

Estimating Effective Thermal Conductivity of Hollow Sphere Foam Materials

Zian Jia, Zhen Wang, David Hwang, and Lifeng Wang

11/6/2017

1. Introduction

- Hollow sphere foams;
- Applications.

2. Methods

- Theoretical modelling;
- Numerical simulation.

3. Results and discussion

- Effect of shell porosity and multilayer shell;
- Effect of sphere packing and binder shape;
- Effect of Interfacial Thermal resistance (ITR).

4. Conclusion

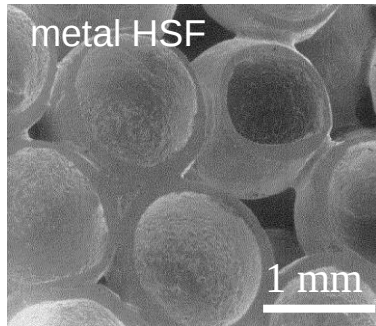
- Guidelines to designs HSFs.

1. Introduction

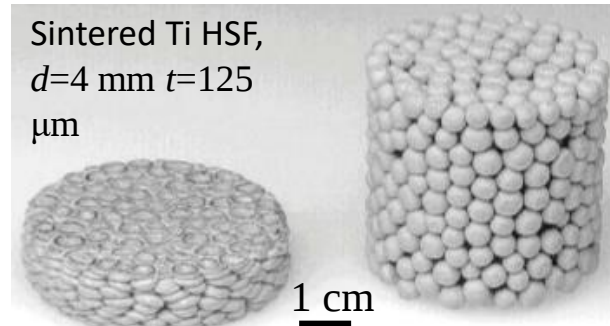


- **Hollow sphere foams (HSF)** are special cellular solids that exhibit **low density, large void space, and large specific surface area.**

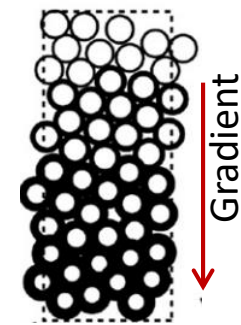
- **Macroscale level ($\sim$ mm)**



Curr. Opin. Solid State Mater. Sci. 3.5 (1998): 474-479.



Adv. Eng. Mater. 2.4 (2000): 192-195.

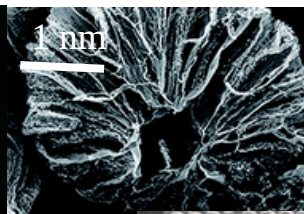


IJMS 52.5 (2010): 680-688.

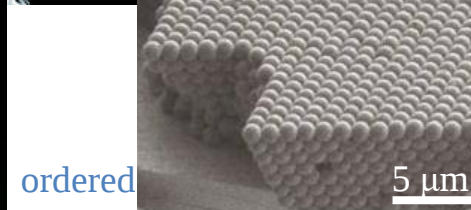
- **Mesoscale level (μ m $\sim$ nm)**



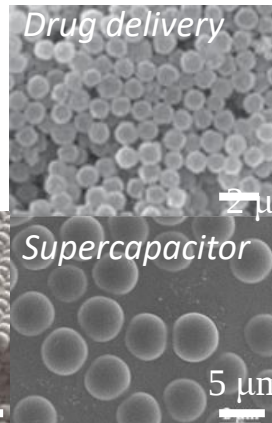
Nature 127.3211 (1931): 741.



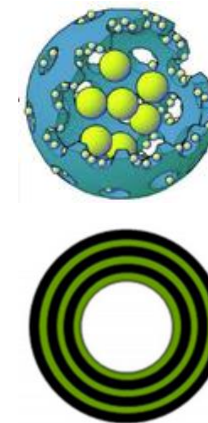
random



ordered



Adv. Mater. 26.20 (2014): 3176-3205.
Electrochim. Acta 173 (2015): 184-192.



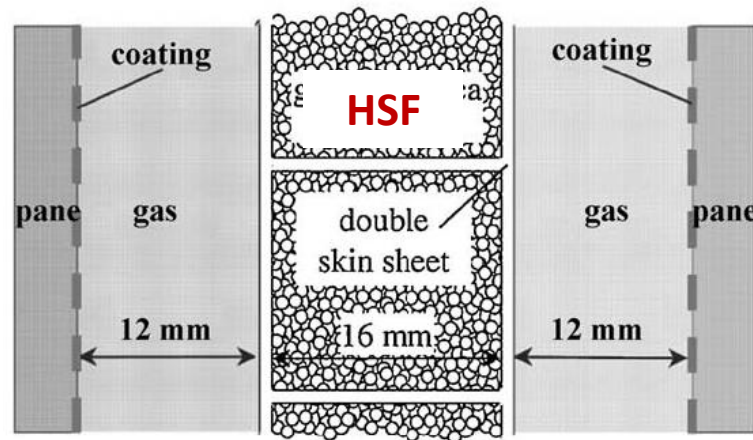
1. Introduction

➤ Hollow sphere foams (HSF) Applications

- Heat exchangers, cooling machines;
- Thermal insulation (offshore pipelines => buoyance);
- Catalyst supports.



Energy and Buildings 43 (2011) 761–769

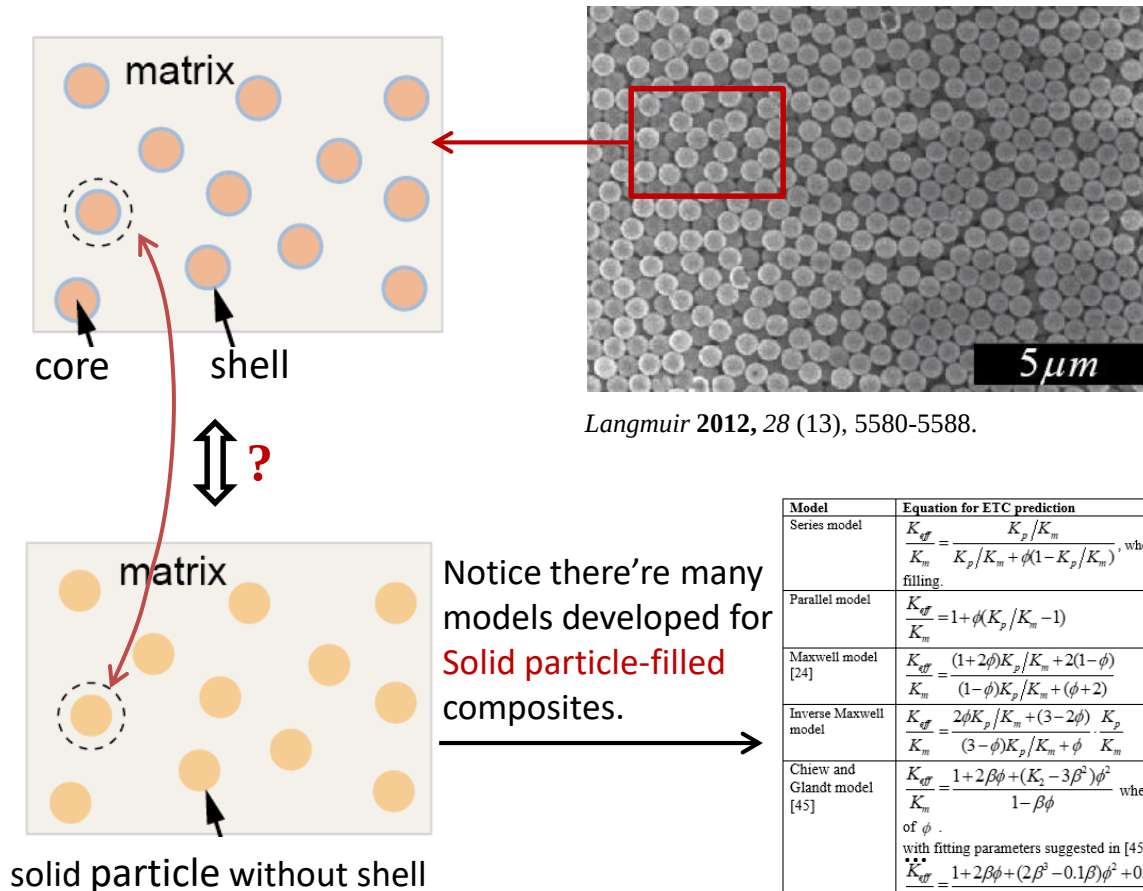


Solar Energy 79 (2005) 131–139

For sphere radius $1\mu\text{m}\sim 10\text{mm}$,
thermal conduction dominates.

