Predicting shim gaps in aircraft assembly with machine learning and sparse sensing

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Abstract

A modern aircraft may require on the order of thousands of custom shims to fill gaps between structural components in the airframe that arise due to manufacturing tolerances adding up across large structures. These shims, whether liquid or solid, are necessary to eliminate gaps, maintain structural performance, and minimize pull-down forces required to bring the aircraft into engineering nominal configuration for peak aerodynamic efficiency. Currently, gap filling is a time-consuming process, involving either expensive by-hand inspection or computations on vast quantities of measurement data from increasingly sophisticated metrology equipment. In either case, this amounts to significant delays in production, with much of the time being spent in the critical path of the aircraft assembly.

In this work, we present an alternative strategy for predictive shimming, based on machine learning and sparse sensing to first learn gap distributions from historical data, and then design optimized sparse sensing strategies to streamline the collection and processing of data. This new approach is based on the assumption that patterns exist in shim distributions across aircraft, and that these patterns may be mined and used to reduce the burden of data collection and processing in future aircraft. Specifically, robust principal component analysis is used to extract low-dimensional patterns in the gap measurements while rejecting outliers. Next, optimized sparse sensors are obtained that are most informative about the dimensions of a new aircraft in these low-dimensional principal components. We demonstrate the success of the proposed approach, known within Boeing as PIXel Identification Despite Uncertainty in Sensor Technology (PIXI-DUST), on historical production data from 54 representative Boeing commercial aircraft. Our algorithm successfully predicts 99\% of the shim gaps within the desired measurement tolerance using around 3\% of the laser scan points that are typically required; all results are rigorously cross-validated.

Keywords: Predictive assembly; Machine learning; Sparse optimization; Sparse sensing; Big data

1. Introduction

Advanced manufacturing is increasingly becoming a data rich endeavor, with big data analytics addressing critical challenges in high-tolerance assembly (Muelaner et al., 2013), operation planning (Lu and Zhang, 1990), quality control (Milo et al., 2015) and supply chains (LaValle et al., 2011). The broad applicability of data science in manufacturing is reviewed in Harding et al. (2006); Lee et al. (2013); Lechevalier et al. (2014); Esmaeilian et al. (2016). Machine learning is a particularly promising tool for extracting actionable patterns in vast quantities of high-dimensional data that are difficult to visualize and/or interpret. Examples of machine learning in manufacturing systems abound, for example using topological data analysis (Guo and Banerjee, 2017), deep learning (Ren et al., 2017), and genetic algorithms to evaluate form tolerances (Sharma et al., 2000). Modern aircraft assembly is at the forefront of integrating big data into manufacturing, with advances in metrology accelerating aircraft manufacturing processes in recent years (Muelaner and Maropoulos, 2010; Jamshidi et al., 2010; Muelaner et al., 2011; Muelaner and Maropoulos, 2011; Martin et al., 2011; Muelaner et al., 2013; Maropoulos et al., 2014), for example in large composite structures (Muelaner and Maropoulos, 2010), in fuselage skin panels (Lowe et al., 2007), and in the wing box (Chouvion et al., 2011).

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Predictive shimming. Aircraft are built to exceedingly high tolerances, with components sourced from around the globe. Even when parts are manufactured to specification, there may be significant gaps between structural components upon assembly. One of the most time-consuming and expensive efforts in part-to-part assembly is the shimming required to bring an aircraft into the engineering nominal shape. Historically, parts have been dry-fit, gaps measured manually, and custom shims manufactured and inserted, often involving disassembly and reassembly. Recent advancements in 3-D scanning have enabled their use for surface measurement prior to assembly, known as predictive shimming (Jamshidi et al., 2010; Muelaner and Maropoulos, 2010; Muelaner et al., 2011; Chouvion et al., 2011; Muelaner and Maropoulos, 2011; Muelaner et al., 2013). There are several patents and papers describing methods of high-tolerance measurement and manufacturing required for predictive shimming (Marsh, 2008; Marsh et al., 2010; Chouvion et al., 2011; Boyl-Davis et al., 2014; Vasquez et al., 2014; Valenzuela et al., 2015; Boyl-Davis et al., 2016; Antolin-Urbaneja et al., 2016). While some cost-effective devices may not provide the fidelity required, higher-fidelity metrology and scanning devices result in overwhelming amounts of data, shifting the burden from time-consuming manual shimming to time-consuming computational processing for predictive shimming. This amounts to significant delays in production, with much of the time being spent in the critical path of assembly. Reducing the burden of data collection and processing, and ultimately reducing delays for optimized aircraft assembly, could have significant financial implications.

