Challenging Question 21

Fractions • Ratio

There were some ten-cent coins and fifty-cent coins in a money pouch. The number of ten-cent coins in the money pouch was $\frac{1}{2}$ of the number of fifty-cent coins. Julia took out 5 fifty-cent coins and exchanged them for ten-cent coins of equivalent value. After that, she put the coins back into the money pouch. The number of fifty-cent coins then became $\frac{5}{8}$ of the number of ten-cent coins. How much money was there in the money pouch?

Put on your thinking cap and think of how you would attempt to solve this problem.
You may use the space here to write down your thoughts.

Try solving it now.
You may use the space here to write down your workings.

Solution 21

Tip

We notice that the ratio of the number of ten-cent coins to the number of fifty-cent coins has changed after some fifty-cent coins were exchanged for ten-cent coins of equivalent value. Hence, we first need to find the number of ten-cent coins added into the money pouch.

We use ratio to present the solution. We notice that everything has changed – there is no constant part, constant total or constant difference. Hence, it is necessary to use units and parts to show the quantities before and after.

Value of 5 fifty-cent coins $= 5 \times \$0.50 = \$2.50$

Number of ten-cent coins to make up \$2.50 $= \dfrac{\$2.50}{\$0.10} = 25$

Before

Ten-cent coins	:	Fifty-cent coins
1	:	2

After

Ten-cent coins	:	Fifty-cent coins
8	:	5

	Ten-cent coins	Fifty-cent coins
Before	1 unit	2 units
Change	+ 25	– 5
After	8 parts	5 parts

$1 \text{ unit} + 25 \text{ coins} = 8 \text{ parts}$
$2 \text{ units} + 50 \text{ coins} = 16 \text{ parts}$
$2 \text{ units} = 16 \text{ parts} - 50 \text{ coins}$

$2 \text{ units} - 5 \text{ coins} = 5 \text{ parts}$
$2 \text{ units} = 5 \text{ parts} + 5 \text{ coins}$

$16 \text{ parts} - 50 \text{ coins} = 5 \text{ parts} + 5 \text{ coins}$
$11 \text{ parts} = 55 \text{ coins}$
$1 \text{ part} = 55 \div 11 = 5 \text{ coins}$

$8 \text{ parts} = 8 \times 5 = 40 \text{ coins}$
$5 \text{ parts} = 5 \times 5 = 25 \text{ coins}$

Value of coins in the money pouch $= (40 \times \$0.10) + (25 \times \$0.50) = \$4 + \$12.50 = \$16.50$

There was \$16.50 in the money pouch.

Working backwards to check your answer

Number of ten-cent coins before the exchange $= 40 - 25 = 15$

Value of ten-cent coins before the exchange $= 15 \times \$0.10 = \$1.50$

Number of fifty-cent coins before the exchange $= 25 + 5 = 30$

Value of fifty-cent coins before the exchange $= 30 \times \$0.50 = \$15$

Before the exchange,

Number of ten-cent coins : Number of fifty-cent coins $= 15 : 30 = 1 : 2$ (checked)

Total value of coins before the exchange $= \$1.50 + \$15 = \$16.50$ (checked)

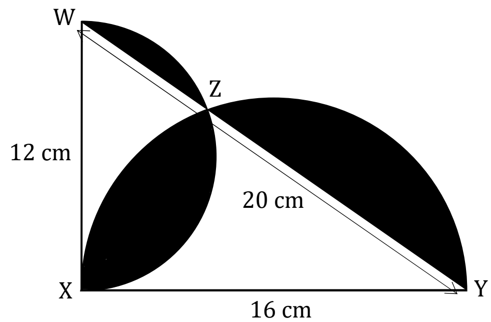
Challenging Question 38

Circles

The figure below is made up of a right-angled triangle WXY and two semicircles with XY and WX as their diameters respectively. The two semicircles and the line WY meet at Z. $WX = 12\text{ cm}$, $XY = 16\text{ cm}$ and $WY = 20\text{ cm}$. Taking $\pi = 3.14$, find

(a) the perimeter of the shaded region

(b) the area of the shaded region



Put on your thinking cap and think of how you would attempt to solve this problem.
You may use the space here to write down your thoughts.

