrium behavior by comparing observed payoffs with those that follow from this incentive compatibility condition. In particular, the predicted payoffs are given by

$$\pi_{obs}(v) \;=\; \pi_{obs}(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} P_{obs}(w)dw,$$

where $\pi_{obs}(\frac{1}{2})$ follows from the condition

$$W_{obs} \;=\; n\int_0^1 \pi_{obs}(v)dF(v)$$

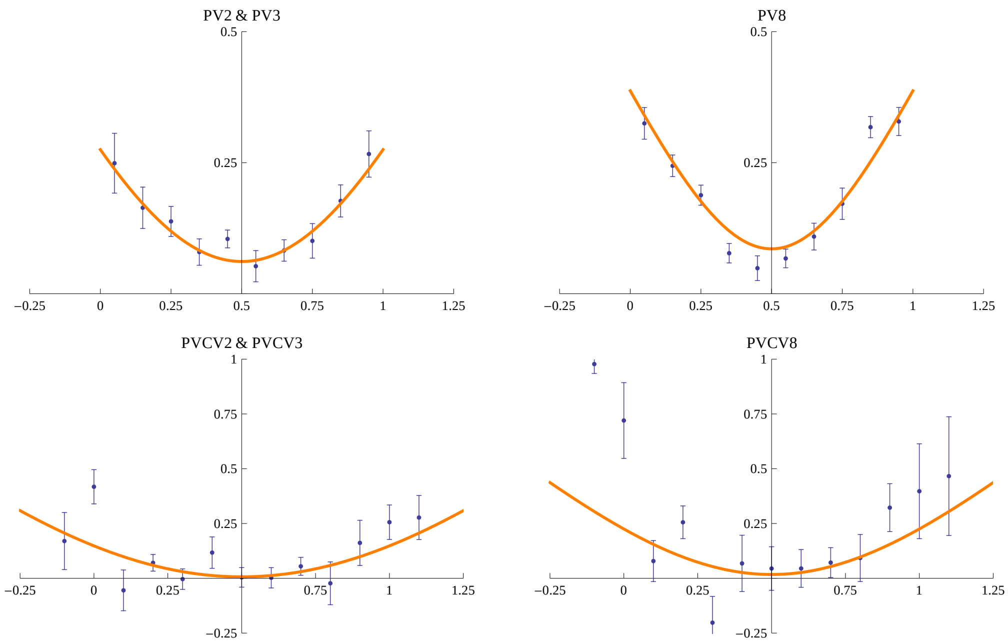

Figure 5: The orange lines show the estimated payoffs of a trader with value v in the PV treatments (top panels) or a trader with summary statistic ξ in the PVCV treatments (bottom panels). The estimated lines are based on the empirical trade functions of Figure 3. The data points with error bars indicate the average observed payoff (plus or minus one standard deviation) for private values (top) or summary statistics (bottom) that are categorized by bins of size 10.

and W_{obs} is the observed surplus in the relevant PV treatment. Analogous expressions for the PVCV treatments follow by replacing the private value v with the summary statistic ξ and $F(v)$ by $G(\xi)$. We can combine the treatments with two and three traders if the observed per-capita surplus is the same in these treatments. With only private values this is the case.[26] With private and common values, the difference in per-capita surplus is only marginally significant.[27] We therefore decided to combine the $n = 2$ and $n = 3$ treatments, also to be able to present the estimated payoff results in a manner parallel to Figure 4.

The orange lines in the top panels of Figure 5 show the results for the PV treatments with $n = 2, 3$ pooled on the left and $n = 8$ on the right. The lines in the bottom panels show analogous results for the PVCV treatments. The fit for the private value treatments is nearly perfect. For the PVCV treatment, observed payoffs are more volatile and there are deviations from theoretical predictions for extreme levels of the summary statistics. Overall the fit is good.

[26]For PV_2 the per-capita surplus is 14.1 (1.0) and for PV_3 it is 13.4 (0.7), where the number in parentheses denotes the standard error based on 180 observations. A simple t-test cannot reject that the per-capita surplus numbers are the same (p-value is 0.55).

[27]For $PVCV_2$ the per-capita surplus is 7.3 (1.2) and for $PVCV_3$ it is 10.1 (0.8), where the number in parentheses denotes the standard error based on 180 observations. A t-test marginally rejects that the per-capita surplus numbers are the same at the 5% level (p-value is 0.046).

Result 11: Observed payoffs are close to their predicted equilibrium levels.

Together, Results 8, 10, and 11 show that Hypothesis 2-3 cannot be rejected and suggest that observed behavior is close to the equilibrium of the private expectations model.

