Frontiers in Spectrum Auction Design

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Abstract

Spectrum auction design has seen a number innovations in the recent years. Regulators have used various types of combinatorial auction formats including simple ascending combinatorial clock auctions and first-price sealed-bid combinatorial auctions. The Simultaneous Multi-Round Auction (SMRA) and the two-stage Combinatorial Clock Auction (CCA) are the most widespread auction formats for spectrum sales to date. We provide an accessible overview of strategic problems in these auction formats and summarize research challenges in this field for a broader audience of readers in industrial organization.

Keywords: auction theory, market design, spectrum auction

1. Introduction

The 1994 sale of radio spectrum for *personal communication services* (PCS) marked a sharp change in policy by the US Federal Communications Commission (FCC). Before turning to auctions the FCC had allocated valuable spectrum on the basis of comparative hearings (also known as *beauty contests*) and lotteries. Nobel laureate Ronald Coase long advocated that market-based mechanisms would improve the allocation of scarce spectrum resources, but his early insights were ignored for decades (Coase, 1959). While there were significant successes in the award of spectrum licenses via auction, there is still no consensus about the best way to auction off spectrum licenses, and many new requirements became known in the last 20 years.

Economic theory provides a well-known solution to the sale of multiple objects in a model with independent and private values and quasi-linear utility functions: the celebrated Vickrey-Clarke-Groves (VCG) mechanism. It is the unique mechanism to provide dominant strategies to bid truthfully.

The result is beautiful, but the mechanism is rarely if ever used (Ausubel and Milgrom, 2006).

There has been a long discussion about appropriate auction mechanisms for the sale of spectrum rights (Porter and Smith, 2006). Since 1994, the Simultaneous Multi-Round Auction (SMRA) has been used worldwide (Milgrom, 2000). The SMRA design was very successful, but also led to a number of strategic problems for bidders (Cramton, 2009b). The *exposure problem* is central and refers to the risk for a bidder to make a loss due to winning only a fraction of the bundle of licenses (or blocks of spectrum) he has bid on at a price which exceeds his valuation of this subset. This has led to the design of combinatorial auctions. The Combinatorial Clock Auction (CCA) is the most wide-spread combinatorial auction design for spectrum sales. The auction format has led to some new issues and currently there is an ongoing debate among regulators, telecoms, consultants, and academics about the future of spectrum auction designs.

In this paper we briefly revisit SMRA and the CCA and some of the known strategic challenges in these auction formats. For this we draw on a new edited volume on spectrum auction design, which covers these and other auction formats in great depth (Bichler and Goeree, 2016). Then we discuss assumptions in game-theoretical models which should be revisited to better reflect requirements of regulators and preferences of bidders in the field.

2. The Simultaneous Multi-Round Auction

Let us first discuss the SMRA, which has been used for selling spectrum licenses for more than 20 years.

2.1. Auction Rules

The SMRA is an extension of the English auction to more than one license. All the licenses are sold at the same time, each with a price associated with it, and the bidders can bid on any one of the licenses. The auction proceeds in rounds, which is a specific period of time in which all bidders can submit bids. After the round is closed, the auctioneer discloses, who is winning and the prices of each license, which coincide with the highest bid submitted on each license. There are differences in the level of information revealed about other bidders' bids. Sometimes all bids are revealed after each round, sometimes only prices of the currently winning bids are published.

The bidding continues until no bidder is willing to raise the bid on any of the licenses any more. In other words, if in one round no new bids are placed, the bidders receive the spectrum for which they hold the highest active bid, then the auction ends with each bidder winning the licenses on which he has the high bid, and paying its bid for any license won.

SMRA uses simple *activity rules* which forces bidders to be active from the start. Monotonicity rules are regularly used, where bidders cannot bid on more licenses in later rounds. This forces bidders to be active from the start. Activity rules can be considered a major innovation of this auction format. Typically, bidders get eligibility points assigned at the start of the auction, which define the number of licenses they are allowed to bid on maximally. If the number of licenses they win in a round and the new bids they submit require less eligibility points than in the last round, then they lose points.

Apart from the activity rules, there are typically additional rules that matter. Auctioneers set *reserve prices* for each license, which describe prices below which an license will not be sold. They need to define *bid increments* and how bid increments might change throughout the auction. A bid increment is the minimum amount by which a bidder needs to increase his bid beyond the ask price in the next round. Sometimes, auctioneers allow for *bid withdrawals* and sometimes bidders get bid waivers, which allow bidders not to bid in a round without losing eligibility points. Finally, auctioneers often set *bidding floors and caps*, which are limits on how much a winner in the auction needs to win at a minimum and how much he can win at most. These rules should avoid unwanted outcomes such as a monopoly after the auction or a winner who wins so little spectrum that it is not sufficient for a viable business.

The auction format is popular, because it is easy to implement and the rules are simple. If the valuations of all bidders were additive, the properties of a single-object ascending auction carry over. Unfortunately, this is rarely the case and bidders have often synergies for specific licenses in a package or their preferences are substitutes. Only if bidders have substitutes preferences and bid straightforwardly, then the SMRA terminates at a Walrasian equilibrium, i.e., an equilibrium with linear prices (Milgrom, 2000). *Straightforward bidding* means that a bidder bids on the bundles of licenses, which together maximize the payoff at the current ask prices in each round. Milgrom (2000) also showed that with at least three bidders and at least one non-substitutes valuation (for example super-additive valuations for a package if licenses) no Walrasian equilibrium exists.