2. Methods: theoretical modelling

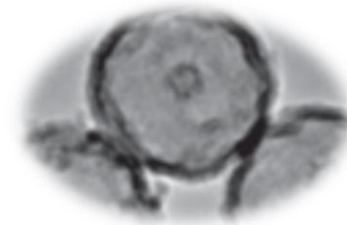
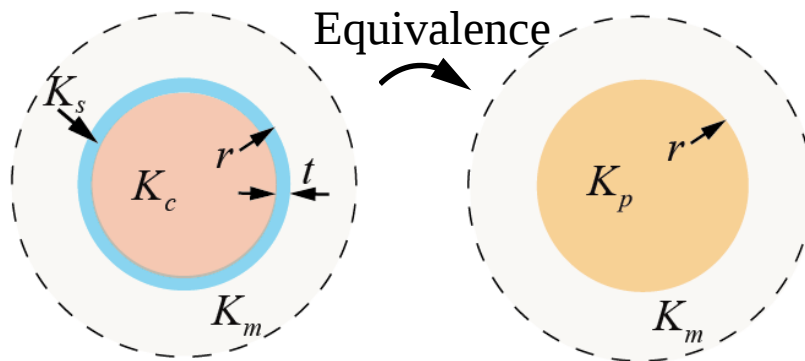
- **Evaluating the effective thermal conductivity (ETC) of HSF**
- composed of three components - core, shell, and matrix.
- Transfer HSF problem to solid-particle problem, which is well studied.



Model	Equation for ETC prediction
Series model	$\frac{K_{eff}}{K_m} = \frac{K_p/K_m}{K_p/K_m + \phi(1 - K_p/K_m)}$, where ϕ is volume fraction of particle filling.
Parallel model	$\frac{K_{eff}}{K_m} = 1 + \phi(K_p/K_m - 1)$
Maxwell model [24]	$\frac{K_{eff}}{K_m} = \frac{(1 + 2\phi)K_p/K_m + 2(1 - \phi)}{(1 - \phi)K_p/K_m + (\phi + 2)}$
Inverse Maxwell model	$\frac{K_{eff}}{K_m} = \frac{2\phi K_p/K_m + (3 - 2\phi)}{(3 - \phi)K_p/K_m + \phi} \cdot \frac{K_p}{K_m}$
Chiew and Glandt model [45]	$\frac{K_{eff}}{K_m} = \frac{1 + 2\beta\phi + (K_2 - 3\beta^2)\phi^2}{1 - \beta\phi}$ where $\beta = \frac{K_p - K_m}{K_p + 2K_m}$, K_2 is a function of ϕ . with fitting parameters suggested in [45]: $\frac{K_{eff}}{K_m} = \frac{1 + 2\beta\phi + (2\beta^2 - 0.1\beta)\phi^2 + 0.05\exp(4.5\beta) \cdot \phi^3}{1 - \beta\phi}$

2. Methods: theoretical modelling

- Equivalent hollow sphere as an effective solid particle



J. Appl. Phys. **1997**, 81 (10), 6692-6699.

Based on **effective media theory**,

$$K_p/K_s = \frac{3 + (K_c/K_s - 1)(1 + 2\nu)}{3 + (K_c/K_s - 1)(1 - \nu)},$$

K_p is the thermal conductivity of the equivalent particle, and $\nu = \frac{r^3}{(r+t)^3}$.

Rev. Mod. Phys. 41.1 (1969): 48.

Interfacial thermal resistance (ITR)

$$\kappa_1 \frac{\partial T^1}{\partial n} = \kappa_2 \frac{\partial T^2}{\partial n} = \beta(T^1 - T^2),$$

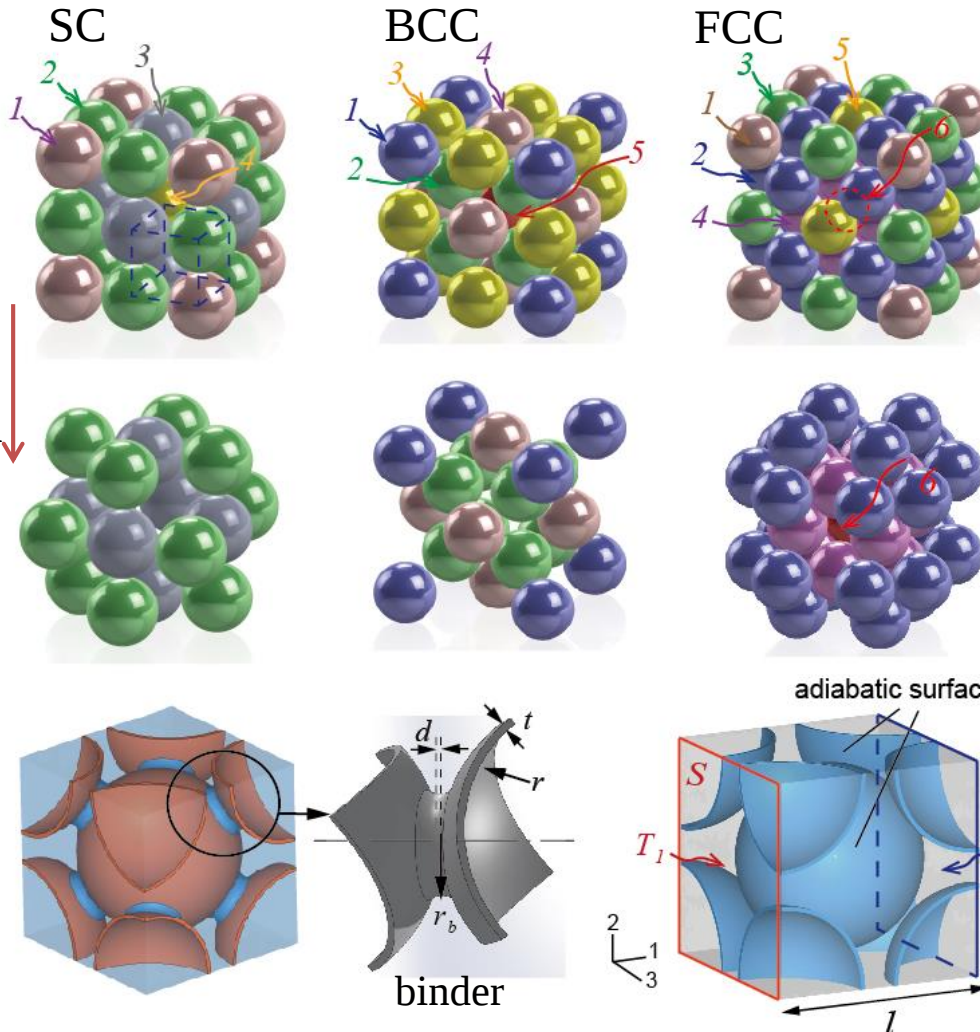
$$R_{Bd} = \beta^{-1} = \lim_{\substack{t \rightarrow 0 \\ K_s \rightarrow 0}} (t/K_s)$$

$$K_{ITC} = \frac{K_c}{1 + \alpha K_c/K_m}, \quad \alpha \stackrel{\text{dimensionless parameter}}{=} R_{Bd} K_m / r$$

$$K_p^{ITC} / K_m = \frac{3 + (s_c - 1)(1 + 2\nu)}{s_m [3 + (s_c - 1)(1 - \nu)] + \alpha [3 + (s_c - 1)(1 + 2\nu)]} \cdot s_c = K_c/K_s, s_m = K_m/K_s$$

2. Methods: numerical simulation

- FEM simulation



Governing Equation

steady heat transfer problem

$$\nabla^2 T = 0$$

$$q = -k \nabla T$$

for cubic symmetric problem, k is a constant.

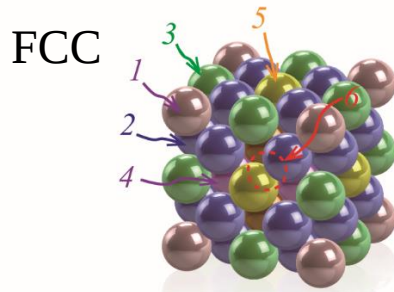
B.C.

specified temperature difference along the $[100]$ direction $T_1 - T_2$.