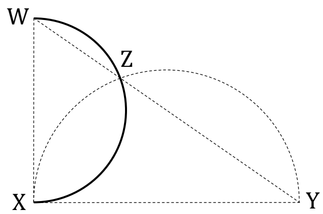
Try solving it now.
You may use the space here to write down your workings.

Solution 38

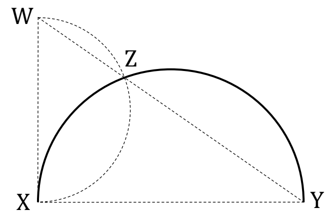
(a)

Tip

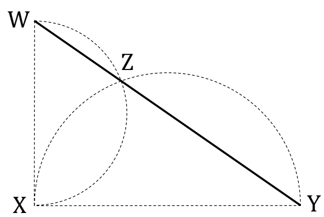
Visualise the shaded region as enclosed by two semicircles and a straight line.



The solid line shows the first semicircle enclosing parts of the shaded region.



The solid line shows the second semicircle enclosing parts of the shaded region.



The solid line shows the straight line enclosing parts of the shaded region.

Perimeter of the shaded region $= \left(\frac{1}{2}\times\pi\times 12\right) + \left(\frac{1}{2}\times\pi\times 16\right) + 20 = 6\pi + 8\pi + 20 = 14\pi + 20$

$= (14\times 3.14) + 20 = 63.96\,\text{cm}$

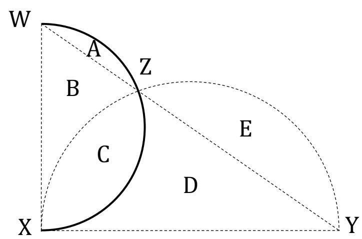
The perimeter of the shaded region is 63.96 cm.

(b)

Tip

Area of the shaded region
= Area of smaller semicircle + Area of bigger semicircle – Area of triangle

To visualize this, consider the figure to be made up of Parts A, B, C, D and E as shown.



Area of smaller semicircle = Area A + Area B + Area C
Area of bigger semicircle = Area C + Area D + Area E
Area of triangle = Area B + Area C + Area D

Area of smaller semicircle + Area of bigger semicircle – Area of triangle
= (Area A + Area B + Area C) + (Area C + Area D + Area E) – (Area B + Area C + Area D)
= Area A + Area C + Area E
= Area of the shaded region

Radius of the smaller semicircle $= 12 \div 2 = 6$ cm

Radius of the bigger semicircle $= 16 \div 2 = 8$ cm

Area of the shaded region $= \left(\frac{1}{2} \times \pi \times 6 \times 6\right) + \left(\frac{1}{2} \times \pi \times 8 \times 8\right) - \left(\frac{1}{2} \times 12 \times 16\right) = 18\pi + 32\pi - 96$

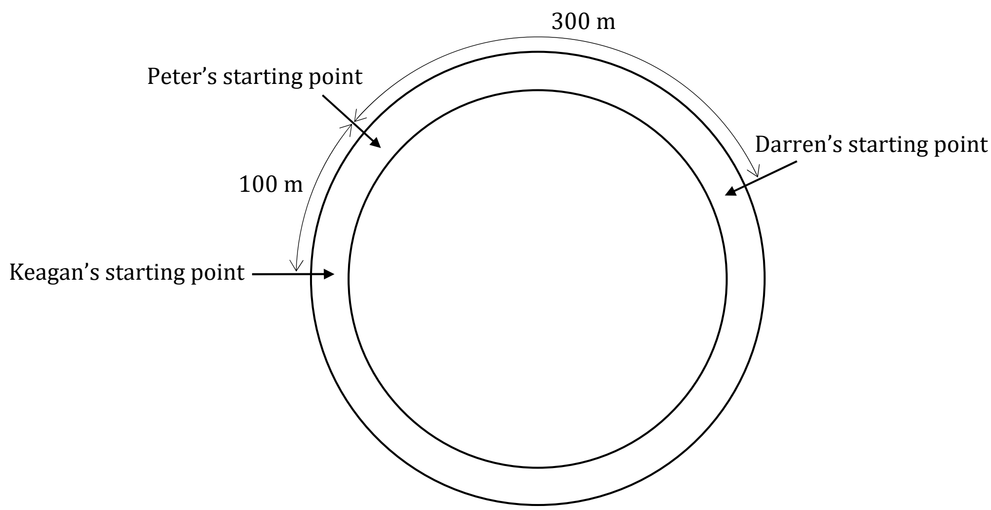
$= 50\pi - 96 = (50 \times 3.14) - 96 = 61\text{ cm}^2$