2.2. Strategic Challenges

Despite the simplicity of its rules there can be considerable strategic complexity in the SMRA when there are synergies between licenses that cover adjacent geographic regions or between licenses. Such complementarities violate the gross substitutes property of valuations. Bidders who compete aggressively for a certain combination of licenses risk being exposed when they end up winning an inferior subset at high prices. When bidders rationally anticipate this *exposure problem*, competition will be suppressed with adverse consequences for the auction's performance. A number of laboratory experiments document the negative impact of the exposure problem on the performance of the SMRA (Brunner et al., 2010; Goeree and Holt, 2010; Kwasnica et al., 2005; Kagel et al., 2010).

Goeree and Lien (2014) report the first Bayes-Nash equilibrium analysis of the exposure problem. They consider a general setup where "local" bidders interested in a single item compete against "global" bidders who wish to aggregate many licenses because of demand complementarities. For instance, suppose there are two local bidders who value items A and B respectively and a single global bidder who only values the package AB. Suppose all bidders' values are independently and uniformly distributed. Goeree and Lien (2014) show that the global bidder's optimal drop-out level when both local bidders are still active is given by

$$B(V) \;=\; \frac{1}{3}\Big(1 + V - \sqrt{1 + 2V - 2V^2}\Big)$$

where V is the global bidder's value for the package AB. When $V = 1$ the global bidder drops out when the prices for A and B reach one-third, i.e. when the cost of the package is only 67% of its value. The intuition is that at a per-unit price of one-third, even if one of the local bidders were to drop out immediately, the global bidder expects the other local bidder to have a value of $\frac{2}{3}$, so staying in the auction is not profitable. Goeree and Lien (2014) show that the exposure problem is even worse when the local bidders do not care which of the two items they win. In this case, the local bidders can switch back and forth between the items ("arbitrage") as is commonly observed in actual spectrum auctions. Now the equilibrium is for a global bidder with $V = 1$ to drop out right away, resulting in zero prices and zero revenue for the seller! The intuition is that, even if one of the local bidders would drop out at zero, the global bidder expects the other local bidder to drive up the price to $\frac{1}{2}$ on both items, so staying in the auction is not profitable.

Other strategic challenges are due to the activity rules. The monotonicity rule does not allow bidders to submit bids on more licenses than in the previous round. Sometimes, a less preferred alternative can have more licenses, which can lead to inefficiencies, in case a bidder is outbid on the preferred allocation. These activity rules lead to *eligibility management* and the *parking* of eligibility points in less desirable licenses, which have been observed in the context of spectrum auctions (Porter and Smith, 2006). Sometimes a bidder might also prefer to bid on a bundle with a higher number of eligibility points rather than the preferred bundle of licenses, in order to have the option to return to it later.

The SMRA also allows for various forms of *signaling and tacit collusion*. Jump bidding is usually seen as a strategy to signal strength and preferences and post threats. Sometimes, even the standing bidder increases his winning bid for the same purpose. However, there are more reasons for jump bids. In later rounds, jump bids are used to avoid ties (Boergers and Dustmann, 2003). *Retaliatory bids* are bids submitted on licenses, which are desired by rivals to force them not to bid on the licenses the bidder desires. For example, if a bidder is interested in license X and another bidder is interested in licenses X and Y, the first bidder can drive up the price of Y, signaling that the second bidder should cease bidding on X. Sometimes bidders might also be interested not to signal interest in a license, as others could take advantage of this interest to park and maintain their own eligibility at no cost, because they know they will be overbid (Salant, 1997). We will provide a more detailed example for tacit collusion in the next subsection.

Also *budget bluffing* is a well known tactic (Porter and Smith, 2006). Bidders typically track the bid exposure of other bidders. The bid exposure is the sum of a bidder's previous round provisionally winning bids plus new bids in a round. This can provide an indication into a competitor's potential budget (Bulow et al., 2009). Bidders can bid above their budget, knowing that they will be outbid on some licenses, in order to fool rivals into believing their budget is larger than it is. Strategies like this lead to a complex decision situation for bidders.

Furthermore, *demand reduction* is an issue. Demand reduction can easily occur in the presence of budget constraints and no complementarities, as the following example illustrates. Suppose, there are two bidders (1 and 2) and two licenses. We assume complete information, where bidder 1 has a valuation of $v_1 = 10$ for each license, and bidder 2 a valuation of $v_2 = 11$, respectively. Bidder 1 has a binding budget constraint of $w_1 = 10$. When bidder 1 reduces demand to 1 license, bidder 2 has an incentive to reduce demand as well. If bidder 2 does not reduce demand, bidder 1 could drive up the price to 10 on the first license until he is overbid, and then drive up the price on the second license. This leaves bidder 2 with a payoff of 11-10=1 for each of the two licenses. In contrast, if bidder 1 reduces demand at zero prices and bidder 2 agrees, they achieve payoffs of 10 and 11 respectively.