ETC is evaluated with Fourier's law:

$$K_{eff} = \frac{\iint_S q ds}{(T_1 - T_2) \cdot l}$$

2. Methods: numerical modelling



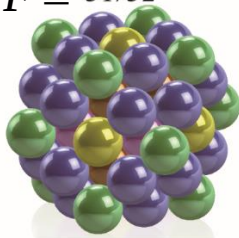
32 particles in one RVE

Different color represents different particle

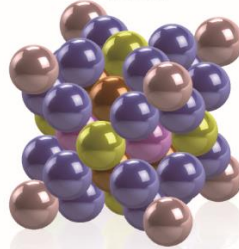
Initial packing fraction: $\phi=0.707$

Number	1	2	3	4	5	6
Effective	1/8	1/2	1/4	1	1/2	1
amount	8	24	12	12	6	1
total	1	12	3	12	3	1

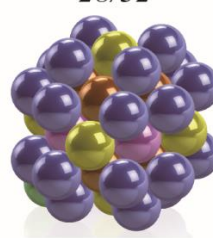
$F = 31/32$



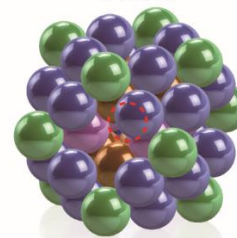
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28/32

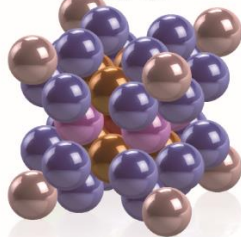


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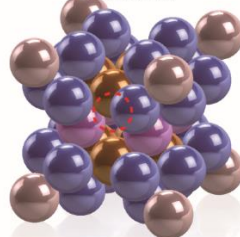


→ 14 different packing arrangements

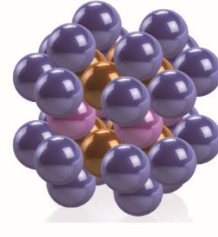
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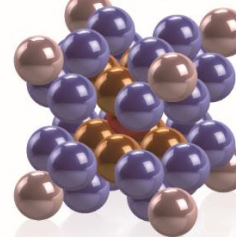
25/32



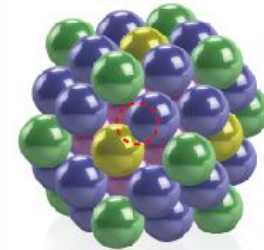
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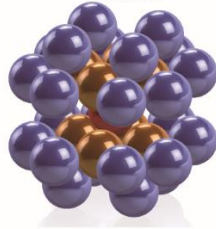
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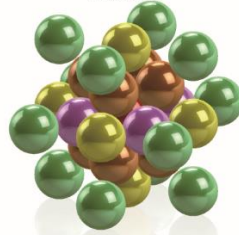
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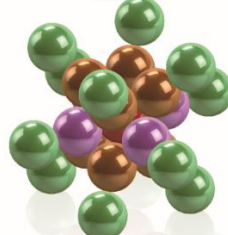
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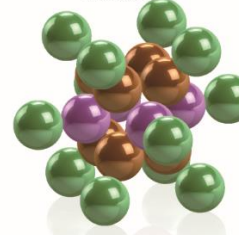
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16/32



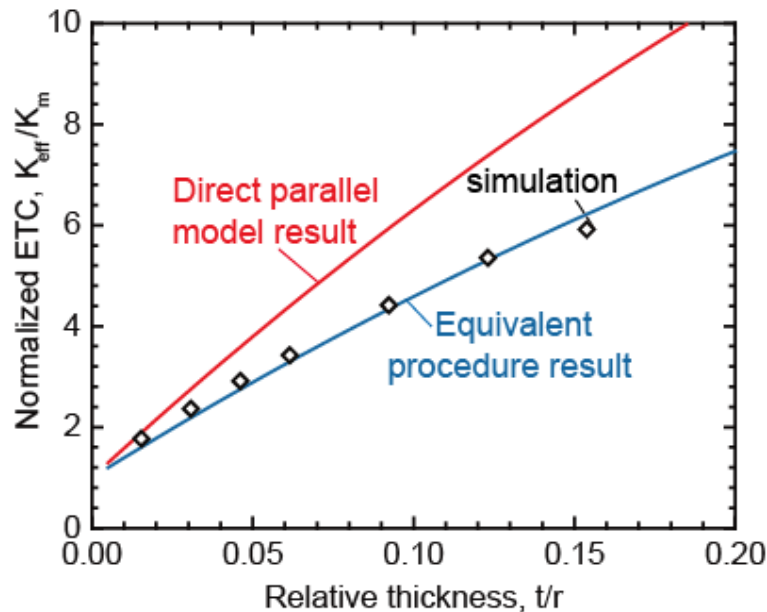
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3. Results

- Verification of the theoretical model**

- compare with a “rule of mixture” model and numerical simulations,
- the proposed equivalent model gives much better consistency.



Shell

- glass (0.8W/m·K),

Matrix and core

- air (0.0257W/m·K)

$$K_c/K_s = K_m/K_s = 1/30$$

Size $r \sim 100 \mu\text{m}$

Binder $r_b = 0.15r$

$$d_b = 0.1r$$

Direct model or “rule of mixture”

$$K_{eff} = \bar{\kappa}_s + \bar{\kappa}_m$$

$$\bar{\kappa}_s = \kappa_s \left(\rho^* / \rho_s \right), \bar{\kappa}_m = \kappa_m \left(1 - \rho^* / \rho_s \right)$$

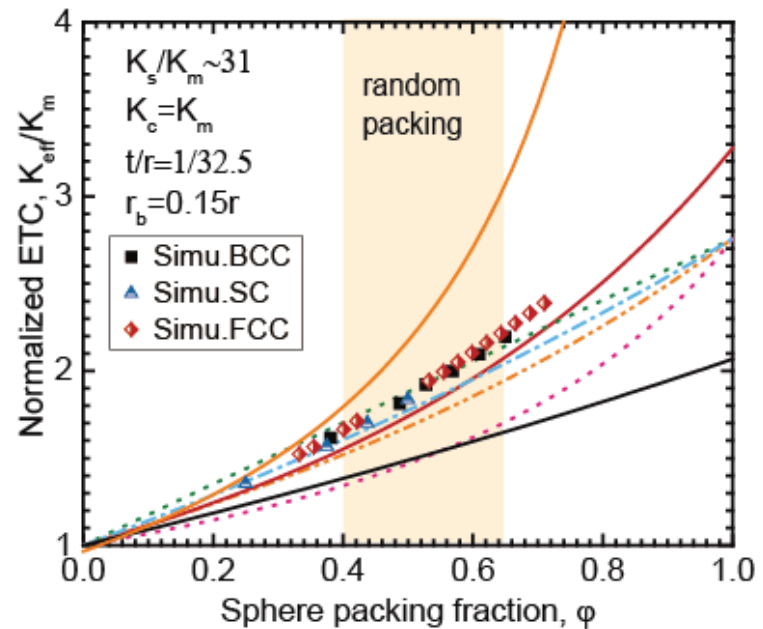
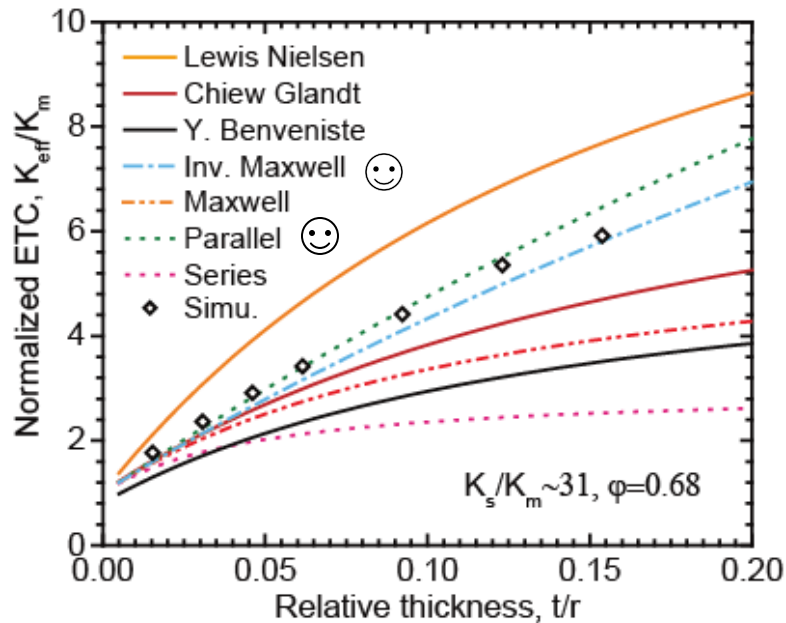
Equivalent procedure

$$K_p/K_s = \frac{3 + (K_c/K_s - 1)(1 + 2\nu)}{3 + (K_c/K_s - 1)(1 - \nu)},$$

$$K_{eff}/K_m = 1 + \phi(K_p/K_m - 1).$$

3. Results

- **Verification of the theoretical model**
 - **Parallel** model and **Inverse Maxwell** model gives the best result.
 - Sphere packing fraction is the main factor that controls ETC, while **packing patterns** has negligible effect.