5. Conclusions

Vernon Smith (2010) reviews the remarkable effectiveness of the continuous double auction to produce competitive equilibrium outcomes in market experiments that employ private values. He notes that despite this empirical success there exists no complete game theoretic explanation: "we cannot model and predict what are subjects routinely accomplish" Smith (2010, p.5). The results reported in this paper warrant a different conclusion. The private expectations model correctly predicts the drop in allocative efficiency when common values are introduced (Result 3), correctly predicts the increase in informational inefficiency (Results 6 and 7), and correctly predicts trade and payoff functions (Results 8, 10, and 11).

While these findings form three reasons to cheer for theory their empirical implications are devastating. In the presence of private and common values, continuous double auction markets result in substantial allocative losses (even with large groups) and prices differ markedly from their rational expectations levels. Observed behavior reveals that traders weigh their private and common value information equally as dictated by incentive compatibility. As a result, allocative and informational inefficiencies are predicted to occur. The experimental results confirm this "inefficient market hypothesis."

One might argue that real markets are larger and information structures more complex. But recall from Section 3 that the inefficiencies that occur with private and common values remain when the number of traders grows large. And while the information technology employed in the experiment was deliberately designed to be simple, all that is needed for inefficiencies to arise more generally is that both private and common values matter. The experiments convincingly demonstrate that subjects are able to combine both pieces of information in an incentive compatible manner. Surely, real traders in real markets will be able to do so too. The consequence is that real markets will be inefficient.

A. Appendix: Proofs

Proof of Proposition 1. Without loss of generality consider a direct mechanism in which traders report values (not necessarily truthfully). The direct mechanism consists of a trade functions, $p_i : [0,1]^n \to [-1,1]$, representing the probability trader i gets a unit (when positive) or gives a unit (when negative) given a profile of reports $(v_1, \ldots, v_n) \in [0,1]^n$, and payment functions $t_i : [0,1]^n \to \mathbb{R}$, which is how much trader i pays (can be positive or negative). The market does not absorb any units of the good nor any money so $\sum_i p_i(v_1, \ldots, v_n) = \sum_i t_i(v_1, \ldots, v_n) = 0$. From trader i's point of view the expected number of units she trades is $P_i(v_i) = E_{v_{-i}}(p_i(v_i, v_{-i}))$ and her expected payment is $T_i(v_i) = E_{v_{-i}}(t_i(v_i, v_{-i}))$ when others report truthfully. As in the rest of the paper, we assume a symmetric setting so that we can drop the player-specific subscripts and simply write $P(v)$ and $T(v)$.

A trader's expected payoff when her true value is v and she reports $\hat{v}$ is

$$\pi(v, \hat{v}) \;=\; vP(\hat{v}) - T(\hat{v})$$

Incentive compatibility requires that if a trader's true value is v she should want to report v and not $\hat{v}$:

$$vP(v) - T(v) \;\geq\; vP(\hat{v}) - T(\hat{v})$$

Likewise, if a trader's true value is $\hat{v}$ then she should want to report $\hat{v}$ and not v

$$\hat{v}P(\hat{v}) - T(\hat{v}) \;\geq\; \hat{v}P(v) - T(v)$$

Together they imply

$$\hat{v}(P(v) - P(\hat{v})) \;\leq\; T(v) - T(\hat{v}) \;\leq\; v(P(v) - P(\hat{v}))$$

As usual, incentive compatibility requires that $P(v)$ is increasing. Furthermore, by considering values that are close, i.e. $\hat{v} = v - \varepsilon$ and taking a Taylor approximation, we get $T'(v) = vP'(v)$, which can be integrated to

$$T(v) \;=\; T(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} yP'(y)dy \;=\; T(\tfrac{1}{2}) + vP(v) - \tfrac{1}{2}P(\tfrac{1}{2}) - \int_{\frac{1}{2}}^{v} P(y)dy$$

A trader's expected equilibrium payoff is thus[28]

$$\pi(v) \;=\; vP(v) - T(v) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} P(y)dy \tag{A.1}$$

Importantly, a trader's expected equilibrium payoff is, up to a constant, determined by the trade function.

Efficient trade is possible if and only if the equilibrium payoffs in (A.1) are non-negative for all values when $P(v)$ is given by the efficient trade function in (1). Since $P(v) > 0$ for $v > \frac{1}{2}$ and $P(v) < 0$ for $v < \frac{1}{2}$, the trader with value $\frac{1}{2}$ is worst off so individual rationality is satisfied if and only if $\pi(\frac{1}{2}) \geq 0$. To determine $\pi(\frac{1}{2})$, note that under efficient trade, those with high

[28] This follows more simply from an envelope theorem argument, i.e. $\pi'(v) = P(v)$, see the main text.