There are additional problems in the presence of budget constraints. Overall budget constraints are often ignored in theory, but they often matter in the field. Let's assume a spectrum sale with only three licenses and two bidders. Bidder 1 wants to win all three licenses. If bidder 2 has a budget limit of w and is willing to spend the whole budget even on a single license, he could drive up prices on all three licenses to w, until he gets overbid. In order for bidder 1 to win all three licenses, he would have to invest 3 times the budget of bidder 2. Note, that in a combinatorial auction it would be sufficient for the bidder to submit a bundle bid on all three licenses, which is $w + \epsilon$, in order to win all three licenses. Weak bidders can also drive up prices on licenses, which are of interest to their rivals, in order to bind the rivals' budget and have less competition on licenses they prefer. This is sometimes referred to as *budget binding*. Brusco and Lopomo (2009) provides a game-theoretical analysis considering complementarities and budget constraints.

In summary, the following strategic considerations come into play when bidders prepare for an SMRA:

- tactics to deal with the exposure problem
- eligibility management
- signaling and tacit collusion
- budget bluffing
- demand reduction
- budget binding

Bidders need to have a good understanding of other bidders' preferences and their financial strength such that they can find a best response to the opposing strategies. As a consequence, telecom operators typically spend months or even years to prepare for high-stakes spectrum auctions using SMRA.

3. The Combinatorial Clock Auction

Combinatorial auctions (CAs) allow for bids on indivisible bundles avoiding the exposure problem. The design of such auctions, however, led to a number of fundamental problems, and many theoretical and experimental contributions during the past 15 years (Cramton et al., 2006). The existing experimental literature comparing SMRAs and CAs suggests that in the presence of significant complementarities in bidders' valuations and a setting with independent private and quasi-linear valuations, combinatorial auctions achieve higher efficiency than simultaneous auctions (Banks et al., 1989; Ledyard et al., 1997; Porter et al., 2003; Kwasnica et al., 2005; Brunner et al., 2010; Goeree and Holt, 2010).

Since 2008 several countries such as the U.K., Ireland, the Netherlands, Denmark, Austria, Switzerland, Canada, and the U.S. have adopted combinatorial auctions for selling spectrum rights (Cramton, 2009b). While the U.S. used an auction format called Hierarchical Package Bidding (HPB) (Goeree and Holt, 2010), which accounts for the large number of regional licenses, many countries used a Combinatorial Clock Auction (CCA) (Maldoom, 2007; Cramton, 2009a), a two-phase auction format with primary bid rounds (aka. clock phase) for price discovery, which is extended by a supplementary bids round (aka. supplementary phase). The CCA design used in those countries is based on the Clock-Proxy auction, which was proposed by Ausubel et al. (2006).[1]

3.1. Auction Process

The auction process consists of a clock phase and a supplementary bids phase. In the *clock phase* or *primary bid rounds*, the auctioneer announces ask prices for all licenses at the beginning of each round. In every round bidders communicate their demand for each license at the current prices. At the end of a round, the auctioneer determines a set of over-demanded licenses for which the bidders' demand exceeds the supply. The price for all over-demanded licenses is increased by a bid increment for the next round. This clock phase continues until there are no over-demanded licenses left. If all bidders follow a straightforward strategy and all licenses were sold after

[1]Note that Porter et al. (2003) have defined a combinatorial clock auction, which is different to the one described in this paper and in Maldoom (2007) and Cramton (2009b), and only consists of a single clock phase.

the clock phase terminates, then the auction outcome is efficient (Ausubel et al., 2006).

The *supplementary stage* is designed to eliminate inefficiency from the single-stage clock phase. In this sealed-bid stage bidders are able to increase bids from the clock phase or submit bids on bundles they have not bid on in the clock phase. Bidders can submit as many bids as they want, but the bid price is restricted subject to the CCA activity rule (see next subsection). Finally, all bids from both phases of the auction are considered in the winner determination and the computation of payments for the winners. The bids by a single bidder are mutually exclusive (i.e., the CCA uses an XOR bidding language).

For the computation of payments, a Vickrey-nearest bidder-optimal core-pricing rule is used (Day and Cramton, 2012) in spectrum auctions, although there have been proposals for other types of core-payments (Erdil and Klemperer, 2010). To illustrate, suppose two local bidders bid \$8 for items A and B respectively, while a single global bidder bids \$10 for the package AB. Then the local bidders each get an item and Vickrey prices are \$2 each. This outcome is not in the core, e.g. the seller and global bidder could block it by settling on a package price higher than \$4. The idea of core-pricing is to make such a blocking coalition impossible, i.e. by imposing that the sum of the item prices is no less than the losing package bid: $p_A + p_B \geq 10$. Bidder optimality resolves part of the resulting indeterminacy by replacing the inequality by an equality, and, finally, a unique set of prices is found by selecting prices that are nearest to the Vickrey prices, which yields $p_A = p_B = 5$ in this example.

Of course, changing the pricing rule has consequences for bidders' behavior. Goeree and Lien (2016) show for a simple example with two local bidders and one global bidder that the change in behavior may be such that core-pricing yields *lower* revenues than the original Vickrey auction. Furthermore, Goeree and Lien (2016) prove that when the Vickrey outcome is not in the core then there exists *no mechanism* that can implement core outcomes. This result has several implications. First, in the presence of value complementarities, it is well-known that a Walrasian equilibrium may not exist and one might conjecture that core prices are the correct generalization (Milgrom, 2000). However, this intuition is wrong as core outcomes are not implementable when Vickrey is not in the core. Second, it implies that straightforward bidding is generally not an equilibrium as it would lead to Walrasian prices, which are not generally implementable.