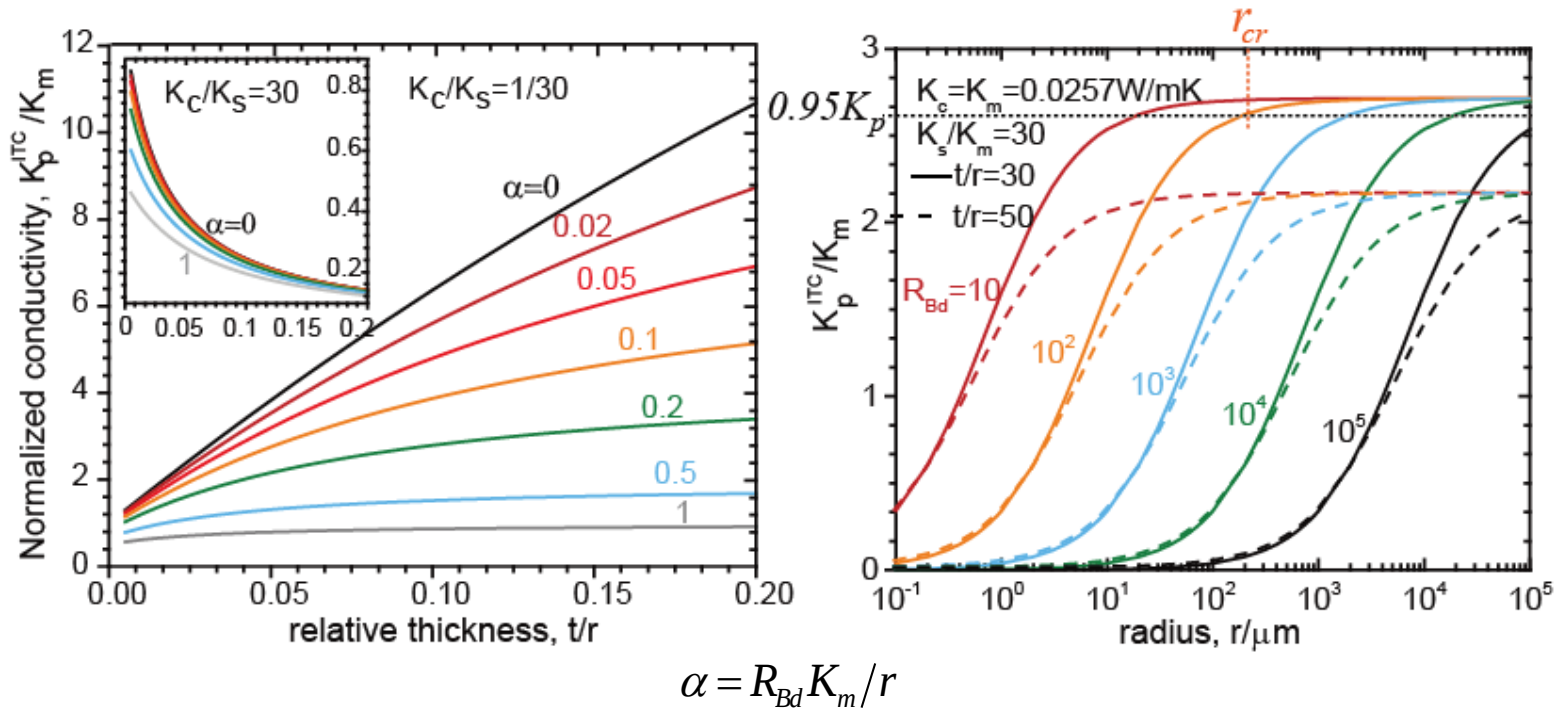


3. Results - theoretical



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- Effect of interfacial thermal resistance (ITR)



- ETC is sensitive to ITR when shells have good conductivity;
- Exists a critical size r_{cr} , larger than which the ITR can be ignored.

$$r_{cr} \approx 20 R_{Bd} K_p$$

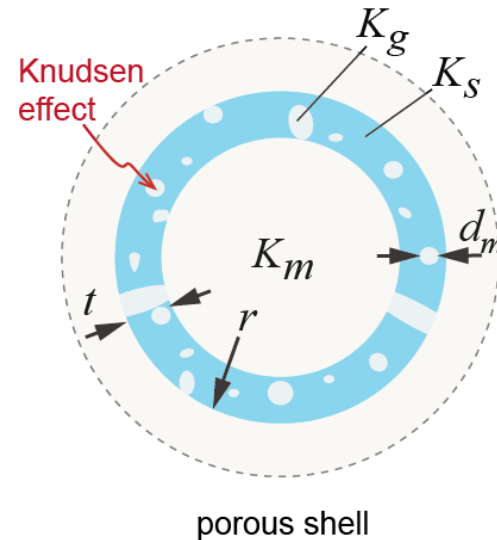
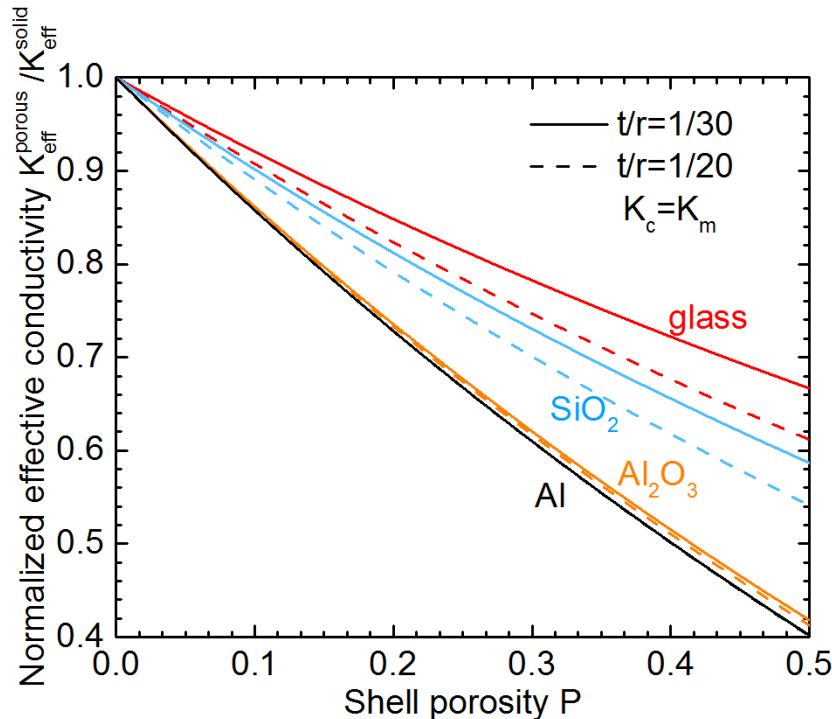
3. Results - theoretical



- Introducing porosity in the shell

- Modelled with two steps, 1) equivalent of the porous shell and 2) using the proposed model;
- Conductivity reduces greatly by partially removing the shell material.

$$\frac{K_{eff}^{shell}}{K_m} = \frac{(1+2P)K_m/K_s + 2(1-P)}{(1-P)K_m/K_s + (P+2)} \quad P \text{ is porosity of shell}$$