Sparse optimization in applications. Digital measurement devices are inherently noisy. Statistical machine-learning can be used to determine when—and where—higher fidelity is required. Additionally, part deformation can occur from measurement to assembly (due to external forces, climate, etc.). While not covered in this paper, machine learning can also be used to augment analysis to predict deformation. Several groups have used dimensionality reduction, such as principal component analysis (PCA) to model deformation in geometric surfaces for improved fitting (Camelio and Hu, 2002; Carnelio and Yim, 2006; Zhang et al., 2006; Lindau et al., 2013). In the automotive industry, PCA was used in combination with sparse sensing to identify key measurement locations for the characterization of compliant part assembly (Camelio and Hu, 2002; Carnelio and Yim, 2006). Such dimensionality reduction methods have also been compared with sparsity promoting techniques for outlier rejection and minimal description of surfaces (Zhang et al., 2006). However, this work is the first to combine RPCA with a scalable greedy sparse sensor optimization for robust predictive shimming in the aerospace industry.

Contributions of this work. In this work, we present an alternative approach to predictive shimming based on machine learning and sparse sensing. Instead of measuring each component of a new aircraft in isolation, we leverage historical production data to learn patterns in the shim gap distributions. In particular, the robust principal component analysis (RPCA) (Candès et al., 2011) provides an estimate of the dominant principal components that is robust to outlier measurements. Robust statistical methods are critically important for evaluating real-world data, as advocated by John W. Tukey in the earliest days of data science (Huber, 2002; Donoho, 2015). RPCA is based on the computationally efficient singular value decomposition (SVD) (Golub and Kahan, 1965), and yields the most correlated spatial structures in the aircraft measurements, identifying areas of high variance across different aircraft. Next, based on the robust principal components obtained from historical data, we design a small subset of key spatial measurement locations that best inform the shim gap distribution of a new aircraft. Our procedure, known within Boeing as PIXel Identification Despite Uncertainty in Sensor Technology (PIXI-DUST), is based on recent advances in convex optimization for sensor placement (Brunton et al., 2016; Manohar et al., 2017a,b). We demonstrate the success of PIXI-DUST on historical production data from 54 Boeing aircraft, predicting 99% of the shim gaps within the desired measurement tolerance using approximately 3% of the available laser scan data. Specific contributions of this work include:

- Machine learning, dimensionality reduction and optimization are used to accelerate high-fidelity, measurement driven aircraft assembly;
- Our novel method extracts features and optimizes gap measurement locations to predict shim gaps in aircraft assembly;
- The proposed algorithm is demonstrated on historic Boeing aircraft production data;
- 99% of shim gaps are predicted within the desired measurement tolerance using 3% of the original laser scan points.
2. Mathematical preliminaries

The results in this work combine robust dimensionality reduction and sparse sensor optimization algorithms to dramatically reduce the number of measurements required to shim a modern aircraft. This section provides a foundation for the methods that will be synthesized and applied throughout the paper.

2.1. Robust principal component analysis

Least-squares regression is highly susceptible to outliers and corrupted data. Principal component analysis (PCA) suffers from the same weakness, making it fragile with respect to outliers. To address this sensitivity, Candès et al. (2011) introduced a robust principal components analysis (RPCA) that decomposes a data matrix $X$ into a low-rank matrix $L$ containing dominant coherent structures, and a sparse matrix $S$ containing outliers and corrupt data:

$$X = L + S. \quad (1)$$

The principal components of $L$ are robust to the outliers and corrupt data in $S$. The RPCA decomposition has tremendous applicability for many modern problems of interest, including video surveillance (Bouwmans and Zahzah, 2014) (where the background objects appear in $L$ and foreground objects appear in $S$), natural language processing (Kondor et al., 2013), matrix completion and face recognition (Wright et al., 2009).

Mathematically, the goal of RPCA is to find matrices $L$ and $S$ that satisfy

$$\min_{L,S} \text{rank}(L) + \|S\|_0 \text{ such that } L + S = X. \quad (2)$$

However, neither the rank($L$) nor the $\|S\|_0$ terms are convex, and this is not a scalable optimization problem. Similar to compressed sensing (Donoho, 2006; Candès et al., 2006), it is possible to solve for the optimal $L$ and $S$ with high probability using a convex relaxation of (2):

$$\min_{L,S} \|L\|_* + \lambda \|S\|_1 \text{ such that } L + S = X. \quad (3)$$

Here, $\| \cdot \|_*$ denotes the nuclear norm, given by the sum of singular values, which is a proxy for rank. The solution to (3) converges to the solution of (2) with high probability if $\lambda = 1/\sqrt{\max(n,m)}$, where $n$ and $m$ are the dimensions of $X$, given that $L$ is low-rank and $S$ is sparse.