3.2. Activity Rules

The CCA combines two auctions in the clock and in the supplementary phase. This requires additional rules setting incentives to bid consistently throughout the two phases. Without activity rules, bidders might not bid in the clock phase, but wait for the other bidders to reveal their preferences, and only bid in the supplementary phase. These activity rules play a crucial role for the bidding strategies, as we will see.

3.2.1. Activity Rules in the Clock Phase

Originally, the clock phase of the CCA employed a simple monotonicity rule which does not allow to increase the size of the package in later rounds as prices increase. It has been shown that with substitutes preferences straightforward bidding is impossible with such an activity rule (Bichler et al., 2011b, 2013a). Later versions use a hybrid activity rule using a monotonicity rule and a revealed preference rule (Ausubel et al., 2006). Revealed preference rules allow bidders to bid straightforward in the clock phase. If they do, then bidders are able to bid on all possible packages up to their true valuation in the supplementary stage (Bichler et al., 2013a).

First, an eligibility points rule is used in the clock phase to enforce activity in the primary bid rounds. The number of bidder's eligibility points is non-increasing between rounds, such that bidders cannot bid on more licenses when the prices rise. A bidder may place a bid on any package that is within its current eligibility. Second, in any round, the bidder is also permitted to bid on a package that exceeds its current eligibility provided that the package satisfies revealed preference with respect to each prior eligibility-reducing round. Bidding on a larger package does not increase the bidder's eligibility in subsequent rounds.

The revealed preference rule works as follows: A package in clock round t satisfies revealed preference with respect to an earlier clock round s for a given bidder if the bidder's package x_t has become relatively less expensive than the package bid on in clock round s, x_s, as clock prices have progressed from the clock prices in clock round s to the clock prices in clock round t. x_s and x_t are vectors where each component describes the number of licenses demanded in the respective category, i.e., region or spectrum band. For example, in a market with three types of licenses or three spectrum bands with licenses of the same quality, a bidder who is interested in a package with 2 licenses in the first band and one license of the third band has a bid $x_t = (2, 0, 1)$ at prices p_t. The revealed preference constraint is:

$$\sum_{i=1}^{m}(x_{t,i} * (p_{t,i} - p_{s,i})) \leq \sum_{i=1}^{m}(x_{s,i} * (p_{t,i} - p_{s,i}))$$

where:

- i indexes the licenses;
- m is the number of licenses;
- $x_{t,i}$ is the quantity of the ith license bid in clock round t;
- $x_{s,i}$ is the quantity of the ith license bid in clock round s;
- $p_{t,i}$ is the clock price of the ith license bid in clock round t; and
- $p_{s,i}$ is the clock price of the ith license bid in clock round s.

A bidder's package, x_t, of clock round t is consistent with revealed preference in the clock rounds if it satisfies the revealed preference constraint with respect to all eligibility-reducing rounds prior to clock round t for the given bidder.

3.2.2. Activity Rules in the Supplementary Phase

Under the activity rule for the supplementary round, there is no limit on the supplementary bid amount for the final clock package. All supplementary bids on packages other than the final clock package must satisfy revealed preference with respect to the final clock round regardless of whether the supplementary bid package is smaller or larger, in terms of eligibility points, than the bidder's eligibility in the final clock round. This is referred to as the *final cap rule.*

In addition, supplementary bids for packages that exceed the bidder's eligibility in the final clock round must satisfy revealed preference with respect to the last clock round in which the bidder was eligible to bid on the package and every subsequent clock round in which the bidder reduced eligibility. This is also called the *relative cap rule.*

Let x denote the package on which the bidder wishes to place a supplementary bid. Let x_s denote the package on which the bidder bid in clock round s and let b_s denote the bidder's highest monetary amount bid in the auction on package x_s, whether the highest amount was placed in a clock round or the supplementary round.

A supplementary bid b on package x satisfies revealed preference with respect to a clock round s, if b is less than or equal to the highest monetary amount bid on the package bid in clock round s, that is, b_s plus the price difference in the respective packages, x and x_s, using the clock prices of clock round s. Algebraically, the revealed preference limit is the condition that:

$$b \leq b_s + \sum_{i=1}^{m} (p_{s,i} * (x_i - x_{s,i}))$$

where:

- x_i is the quantity of the ith license in package x;
- b is the maximum monetary amount of the supplementary bid on package x; and
- b_s is the highest monetary amount bid on package x either in a clock round or in the supplementary round.

In addition, for supplementary bid package x, let $t(x)$ denote the last clock round in which the bidder's eligibility was at least the number of eligibility points associated with package x.

A given bidder's collection of supplementary bids is consistent with the revealed preference limit if the supplementary bid for package x, with a monetary amount b for the given bidder satisfies the following condition: for any package x, the monetary amount b must satisfy the revealed preference constraint, as specified above with respect to the final clock round and with respect to every eligibility-reducing round equal to $t(x)$ or later.

Note that, in the application of the formula above, the package x_s may itself be subject to a revealed preference constraint with respect to another package. Thus, the rule may have the effect of creating a chain of constraints on the monetary amount of a supplementary bid for a package x relative to the monetary amounts of other clock bids or supplementary bids.