3. Results - theoretical

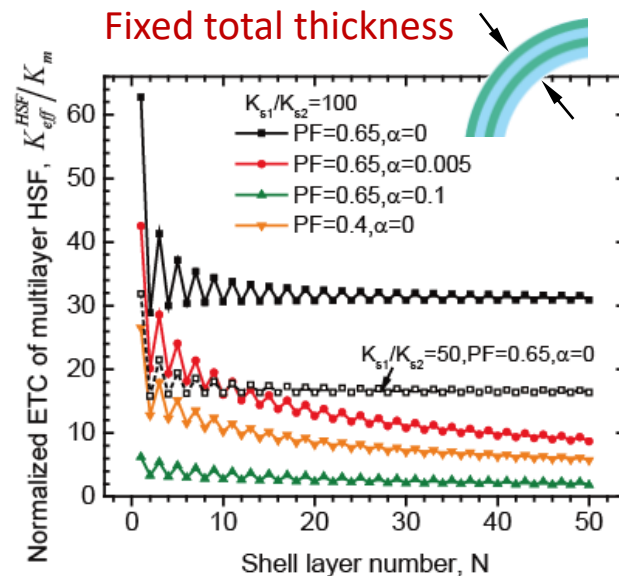
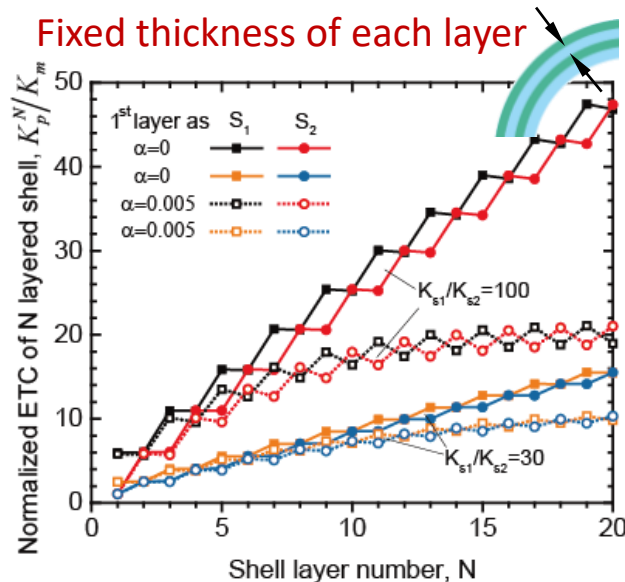
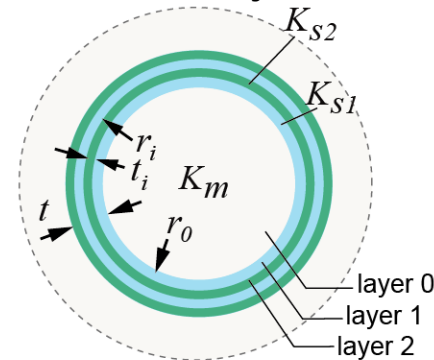


Multilayer shells - supercapacitor

-the normalized ETC of a hollow sphere with i to $i+1$ layer included can be written in a recursive relation:

$$\frac{K_{eff}^{i+1}}{K_m} = f\left(\frac{K_{eff}^i}{K_{s_j}}, \frac{K_m}{K_{s_j}}, \alpha_i, \frac{t_i}{r_i}\right) \quad 0 < i < N$$

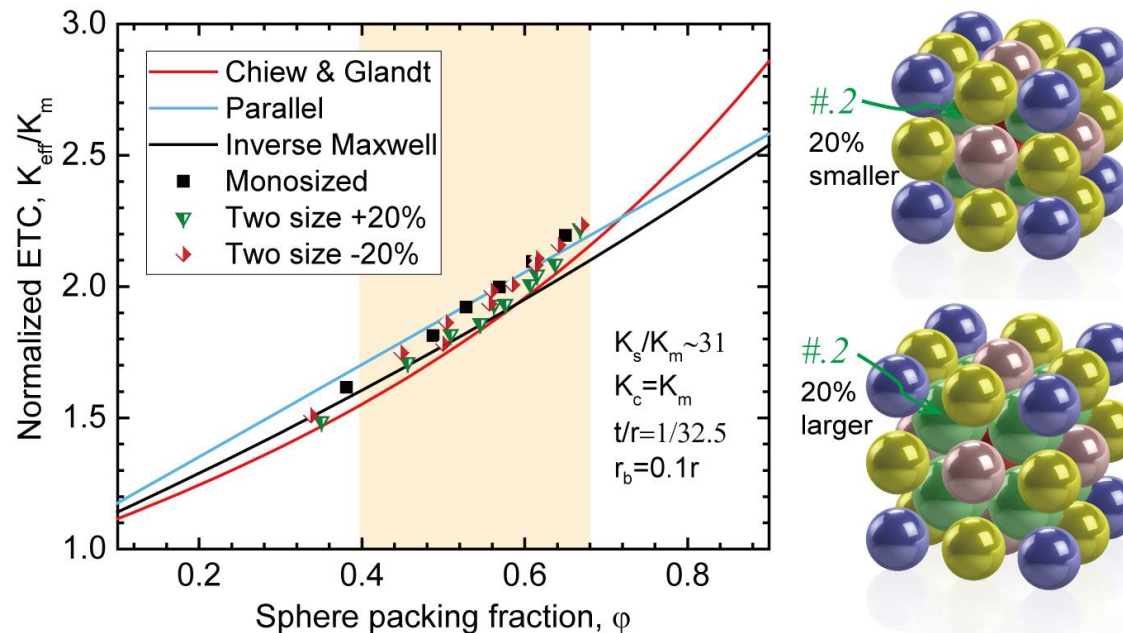
$$\sum_{i=1}^N t_i = t \quad \text{and} \quad \begin{cases} j=1, \text{ odd } i \\ j=2, \text{ even } i \end{cases}$$



-a proper total layer number and material sequence can be designed to tune the ETC of multilayer shell foams.

3. Results - theory and simulations

- How about foams with size distributions?

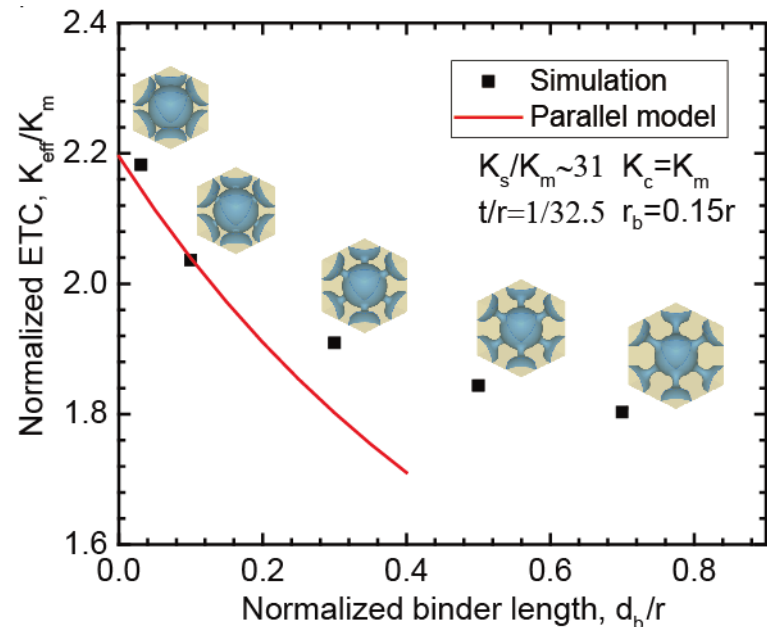
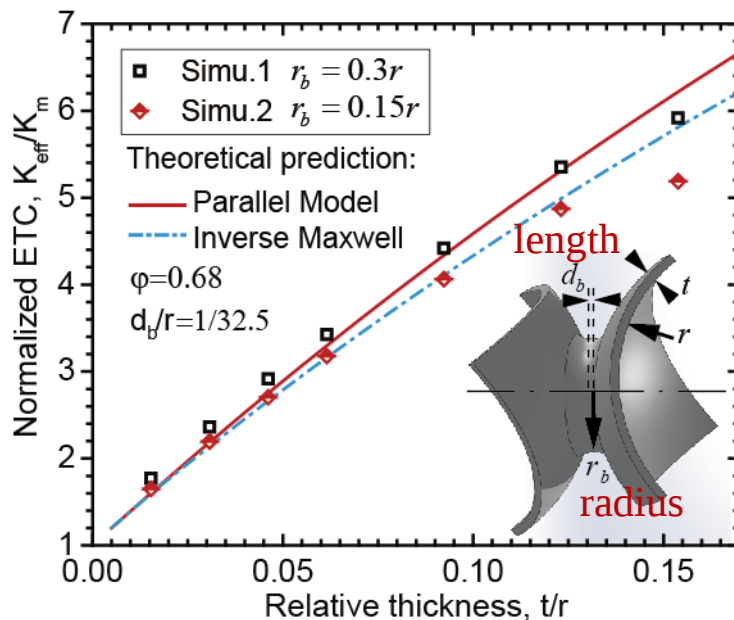


Sphere size distribution also have weak influence on the ETC, suggesting that packing fraction is the main parameter that controls the ETC.



3. Results - theory and simulations

- How does binder shape affects ETC?

