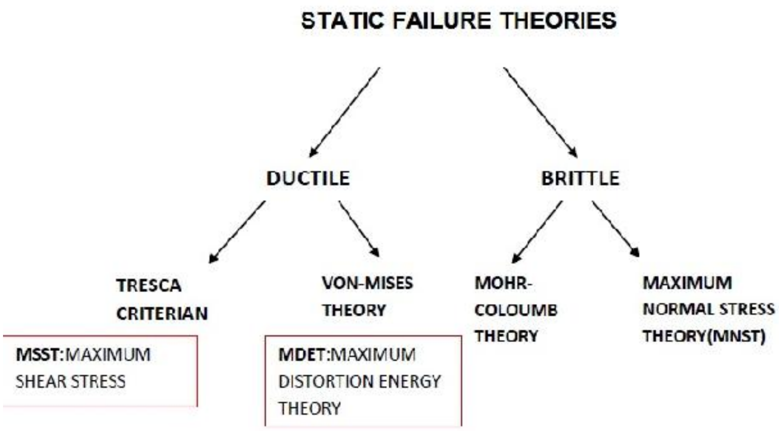
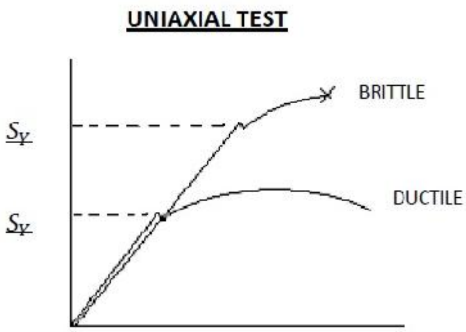
STATIC FAILURE THEORIES





MAXIMUM NORMAL STRESS THEORY

For maximum normal stress theory, the failure occurs when one of the principal stresses ($\sigma 1$, $\sigma 2 and\ \sigma 3$) equals to the yield strength.

$$\sigma 1 > \sigma 2 > \sigma 3$$

Failure occurs when either $\sigma 1 = St$ or $\sigma 3 = -Sc$,where St is strength in tension and Sc is strength in compression.

MOHR-COULOMB THEORY

The Coulomb-Mohr theory or internal friction theory assumes that the critical shearing stress is related to internal friction.

MAXIMUM DISTORTION ENERGY THEORY (VON-MISES THEORY)

The maximum distortion energy theory, also known as the Von Mises theory, was proposed by M.T.Huber in 1904 and further developed by R.von Mises(1913).In this theory failure by yielding occurs when at any point in the body ,the distortion energy per unit volume in a state of combined stress
becomes equal to that associated with yielding in a simple tension test. STRAIN ENERGY Generally strain energy U is obtained by this equation.

$$U = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3)$$

$$\varepsilon_2 = \frac{1}{E}(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3)$$

$$\varepsilon_3 = \frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3)$$

Then, substituting these three equations in to general strain energy equation:

$$U = \frac{1}{2}\sigma_1\frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) + \frac{1}{2}\sigma_2\frac{1}{E}(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3) + \frac{1}{2}\sigma_3\frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3)$$

HYDRO STATIC STRESS

The hydrostatic stress(σh)causes a change in the volume.

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Strain energy associated with the hydrostatic stress:

$$U_h = \frac{1}{2E}[\sigma_h^2 + \sigma_h^2 + \sigma_h^2 - 2\nu(\sigma_h\sigma_h + \sigma_h\sigma_h + \sigma_h\sigma_h)] = \frac{3(1-2\nu)}{2E}\sigma_h^2$$

Then distortional energy $U_d = U - U_h$

From previous equations: $U_d = \frac{1+\nu}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1]$

Then yielding will occur at this condition:

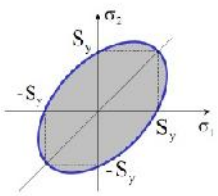
$$U_d = \frac{1+\nu}{3E}S_Y$$

$$\sigma_{eff} = S_Y = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$\sigma_{eff} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

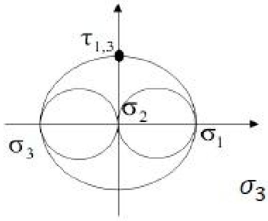
For plane stress condition $\sigma_3 = 0$

$$\sigma_{eff} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_1\sigma_2}$$

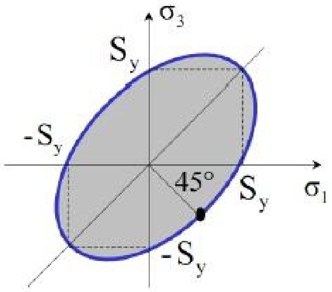


If the state stress is in this area then the material will not yield.