The area of the shaded region is 61 cm^2.

Challenging Question 52

Speed

Keagan, Peter and Darren took part in a race. They were each positioned at different starting points along a circular track at the start of the race. Darren was 300 m ahead of Peter and Peter was 100 m ahead of Keagan. All of them had to run in the clockwise direction. They started the race at 8 a.m. Keagan overtook Peter in 2 minutes. In another 2 minutes, Keagan overtook Darren. If Peter's speed was 140 m/min, at what time did Peter overtake Darren?



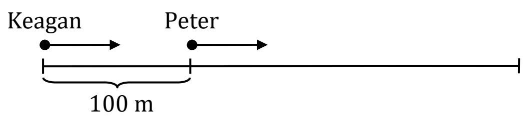
Put on your thinking cap and think of how you would attempt to solve this problem.
You may use the space here to write down your thoughts.

Try solving it now.
You may use the space here to write down your workings.

Solution 52

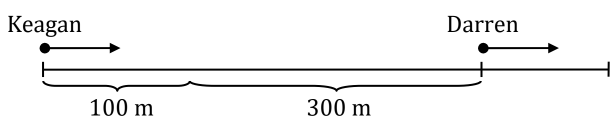
Tip

It may seem difficult to examine a situation with 3 subjects moving at the same time. The trick is to examine only 2 subjects at a time. Drawing diagrams to show the relative positions of the subjects will be very helpful.



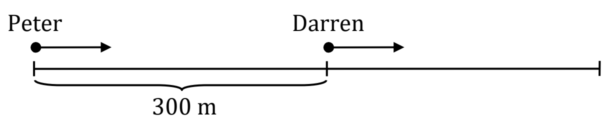
Difference between Keagan's speed and Peter's speed $= \dfrac{100}{2} = 50\text{ m/min}$

Keagan's speed $= 140 + 50 = 190\text{ m/min}$



Difference between Keagan's speed and Darren's speed $= \dfrac{100+300}{2+2} = 100\text{ m/min}$

Darren's speed $= 190 - 100 = 90\text{ m/min}$



Difference between Peter's speed and Darren's speed $= 140 - 90 = 50\text{ m/min}$

Time taken for Peter to overtake Darren $= \dfrac{300}{50} = 6\text{ min}$

Peter overtook Darren at 8.06 a.m.

<u>Working backwards to check your answer</u>

Distance ran by Darren in 6 minutes $= 90 \times 6 = 540$ m
Distance ran by Peter in 6 minutes $= 140 \times 6 = 840$ m
Distance between Darren and Peter at the start $= 840 - 540 = 300$ m (checked)

Distance ran by Keagan in 2 minutes $= 190 \times 2 = 380$ m
Distance ran by Peter in 2 minutes $= 140 \times 2 = 280$ m
Distance between Keagan and Peter at the start $= 380 - 280 = 100$ m (checked)

Distance ran by Keagan in 4 minutes $= 190 \times 4 = 760$ m
Distance ran by Darren in 4 minutes $= 90 \times 4 = 360$ m
Distance between Keagan and Darren at the start $= 760 - 360 = 400$ m (checked)