3.3. Strategic Challenges

An equilibrium analysis of a CCA with all its detailed rules is difficult. However, there are a number of papers, who analyzed simplified environments game-theoretically. Levin and Skrzypacz (2014) showed that truthful bidding is not dominant in an environment with homogeneous goods, and that there is a wide range of ex post equilibria with demand expansion, demand reduction and predation. In the following, we focus on possibilities to raise rivals' costs in a CCA. These possibilities arise due to the payment rule, which charges bidders differential payments, and the possibility to submit safe supplementary bids, i.e., bids which will definitely be losing, but possibly impact the payments of competitors (Bichler et al., 2011b, 2013a). Janssen and Karamychev (2013) provides motivation for spiteful bidding behavior and a game-theoretical model with complete information, where bidders raise rivals' cost. These strategic challenges are due to the non-anonymous payment rule and the possibility of submitting safe supplementary bids, which we will discuss in the following.

3.3.1. Non-Anonymous Payments

Neither the two-stage CCA nor the VCG mechanism have anonymous prices. Let us provide a simple example to illustrate this well-known fact. Suppose there are two bidders and two homogeneous units of one license. Bidder 1 submits a bid of \$5 on one unit, while bidder 2 submits a bid of \$5 on one unit and a bid of \$9 on two units. Each bidder wins one unit, but bidder 1 pays \$4 and bidder 2 pays zero.

This difference is due to the asymmetry of bidders, and this asymmetry leads to a violation of the law of one price, a criterion, which is often seen desirable in market design. Although arbitrage is avoided as bidders typically cannot sell licenses among each other immediately after a spectrum auction, different prices for the same spectrum are difficult to justify in the public and violate the anonymity of prices. This has become a topic of debate in the Swiss spectrum auction in 2012, a CCA where two bidders paid substantially different prices for almost the same allocation.

3.3.2. Safe Supplementary Bids

After the clock rounds, if a bidder has a standing bid on his most preferred bundle, he might not have an incentive to bid truthfully in the supplementary phase, because he can submit a bid price, which is sufficient to win this standing bid with certainty. The following two theorems define "safe supplementary bids", which cannot become losing based on the final cap activity rule if the bidders have a standing bid after the primary bid rounds. These bids also introduce a possibility for riskless spiteful bidding, as we will see later.

Theorem 1. *(Bichler et al., 2013a) If demand equals supply in the final primary bid round, any single supplementary bid $b_j^s(x_j) > b_j^p(x_j)$ cannot become losing.*

Here, $b_j^s(x_j)$ describes a supplementary bid of a bidder $j \in \mathcal{I}$ on a package x_j, while $b_j^p(x_j)$ is the standing bid of bidder j from the primary bid rounds. The intuition is that the supplementary bids of competitors on their standing bundle bid from the final primary bid round does not impact the safe supplementary bid of a bidder $j \in \mathcal{I}$. Any additional licenses added by competitors to their standing bundle bid cannot increase the supplementary bid price by more than the ask price in the last of the primary bid rounds. If the bidder submits additional supplementary bids on packages not containing x_j, his bid $b_j^s(x_j)$ can well become losing, as can easily be shown by examples. The activity rule also applies to bundles which are smaller than the standing bid of the last primary bid round.

If there is excess supply in the last round of the primary bid phase, a last primary round bid $b_j^p(x_j)$ can become losing, because even if no supplementary bids were submitted, the auctioneer conducts an optimization with all bids submitted at the end, which might displace $b_j^p(x_j)$. This raises the question for the safe supplementary bid $b_j^s(x_j)$, which ensures that the bidder j wins the bundle x_j of his standing bid from the primary round after the supplementary bids phase. We will denote the price vector of the last primary bid round as α.

Theorem 2. *(Bichler et al., 2013a) If there is zero demand on bundle y after the last primary bid round, a single supplementary bid of a standing bidder $b_j^s(x_j) > b_j^p(x_j) + \alpha y$ cannot become losing.*

Let's take the example with four bidders (B1 to B4) described in Table 1 to illustrate this point. There is a supply of 6 units of a single license. The number in brackets after round 1 to 3 is the ask price for the licenses in this round. There is excess demand until round 3, when bidders 2, 3 and 4 reduce to a demand of zero. In the supplementary bid round (S) these two bidders increase their last bid to a maximum of 300 Euro for 3 licenses. They would become winning, while the standing bid of bidder 1 after the primary bid rounds is displaced. In the example, bidder B1 only needs to increase his bid price by 100 \$ and not by $\alpha * 4 = 400\$$. This is the difference between the allocation with B3 and B4, and the best allocation with B1's bid winning,

	B1	B2	B3	B4
Round 1 (80)	2	2	3	3
Round 2 (90)	2	2	3	3
Round 3 (100)	2	0	0	0
Round S	2(200)		3(300)	3(300)

Table 1: Example with supplementary bid phase (S)

which is B1's bid on 2 licenses for 200 $ and the bid by bidder B3 or B4 on 3 units.

As a consequence of safe supplementary bids, bidders can submit riskless spiteful bids to drive up payments of other bidders. Bidders in spectrum markets may spitefully prefer that their rivals earn a lower surplus. This is different from the expected utility maximizers typically assumed in the literature.

Spiteful bidding has been analyzed by Morgan et al. (2003) and Brandt et al. (2007), who show that the expected revenue in second-price auctions is higher than the revenue in first-price auctions with spiteful bidders in a Bayes Nash equilibrium. While spiteful bidding is possible in any auction, the two-stage CCA provides possibilities to submit spiteful supplementary bids with no risk of actually winning such a bid, if all licenses are sold after the primary bid rounds and the standing bidders only want to win their standing bid in the supplementary bids round with a small bid increment. The latter is a relatively mild assumption.