For pure shear condition



$$\sigma_3 = -\sigma_1$$

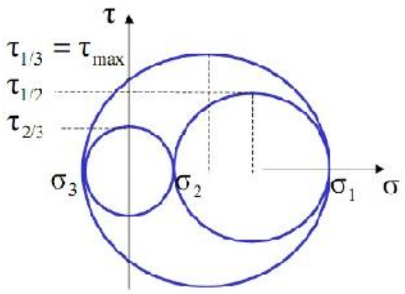


$$\sigma_{eff} = \sqrt{\sigma_1{}^2 + \sigma_3{}^2 - \sigma_1\sigma_3} = \sqrt{3\sigma_1{}^2} = \sqrt{3\tau_{max}{}^2} = S_Y$$

$$\tau_{max} = 0.577S_Y$$

MAXIMUM SHEAR STRESS THEORY

The maximum shearing stress theory is an outgrowth of the experimental observation that a ductile material yields as a result of slip or shear along crystalline planes. Yielding begins whenever the maximum shear stress in a part equals to the maximum shear stress in a tension test specimen that beings to yield.



$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_{1/3} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

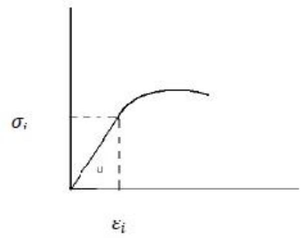
$$\tau_{max} = S_{YS} = \text{yield strength in shear} = \frac{S_Y}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Then , $S_Y = \sigma_1 - \sigma_3$

	Elasticity		Materials
-brittle (**MNST**)	$\sigma_1(\sigma_x)$	=	$\frac{S_Y}{N}$ where N is safety factor
-ductile (**MSST**)	τ_{max}	=	S_{YS}(yield strength in shear)
	$\frac{\sigma_1 - \sigma_3}{2}$	=	$\frac{S_Y}{2}$
	$\sigma_1 - \sigma_3$	=	$\frac{S_Y}{N}$
(**MDET**)	σ_{eff}	=	$\frac{S_Y}{N}$

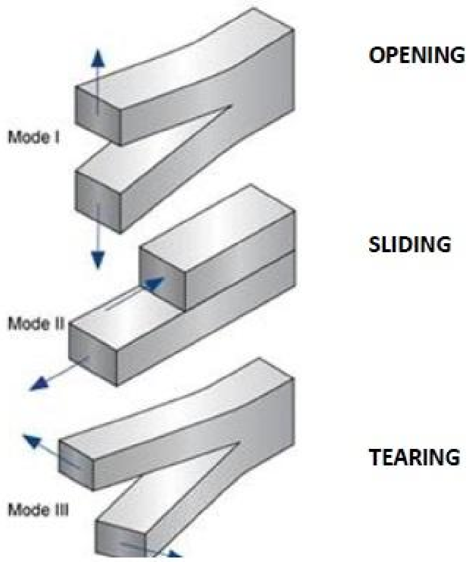
Strain Energy:



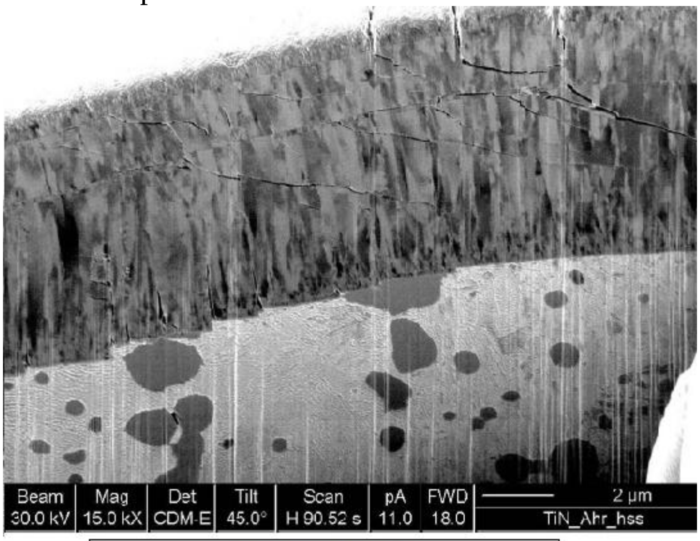
$$\frac{1}{2}\sigma_{ij}\,\varepsilon_{ij} = U$$

Design Margin=M= $\frac{S_Y - \sigma_{eff}N}{S_Y}$

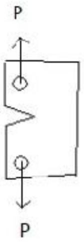
FRACTURE MODES

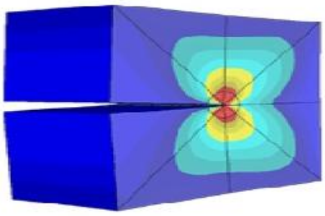


Fracture is defined as the separation of a part into two or more pieces. The mechanisms of brittle fracture are the concern of fracture mechanics ,which is based on a stress analysis in the vicinity of a crack or defect of unknown small radius in a part.



MULTIPLE FRACTURES





Stress Intensity Factor:

In the fracture mechanics approach a stress intensity factor,KI , is evaluated. This can be thought of as a measure of the effective local stress at the crack root. $KI = \beta\sigma \pi a$

Where,

σ=normal stress,

$\beta = geometry\ factor\ wic\ depends\ on\ a/w,\ a = crack\ lengt$h (or half crack length), w=member width (or half width of member)

Fracture Toughness:

In a toughness test of a given material, the stress – intensity factor at which a crack will propagate is measured. This is the critical stress intensity factor, referred to as the fracture toughness and denoted by the symbol K_{IC}.

N=K_{IC}/K_I (N=factor of safety)