values buy from those with low values, e.g. if there are four bidders with values $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ then the total gains from trade are simply $+\frac{4}{5}+\frac{3}{5}-\frac{2}{5}-\frac{1}{5}$. More generally, if we order values in increasing order, i.e. $v_{[1]} \leq \ldots \leq v_{[n]}$, the total gains from trade are

$$W \;=\; \sum_{k=1}^{n} \text{sign}(2k-n-1)v_{[k]}$$

where $\text{sign}(x) = -1$ for $x < 0$, $\text{sign}(x) = 0$ for $x = 0$, and $\text{sign}(x) = 1$ for $x > 0$. The worst payoff, $\pi(\frac{1}{2})$, now follows by equating the sum of traders' ex ante expected payoffs to the ex ante expected total gains from trade

$$\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, v_1 \leq \cdots \leq v_n) \;=\; n\int_0^1 \pi(v)dF(v),$$

This yields $\pi(\frac{1}{2}) = \frac{1}{12}$ for $n = 2, 3$ and $\pi(\frac{1}{2}) = \frac{187}{2304}$ for $n = 8$. In other words, in the private values treatments, incentive compatible, individual rational, efficient trade is possible. *Q.E.D.*

Proof of Proposition 3. Consider, as in the proof of Proposition 1, a direct mechanism where traders report (not necessarily truthfully). Trader i with value v_i and common-value signal θ_i, and, hence, summary statistic $\xi_i = v_i + \theta_i$, faces the maximization problem

$$\max_{r_i} E_{v_{-i},\theta_{-i}}\left((\xi_i + \sum_{j\neq i}\theta_j)p_i(r_i, r_{-i}(v_{-i},\theta_{-i})) - t_i(r_i, r_{-i}(v_{-i},\theta_{-i}))\right)$$

where r_i is trader i's report and r_{-i} those of others. Note that the maximization problem takes the form

$$\max_{r_i}\left(\xi_i \times \text{function}(r_i) + \text{another function}(r_i)\right)$$

and the first-order condition for profit maximization will determine the report r_i as a function of ξ_i. Next, we replicate the steps of Proposition 1, replacing reports based on values by reports based on summary statistics. This yields

$$\pi(\xi) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{\xi} P(\eta)d\eta,$$

Truthful revelation of the summary statistic is possible if this payoff is non-negative for all values of ξ, in particular, $\xi = \frac{1}{2}$. Now the total gains from trade are

$$W \;=\; \sum_{k=1}^{n} \text{sign}(2k-n-1)v_{\xi[k]}$$

and the worst payoff again follows by equating the sum of traders' ex ante expected payoffs to the ex ante expected total gains from trade

$$\sum_{k=1}^{n} \text{sign}(2k-n-1)E(v_k \,|\, \xi_1 \leq \cdots \leq \xi_n) \;=\; n\int_{-\frac{1}{4}}^{\frac{5}{4}} \pi(\xi)dG(\xi).$$

This yields $\pi(\frac{1}{2}) = \frac{1}{16}$ for $n = 2, 3$ and $\pi(\frac{1}{2}) = \frac{8237}{131072}$ for $n = 8$. *Q.E.D.*

Proof of Proposition 4. We consider here the private values case – the proof for the private plus common values case is similar. First, note that while the total gains from trade are larger with three bidders, the per-capita surplus is the same. In detail, with uniformly distributed values and $n = 2$, the highest order statistic is at $2/3$ and the lowest order statistic is at $1/3$. So the total gains from trade is $1/3$, and the per-capita surplus is $1/2 \times 1/3 = 1/6$. With $n = 3$ the highest order statistic is $3/4$ and the lowest order statistic is $1/4$, so the total gains from trade are $1/2$ and the per-capita surplus is $1/3 \times 1/2 = 1/6$.

Next, for $n = 2$ and uniform values, the efficient trade function is $P(v) = 2v - 1$. (Since a trader with value v buys with chance v and sells with chance $1 - v$, so the expected number of units she buys is $v(+1) + (1 - v)(-1) = 2v - 1$.) The expected equilibrium payoff of a trader with value v is then

$$\pi(v) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} P(y)dy \;=\; \tfrac{1}{12} + (v - \tfrac{1}{2})^2$$

where $\pi(\frac{1}{2}) = \frac{1}{12}$ was determined in the proof of Proposition 1. For $n = 3$ the trade function is $P(v) = v^2(+1) + (1 - v)^2(-1) = 2v - 1$, so

$$\pi(v) \;=\; \pi(\tfrac{1}{2}) + \int_{\frac{1}{2}}^{v} P(y)dy \;=\; \tfrac{1}{12} + (v - \tfrac{1}{2})^2$$

with $\pi(\frac{1}{2}) = \frac{1}{12}$ determined in the proof of Proposition 1. So the trade functions and the expected equilibrium payoffs are the same for $n = 2, 3$. *Q.E.D.*

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