Simple examples suggests that there are situations where the clock auction reveals enough information for a bidder to increase the Vickrey price of other bidders by losing bids, and therefore also the Vickrey-closest core-selecting payment of all bidders. Not revealing excess supply after the clock rounds can mitigate the problem, but, depending on the history of primary round bids, there might still be a risk of spiteful bids.

The Austrian Auction in 2013 is interesting for this reason. In this auction bidders could potentially submit up to 12,810 package bids (considering caps) on the 800 MHz, 900 MHz, and 1800 MHz bands, but they were limited to 2000 bids in the supplementary phase. The regulator reported that the three bidders actually submitted 4000 supplementary bids in total. The regulator also disclosed that most of these bids were submitted on very large packages up to the price limits imposed by the activity rule. This large number of supplementary bids can be seen as one reason for the high prices paid in Austria. The attempt to drive up prices of other bidders and avoid having to pay more for an allocation than ones competitors can serve as an explanation for this bidding behavior. Of course, if all bidders follow this strategy, this leads to a strategic situation similar to a prisoners' dilemma. If none of the bidders submitted high supplementary bids on these large packages, they would all have had to pay less (see Kroemer et al. (2016)). Some implementations of the CCA do not reveal the level of excess supply in the last clock rounds, such that it is harder to determine safe supplementary bids.

4. Revisiting the Environment of Spectrum Auction Markets

The first sections summarized wide-spread market designs for spectrum sales and strategic challenges that arise in these auction formats. A lot has been learned about spectrum auctions in the recent years since the first auctions have been organized, and it is time to step back and look at requirements for the allocation problem in spectrum auctions that might not have deemed central in the mid-90s, but turn out to be important to the market participants. Ultimately, market design is a modeling exercise and we will only be able to derive adequate market mechanisms, if we model the preferences of market participants appropriately. In what follows, we want to discuss objectives of regulators and bidders in spectrum auction markets and discuss differences from assumptions in standard auction theory.

4.1. The Regulator's Objectives

Let us first assume that bidders have independent and private valuations. Even under these idealized assumptions, regulators face a number of problems, which are due to the fact that the preferences of bidders include complements and substitutes. Such valuation functions have motivated the use of combinatorial auctions, which have some inherent complexities. By now, it is well accepted, that complements in bidder valuations matter. We will start with some fundamental problems arising from welfare maximization in the presence of complementary valuations, before we discuss how welfare maximization relates to the policy goals of regulators.

4.1.1. Computational Complexity and Approximation

Even if we assume simple payoff-maximization of bidders, the allocation problem is a hard computational problem if bidders are allowed to express complements and substitutes as in a combinatorial auction. Modern day optimization software allows solving real-world instances to optimality, but very large markets such as in the USA might still be a challenge. For some auctions such as the incentive auctions in the USA,[2] the regulator cannot aim for welfare maximization, but needs to restrict to approximations of the welfare-maximizing allocation. This has led to fruitful research in computer science on *approximation mechanisms*, and new mechanisms which maintain strategy-proofness, but relax the goal of maximizing social welfare (Nisan and Ronen, 2001). By now, we know worst-case bounds of approximation algorithms for a number of problem types, which still exhibit strong game-theoretical solution concepts.

4.1.2. Communication Complexity and Compact Bid Languages

Communication complexity turned out to be an equally fundamental problem. Communication complexity refers to the amount of information that bidders need to communicate to the auctioneer for him to make an efficient allocation. For some spectrum auctions, such as in the Canadian auction in 2014, there were around 100 licenses for sale. Bidders cannot enumerate all possible packages (2^{100} ignoring caps and floors) and the selection of package bids by bidders can have a substantial impact on allocation and prices. This leads to a considerable level of randomness in the allocation. Simplification has been introduced as a guiding principle in market design (Milgrom, 2010), and regulators need to be aware of the fact that higher expressiveness of the bid language does not always lead to higher efficiency. Bichler et al. (2014) showed that *compact bid languages* can have a substantial impact on efficiency in larger auctions with many items, but it has largely been ignored in spectrum auction design. Compact bid languages leverage prior information about the structure of the bidders' preferences and elicit these with a small number of parameters. Examples are hierarchical package bidding (Goeree and Holt, 2010), which reduces the packages allowed in the auction to a hierarchy, or domain-specific languages as they are used in procurement auctions (Bichler et al., 2011a).

4.1.3. Policy Goals and Allocation Constraints

Even if a regulator is able to compute the welfare maximizing allocation and bidders express their valuations truthfully, welfare maximization might

[2] https://www.fcc.gov/about-fcc/fcc-initiatives/incentive-auctions

not be what the regulator wants. The welfare maximizing allocation could well be a monopoly. However, regulators are concerned with efficiency of the downstream market not with welfare maximization in the auction market. They need to strike a balance between incentives for investments and enough competition in the end consumer market such that there are low prices for the end consumer and good quality of service. Caps and set-aside licenses are frequently used by regulators to avoid unwanted allocations or encourage participation by additional companies. It is important that regulators are able to implement policy decisions in the mechanism to avoid unwanted outcomes. While it is simple to consider *allocation constraints* in an optimization model computing the optimal allocation, such constraints have received little attention in the auction design literature, in particular with ascending auction designs (Petrakis et al., 2013).