The problem in (2) is known as principal component pursuit (PCP), and may be solved using the augmented Lagrange multiplier (ALM) algorithm. The augmented Lagrangian may be constructed as:

$$\mathcal{L}(L, S, Y) = \|L\|_* + \lambda \|S\|_1 + \langle Y, X - L - S \rangle + \mu/2 \|X - L - S\|_F^2. \quad (4)$$

A general solution would solve for the $L_k$ and $S_k$ that minimize $\mathcal{L}$, update the Lagrange multipliers $Y_{k+1} = Y_k + \mu(X - L_k - S_k)$, and iterate until the solution converges. This is outlined in Algorithm 1. For this specific system, the alternating directions method (ADM) (Lin et al., 2010; Yuan and Yang, 2009) provides a simple procedure to find $L$ and $S$. The parameter $\mu$ is discussed more in Yuan and Yang (2009) and Candès et al. (2011).

In the following, RPCA will be used to develop low-dimensional representations for high-dimensional aircraft metrology data (e.g., laser scans or point cloud measurements). In particular, the left singular vectors $\Phi$ of the low-rank matrix $L$ provide robust principal components, and are computed via the SVD:

$$L = \Phi D V^*. \quad (5)$$

These low-rank coherent patterns will then facilitate sparse sensing strategies.
Algorithm 1 Inexact augmented Lagrange multiplier method for Robust PCA
Usage: inexact_ALM_RPCA( X, λ )

1: procedure inexact_ALM_RPCA( X, λ )
2:   Initialize X, L, S, µ
3:   while not converged do
4:     minimize L(L, S, Y) by updating L and S only once
5:     Y = Y + µ(X - L - S)
6:   update µ
7:   end while
8: return L, S
9: end procedure

2.2. Sparse sensor optimization

Despite the growing abundance of measurement data across the engineering sciences, systems are often fundamentally low-dimensional, exhibiting a few dominant features that may be extracted using dimensionality reduction, such as the RPCA above. The existence of low-rank patterns also facilitates sparse sampling, as there are only a few important degrees of freedom that must be characterized, regardless of the ambient measurement dimension. In the past decades, tremendous advances have been made for the sparse sampling of low-rank systems.

Compressed sensing (Donoho, 2006; Candès et al., 2006) provides powerful mathematical guarantees for the reconstruction of high-dimensional data that is sparse in a universal encoding basis, such as a Fourier or wavelet basis. The resulting algorithms are general, enabling the reconstruction of unknown signals from surprisingly few measurements. However, if additional information is available about the structure of the unknown signal (e.g., it is an image of a human face, or a point cloud scan of a particular aircraft part), then the number of measurements required may be further reduced, often dramatically. In these cases, a tailored basis may be constructed, for example using PCA or RPCA, and sparse sensors can then be optimized to identify the few relevant coefficients in this basis. These methods are generally referred to as gappy sampling (Everson and Sirovich, 1995; Willcox, 2006) or empirical interpolation methods (EIM) (Barrault et al., 2004; Chaturantabut and Sorensen, 2010; Drmac and Gugercin, 2016), including the discrete empirical interpolation method, or DEIM. Sparse sampling in a tailored basis has been widely applied to problems in the imaging sciences as well as to develop reduced order models of complex physics, such as unsteady fluid dynamics. We briefly summarize these various sparse sampling methods in Sec. 3, following the review of Manohar et al. (2017a).

3. Methods

In this work, we accelerate the predictive shimming of aircraft by combining sparse sensor optimization with a robust low-dimensional representation of the historical manufacturing data obtained from RPCA. The resulting method, referred to as PIxel Identification Despite Uncertainty in Sensor Technology (PIXI-DUST), is shown schematically in Fig. 1. The term uncertainty in sensor technology indicates that the resulting algorithm should be agnostic to the specific metrology technology.

3.1. Reconstruction of high-dimensional states from sparse measurements

High-dimensional scan data is transformed to a low-dimensional representation using the RPCA basis of features Φr ∈ Rn×r

\[ \begin{align*} \mathbb{R}^n \xrightarrow{\Phi_r} \mathbb{R}^r, \end{align*} \]

where Φr is obtained from the first r left singular vectors of the low-rank matrix L in (5). The rank r of the library Φr is often much smaller than the data dimension n. Explicitly, each data instance x ∈ Rn can be written

\[ x = \sum_{k=1}^{r} a_k \phi_k = \Phi_r a, \]

4
where the basis coefficients $a \in \mathbb{R}^r$ provide its low-dimensional representation.