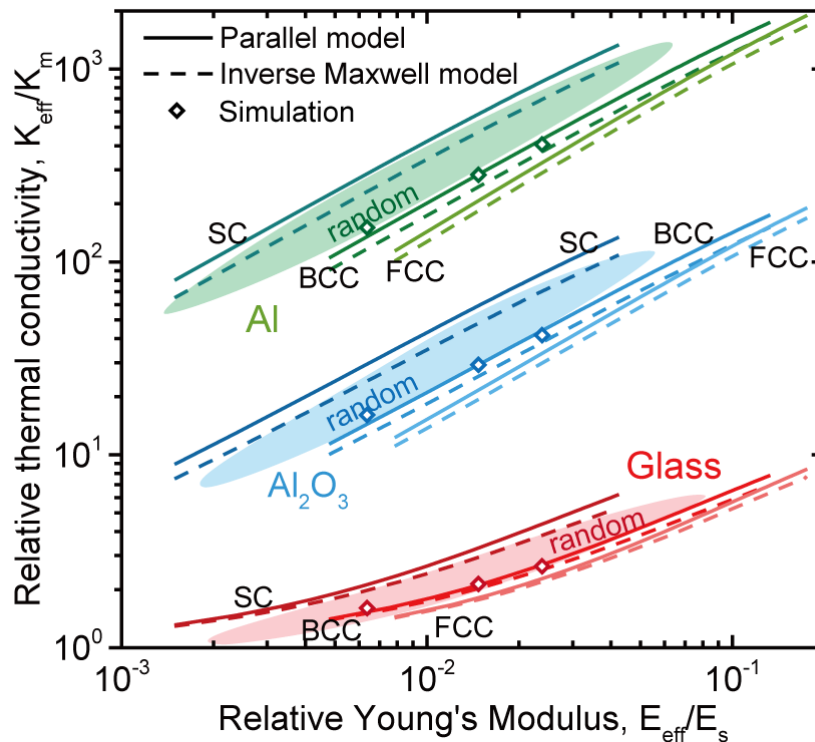


Binder radius r_b has relative small influence;
While the binder length d_b have a more noticeable influence.



3. Results and discussion:

- **Materials property chart**
- Relative thermal conductivity vs relative Young's modulus;
- Using Gibson's fitting equation for mechanical prediction;
- Wide design space.



Mater. Sci. and Eng. A **2003**, 347 (1), 70-85.

$$E_{eff} = [3.85 \times 10^{-3} \theta^2 - 0.294 \theta + 8.33] \times \left(\frac{t}{r} \right)^{1.4 \times 10^{-2} \theta - 1.39}$$

$$\tan \theta = r_b / h_b$$

$$t/r = 0.01 \sim 0.2$$



4. Conclusion:

- We've proposed an equivalence model that gives accurate prediction of the ETC of HSFs;
- The model considers the effects of porous shells, multi-layered shells, and interfacial thermal resistance;
- Two size effects, the Knudsen effect and the interfacial thermal resistance effect, are further evaluated, extending the model to nanoscale foams.

Guidelines for designing HSFs:

- A small size is essential to achieve extremely thermal insulation materials.
- Lower packing fractions, larger shell porosity, longer binder length and higher interfacial thermal resistance lead to lower ETC;
- Packing patterns, binder radius, and sphere size distribution have weak influence.



Thank you for your attention!

Acknowledgement:



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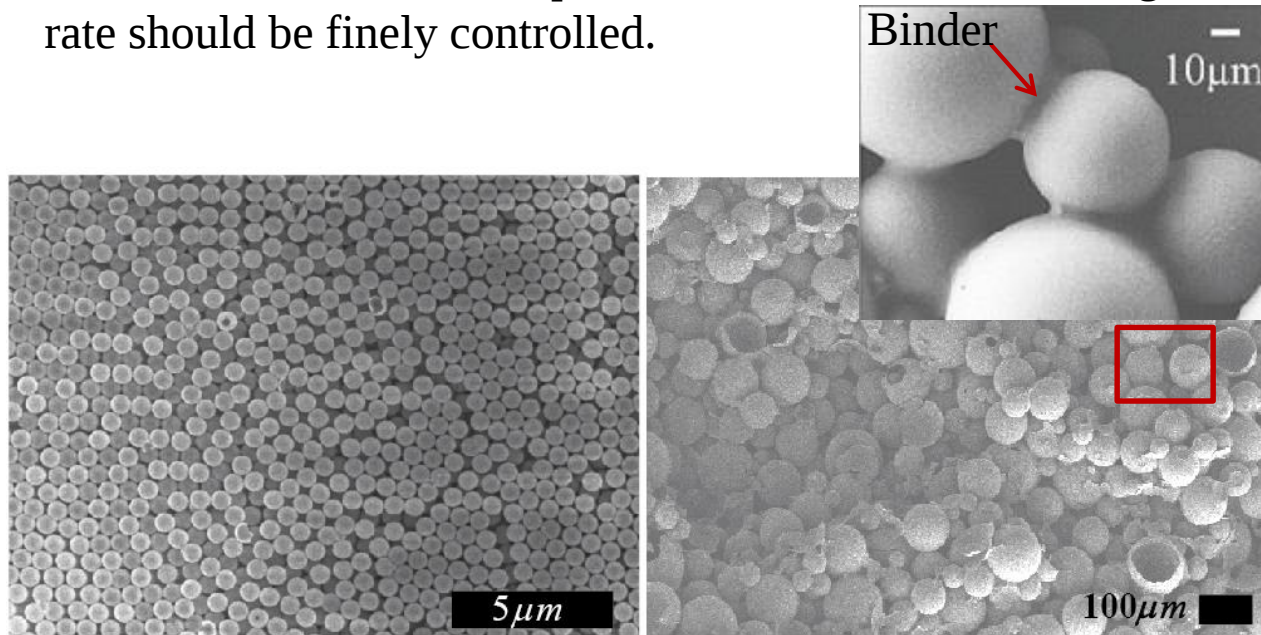
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Supporting material

1. Introduction

- **Problem description**
- Heat transfer in HSF is of particular interest when cooling or heating rate should be finely controlled.



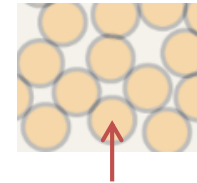
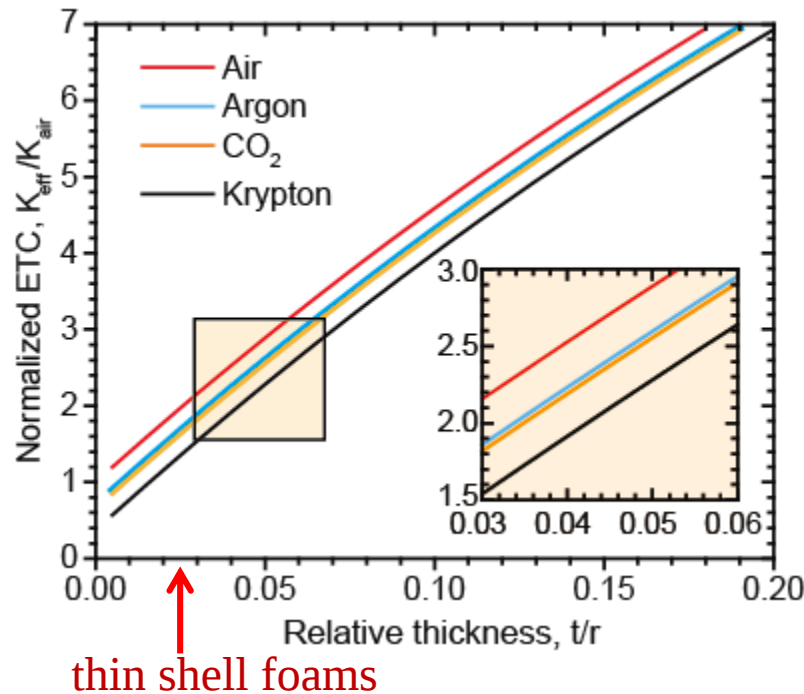
thermal radiation can be estimated using the unit cell method³¹, which gives $K^r = 8F_E^* \sigma \bar{T}^3$, where $\bar{T}$ is the mean temperature, σ is the Stefan-Boltzmann constant, F_E^* is radiation exchange factor ($F_E^* = 1/(P^{-1} - 1)$), and P is porosity. For porous materials $F_E^* = 0.2 \sim 0.6$, when cell length $\sim 100 \mu\text{m}$, $\bar{T} \sim 300\text{K}$, K^r is estimated as $0.8 \sim 2.4 \times 10^{-4} \text{ W/m} \cdot \text{K}$.