4.2. The Bidders' Preferences

The standard models used to advocate the use of auctions for the sale of spectrum licenses are based on the assumption of independent and private valuations and bidders having a quasi-linear utility function. While these assumptions appear like a reasonable approximation of bidders in spectrum markets, models based on these idealized assumptions might lead to wrong advice for both, bidders and regulators.

4.2.1. Value Uncertainty, Value Interdependencies, and Endogeneity of Values

Bidders spend substantial time estimating the net present value of certain packages of licenses. Such estimates are highly uncertain. Bulow et al. (2009) show that revenues in spectrum auctions are hard to predict and that even forecasts made just prior to an auction by investment banks tend to have high variance of several billion dollar in the US. For example, prior to the AWS auction in the US, analyst estimates of auction revenue ranged from \$7 billion to \$15 billion. Calculating a value of spectrum for a single bidder requires consideration of total market population, market penetration rates, market share, average revenue per unit, customer acquisition and activation costs, customer deactivations, and many more factors (Korczyk, 2008), and the estimated net present values are highly uncertain. For example, the advent of media streaming and smart phones has probably led to a substantial change in valuations, compared to those that companies had in 2000.

Value uncertainty can lead to problems in sealed-bid auctions. For example, in a first-price sealed-bid combinatorial auction in Norway 2013, one of the incumbents was bidding too low such that he did not become a winner in the auction. Later he had to leave the market.[3] Commentators argued that this would not have happened if it was an ascending auction and the bidder had a chance to react to the bids of others. Some also argue that the assumption of independent and private valuations might be too strong in spectrum auction markets (Goeree and Offerman, 2003). Unfortunately, negative results show that mechanisms satisfying strong solution concepts such ex-post implementations cannot be achieved in general Jehiel et al. (2006).

It is also important to note that the way how spectrum is awarded has an impact on the valuations. If a telecom knows that the next award will be an efficient auction again, he might have more incentives to invest and consequently a higher value than if the award will be via lottery. So the valuations are endogenous to the auction mechanism. On the other hand, a perfectly efficient auction might deter weaker bidders with a lower budget from participating at all, possibly leading to a monopoly or oligopoly of only a few telecoms where there could be a competitive end consumer market (Klemperer, 2002). Ultimately, these are questions of industrial organization and the overall objectives of the regulator must not be confused with welfare maximization.

4.3. Allocative Externalities and Non-Anonymous Pricing

For telecoms in many markets the entire allocation matters, not only the package that a bidder wins and the price he pays. For example, the number of competitors and also their allocations can have a substantial impact on the revenues in the downstream market. End consumers pay a premium for the telecom with the best network, and this is relative to the spectrum holdings of competitors. The provider with the best network is able to charge higher prices to end consumers eventually leading to higher revenues. In other words, the net present value of a package of licenses can be substantially different depending on the allocation of competitors.

In the German spectrum auction in 2000 six bidders could have closed the auction if they all reduced demand to two units at a revenue of EUR 30 bn.,

[3] http://www.policytracker.com/free-content/blogs/toby-youell/norway-is-now-a-two-player-mobile-market-for-the-time-being-at-least

but two bidders eventually drove up the revenue to EUR 50 bn. This was described as an attempt to drive out another bidder from the downstream market, and it shows that externalities can be substantial. Bichler et al. (2016) discuss the impact of allocative externalities in the German spectrum auction in 2015.

Auction design with externalities has received little attention as of yet (Jehiel and Moldovanu, 2005). The VCG mechanism would still determine the efficient allocation in dominant strategies, if bidders could express their preferences for all possible allocations. This, however, is unreasonable to assume in realistic markets due to the combinatorial explosion of possible allocations. Therefore, it is interesting to understand how bidders would bid in standard auction formats in the presence of allocative externalities.

Allocative externalities refers to situations where the valuation for objects depends on who obtains which objects. However, it also matters for telecoms, how much their competitors pay for a license. Payments in the Vickrey-Clarke-Groves mechanism and in the Combinatorial Clock Auction are non-anonymous. As we discussed earlier, in the Swiss Combinatorial Clock Auction in 2012 one bidder payed substantially more than another for almost the same allocation of spectrum licenses (Kroemer et al., 2016). In high-stakes spectrum auctions payments are in the billions of dollars, and a much higher payment in a spectrum auction binds budget of a competitor, which can be a disadvantage in the downstream market. Spiteful bidding and according strategies to raise rivals' costs have been observed in spectrum auctions and analyzed theoretically (Janssen and Karamychev, 2013). Such motives differ from the traditional independent private values model.

4.4. Principal-Agent Relationships and Budget Constraints

If financial markets were perfect, there would be no budget constraints preventing telecoms from acquiring licenses. In reality, *budget constraints* are almost always an issue and they violate the quasi-linear utility functions typically assumed in mechanism design. Such budget constraints defy strategy-proof mechanisms, even if bidders maximize payoff and they have independent and private valuations (Dobzinski et al., 2008). It is important to understand, how these budgets are determined.