3. Results - theoretical



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- Using low thermal conductivity gas filling?

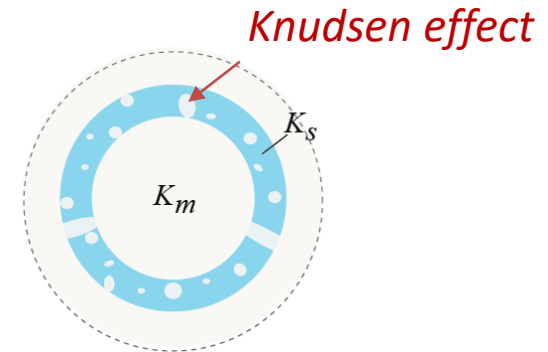
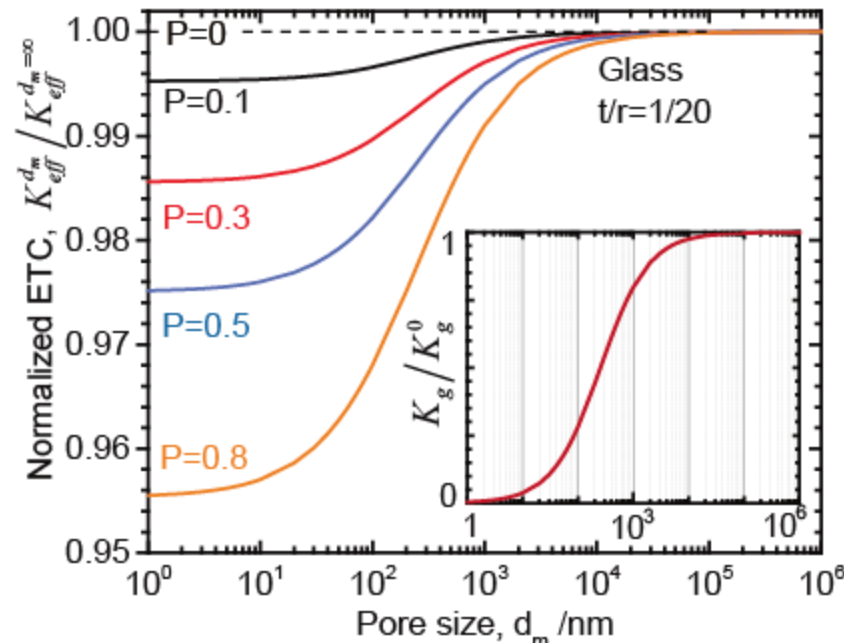


Air 25.7 mW/m·K
Argon 17.9 mW/m·K
Krypton 9.5 mW/m·K
CO₂ 16.8 mW/m·K

Use low thermal conductivity filling gas for **thin shell foams**!

3. Results - theoretical

- Knudsen effect in the pores of porous foams



Knudsen effect

a reduction of thermal conductivity in gases when the size of the cavity encompassing the gas becomes comparable to the mean free path.

The ETC reduction induced by Knudsen effect of inside the porous shell is small, because the pores in the shell have both a small volume fraction and a small K_g^0 / K_s .

K_g^0 is thermal conductivity of a free gas.

Theoretical prediction of effective thermal conductivity:

- Dimensional analysis result**

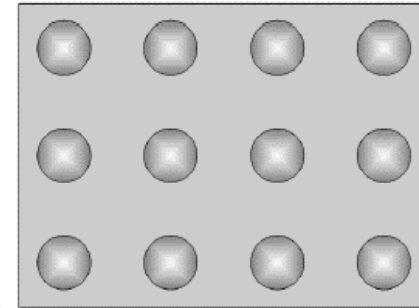
-for foam material

$$\bar{K}_{eff} = \bar{K}_s + \bar{K}_g + \bar{K}_c + \bar{K}_r$$

$$\bar{K}_s = \eta \bar{K}_s (\rho^* / \rho_s)^\alpha, \bar{K}_g = \eta' \bar{K}_g (1 - \rho^* / \rho_s)$$

- Effective thermal conductivity models**

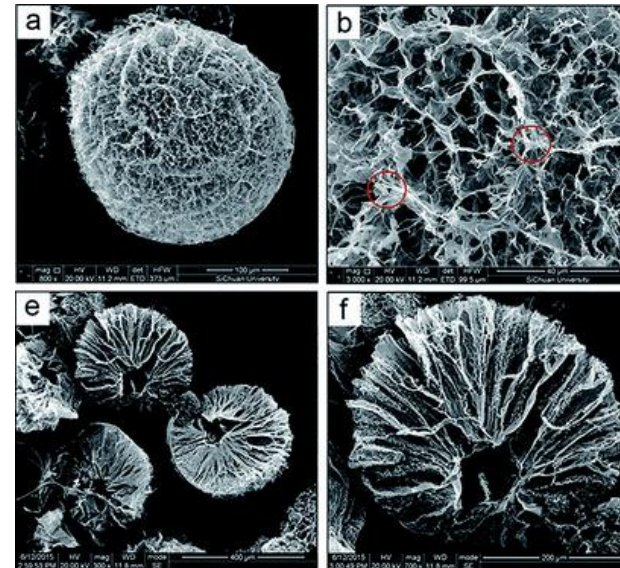
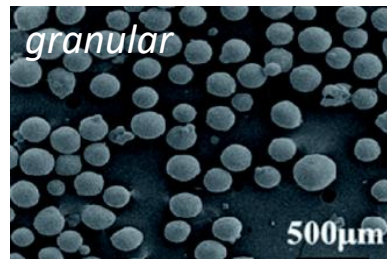
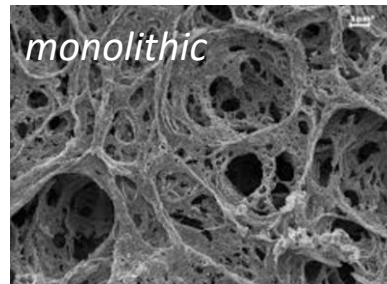
MODEL	EFFECTIVE THERMAL CONDUCTIVITY MODELS
SERIES MODEL	$\frac{K_{eff}}{K_m} = \frac{K_p / K_m}{K_p / K_m + \phi(1 - K_p / K_m)}$, ϕ is volume fraction of particle filling.
PARALLEL MODEL	$\frac{K_{eff}}{K_m} = 1 + \phi(K_p / K_m - 1)$,
MAXWELL MODEL[17]	$\frac{K_{eff}}{K_m} = \frac{(1 + 2\phi)K_p / K_m + 2(1 - \phi)}{(1 - \phi)K_p / K_m + (\phi + 2)}$,
INVERSE MAXWELL MODEL	$\frac{K_{eff}}{K_m} = \frac{(1 + 2\phi)K_p / K_m + 3}{(2 - \phi)K_p / K_m + \phi} \cdot \frac{K_p}{K_m}$,
CHIEW AND GLANDT MODEL[38]	$\frac{K_{eff}}{K_m} = \frac{1 + 2\beta\phi + (K_2 - 3\beta^2)\phi^2}{1 - \beta\phi}$ where $\beta = \frac{K_p - K_m}{K_p + 2K_m}$, K_2 is a function of ϕ . with fitting parameters: $\frac{K_{eff}}{K_m} = \frac{1 + 2\beta\phi + (2\beta^3 - 0.1\beta)\phi^2 + 0.05\exp(4.5\beta) \cdot \phi^3}{1 - \beta\phi}$,
Y. BENVENISTE MODEL[21]	$\frac{K_{eff}}{K_m} = \frac{2\alpha(1 - \phi) + [(1 + 2\phi) + K_p / K_m (2 - 2\phi)]}{\alpha(2 + \phi) + [(1 - \phi) + K_p / K_m (2 + 2\phi)]}$, For perfect interface, $\alpha \rightarrow 0$ $\frac{K_{eff}}{K_m} = \frac{(1 + 2\phi) + K_p / K_m (2 - 2\phi)}{(1 - \phi) + K_p / K_m (2 + 2\phi)}$
LEWIS-NIELSEN MODEL[24]	$K_{eff} = \frac{1 + AB\phi}{1 - B\psi\phi}$, A is shape factor, for sphere A=1.5, $B = \frac{K_p / K_m - 1}{K_p / K_m + A}$, $\psi = 1 + \frac{1 - \phi_m}{\phi_m^2} \phi$, ϕ_m is the maximum filler volume fraction.



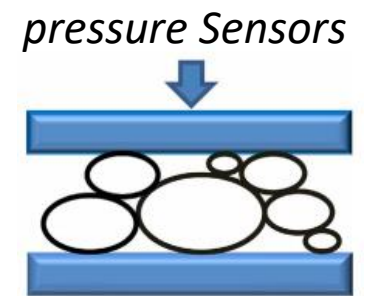
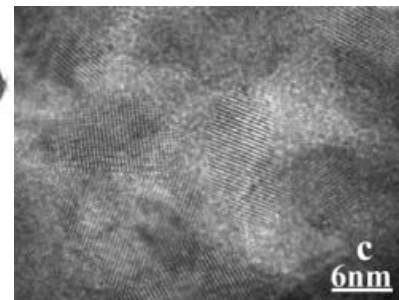
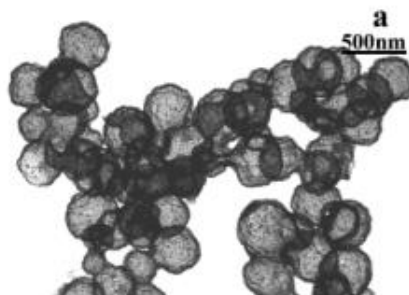
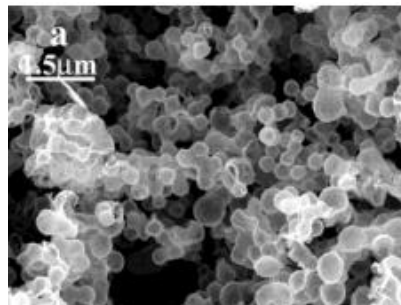
Nearly all models are
for Solid particles
filled materials,
Not for hollow spheres
filled Materials.

Applications:

➤ mesoscale level (10~100nm)

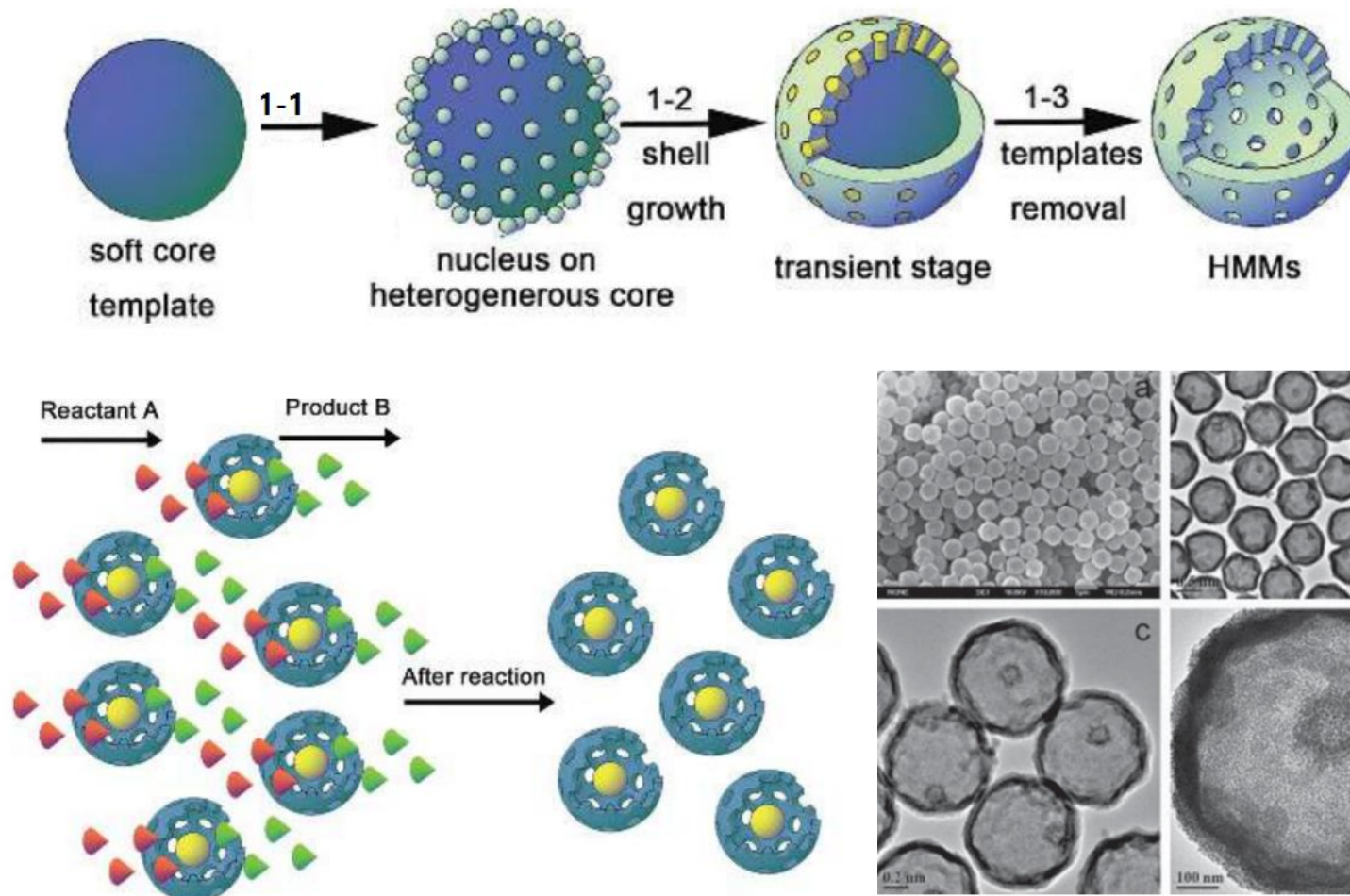


Highly Sensitive WO_3 Hollow-Sphere Gas Sensors



Applications:

- **mesoscale level** (mesoscale hollow-structured materials)



Applications:

➤ mesoscale level (mesoscale hollow-structured materials)

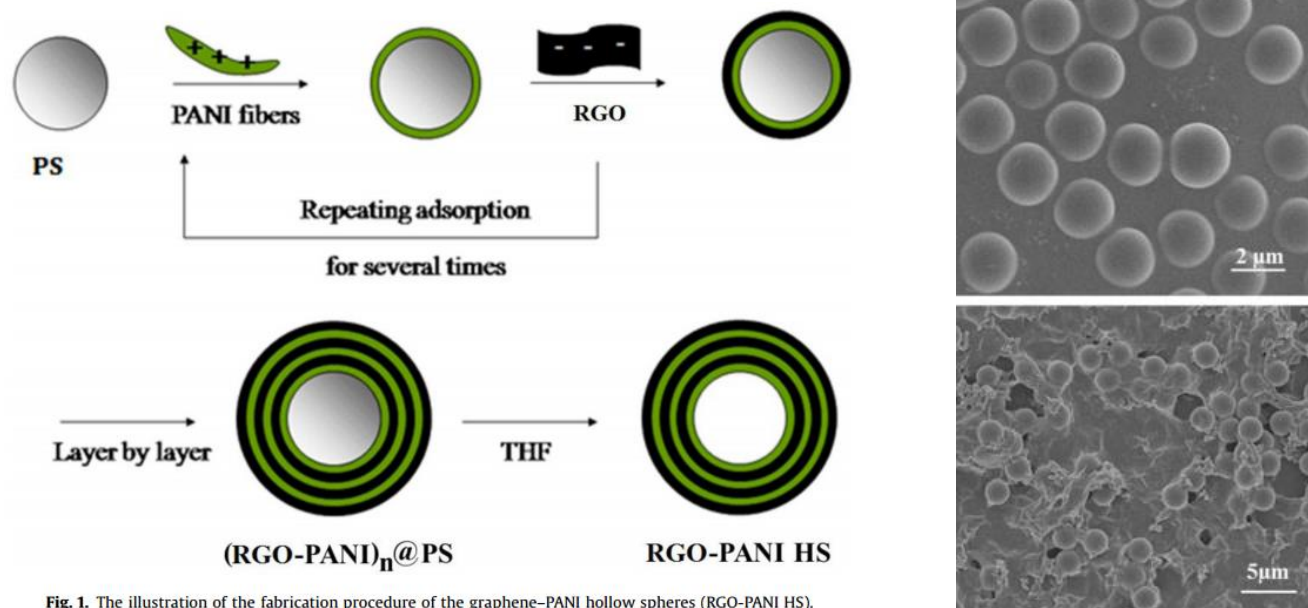


Fig. 1. The illustration of the fabrication procedure of the graphene-PANI hollow spheres (RGO-PANI HS).

Supercapacitor

Shell Spherical Colloids with Movable Cores