Bidding firms often exhibit principal-agent relationships, where the management is the agent and the board of directors or the stakeholder can be seen as the principal. The agent typically as a good estimate of the value of a particular package of licenses, while the principal has not. In contrast, the principal wants to maximize payoff, but the agents often have empire building motives and they prefer more valuable packages to less valuabe ones. Agents try to win their most preferred package within a budget constraint, which is different to payoff-maximization. The payments are usually in the billions and need to be paid by the principal in these environments. Paulsen and Bichler (2015) show that there are environments where the agents bid more aggressive than a principal would do in equilibrium. This can lead to adverse selection and inefficient outcomes. Although, it can be seen as the responsibility of the principal to set incentives for payoff maximization in the bidding team, the hidden information problem makes the design of optimal contracts between principal and agent very difficult in practice.

5. Discussion

The design of spectrum auctions has seen considerable progress, but the journey has just begun. Mechanism design imposes *strategy-proofness* as a constraint first and then tries to satisfy other design desiderata such as efficiency or revenue. Most of the literature draws on direct revelation mechanisms based on the revelation principle (Gibbard, 1973), and iterative processes did not play a central role. In the past decade, it became obvious that the objectives of the regulator and the utility functions of telecoms are differ from those traditionally discussed in auction theory. These differences require us to rethink the auction process, the bid language, and the payment rules used in spectrum auctions.

- Auction process: sealed-bid vs. iterative
  - The revelation principle has focused much of the literature in mechanism design on direct revelation mechanisms. Even in the recent years, sealed-bid auctions have actually been used in a number of countries for selling spectrum. Iterative auctions have several advantages when bidders have *value uncertainties and value interdependencies*. Milgrom and Weber (1982) write "... when bidders are uncertain about their valuations, they can acquire useful information by scrutinizing the bidding behavior of their competitors during the course of an [iterative] auction. That extra information weakens the winner's curse and leads to more aggressive bidding in the [iterative] auction, which accounts for the higher expected price."

  - Iterative auctions also make it easier for a board of directors to control the management bidding in a spectrum auction, which might have incentives different from payoff-maximization due to hidden information in a *principal-agent relationship* within the firm (Paulsen and Bichler, 2015).
  - Finally, iterative auctions with high transparency about the winners in each round allow addressing *allocative externalities*, because bidders see an allocation emerge and can veto this allocation with high bids, if it is not in their interest (Bichler et al., 2016). Of course, the level of transparency in an auction needs to be decided with care and depends also on the competitive situation in a market and the likelihood of tacit collusion.

- Bid language: expressiveness vs. compactness
  - Telecoms have *complex preferences* for spectrum licenses including complements and substitutes. Combinatorial auctions provide a solution as they allow bidders to fully specify their preferences. However, a fully enumerative bid language, which allows bidders to submit bids on every possible package suffers from the fact that bidders will only specify bids for a small subset of the exponentially many packages, which can lead to substantial inefficiencies. A compact bid language is less demanding in that it lets bidders specify packages of licenses with high synergies, but does not require an exponentially large set of bids. Hierarchical package bidding (HPB) is one example for a compact bidding language with regional licenses (Goeree and Holt, 2010). Compact bid languages can also be discussed for the award of national licenses (Bichler et al., 2014).
  - Also regulators need to be able to express their preferences and constraints. For example, allocation constraints can be used in the winner determination to avoid very unequal distributions of spectrum, when the policy goal is to achieve a competitive end-consumer market.

- Payment rules: non-anonymous vs. anonymous
  - The Vickrey-Clarke-Groves mechanism can be considered the central result in mechanism design, but it is based on the assumption

> of independent and private valuations in isolated markets. Auctions where *bidders compete in a downstream market* are different. If one bidder has to pay considerably more than another for a similar allocation, as it can be the case with Vickrey-Clarke-Groves mechanisms or the CCA, then this is perceived as undesirable by many participants. This also relates to notions of fairness such as equitability, which need to be taken seriously. Anonymous prices as they are used in SMRA, HPB, or the single-stage Combinatorial Clock Auction have advantages in this respect, even if they do not necessarily lead to full efficiency in the traditional independent and private values model (Bichler et al., 2013b).

Every theoretical model has assumptions, and it is important to have these assumptions in mind, when we provide policy advice based on such models. Models which are based on bidders with independent and private valuations and auctioneers, who want to maximize allocative efficiency of the auction market, might not provide the right solution for policy makers and regulators. In spectrum auction markets, we typically have allocative externalities, we find high value uncertainties for bidders, and regulators want to achieve a competitive and sustainable downstream market, which is different from allocative efficiency of the auction market. Market design is an engineering discipline and as so often in engineering there are conflicting objectives and one needs to find a satisficing solution (Simon, 1991). We argue that the design of the auction process (iterative vs. sealed-bid), the bid language (compact vs. fully enumerative), and the payment rule (anonymous vs. non-anonymous) need to be revisited for future auction designs.

There might also not be a single optimal auction design for all types of spectrum sales. Large markets with many bidders and regional licenses such as in the USA and in Canada are different to small national markets with a few bidders only. These differences in the market environment will lead to different bid languages and also pricing rules. Regulators also need to consider the specific market environment and the financial strength of bidders in the market. Even if there is no one-size-fits-all auction design for all of these markets, it might well be possible to develop market designs addressing the goals and requirements of certain market types. As a community, we need to think about robust auction designs, which consider the preferences of telecom companies, and ultimately help regulators achieve their policy goals.

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