**Problem Statement.** Suppose we cannot access the full state $x$, but only $p$ point measurements of $x$ within the *observation* vector $y \in \mathbb{R}^n$

$$y = Cx = C\Phi_r a$$

where $C \in \mathbb{R}^{p \times n}$ is the point measurement operator and $\gamma_i$ are the indices of the measurement locations. Here, we are imposing the following structure on the measurement matrix $C \in \mathbb{R}^{p \times n}$

$$C = [e_{\gamma_1} \mid e_{\gamma_2} \mid \ldots \mid e_{\gamma_p}]^T,$$

where $\gamma_i \in \{1, \ldots, n\}$ indexes the high-dimensional measurement space, and $e_{\gamma_i}$ are the canonical unit vectors consisting of all zeros except for a unit entry at $\gamma_i$. This guarantees that each row of $C$ only measures from a single spatial location, corresponding to a point sensor. Then, the reconstruction of the remaining state reduces to a least squares estimation problem for the coefficients

$$\hat{a} = (C\Phi)^\dagger y,$$

where $\dagger$ is the Moore-Penrose pseudoinverse. This procedure, originally known as *gappy POD* (Everson and Sirovich, 1995), was first used to reconstruct facial images from a subsampled pixel mask. Importantly, it permits a drastic reduction in computation by solving for $r \ll n$ unknowns. The full state estimate $\hat{x}$ is subsequently recovered using the feature basis

$$\hat{x} = \Phi_r \hat{a}.$$
Because of tight manufacturing tolerances and a high degree of reproducibility from part to part, it is assumed that the matrix $X$ possesses low-rank structure. As described above, there may be sparse outliers that will corrupt these coherent features, motivating the use of RPCA to extract the dominant features. The data matrix is decomposed as $X = L + S$, and then the low-rank matrix $L$ is decomposed into coherent features given by the matrix $Φ$, as in (5). We generally use the first $r$ dominant columns of $Φ$, corresponding to the $r$ features that explain the most variance in the data. The SVD provides the best rank-$r$ least squares approximation for a given rank $r$ by solving the following optimization problem

$$\min_{L} \|L - \hat{L}\|_2 \text{ subject to } \text{rank}(L) = r.$$ 

The explicit minimizer of this expression is given by the $r$-truncated SVD of $L = Φ_r D_r V_r^T$. The target rank $r$ must be carefully chosen so that the selected features only include meaningful patterns and discard noise corresponding to small singular values. The subsequent left singular vectors $Φ_r$ are the desired low-rank features that span the columns of $L$. The truncation parameter $r$ ultimately determines the minimal number of observations necessary for a well-posed inverse problem (10), and is determined using the optimal singular value truncation threshold of Gavish and Donoho (2014).

3.3. Proposed sensor selection algorithm

We now turn to the problem of optimizing gap measurements for reconstruction via the linear inverse operation. We modify the observations to include i.i.d. measurement white noise $ξ \sim N(0, \sigma^2 I)$

$$y = CΦ_r a + ξ. \quad (13)$$

The goal of sensor selection is to quantify the expected error as a function of the measurement locations, and then formulate the optimal sensor selection $C$ as the unique minimizer of this expected error. We quantify the distribution of the estimation error $a - \hat{a}$ using the $η$-confidence ellipsoid, $ε_η$, that contains $a - \hat{a}$ with probability $η$. We characterize the “smallness” of this ellipsoid by its volume

$$\text{vol}(ε_η) = α_{η,r} \det Σ^{\frac{1}{2}}, \quad (14)$$

where $α_{η,r}$ only depends on $η$ and $r$, and $Σ$ is the matrix of error covariances between $a$ and the least squares estimate $\hat{a}$ (10)

$$Σ = \text{Var}(a - \hat{a}) = \sigma^2 [(CΦ_r)^T CΦ_r]^{-1}. \quad (15)$$

Note that decreasing the matrix volume or determinant of $Σ$ would simultaneously shrink all components of the error covariance. Although the features $Φ_r$ and noise $ξ$ are predetermined, we can design point measurements $C$ to minimize the volume of the error covariance. Optimization of the determinant/volume, also known as the $D$-optimal experimental design criterion, is formulated as follows

$$\max_{C} \log \det \left[ (CΦ_r)^T CΦ_r \right]. \quad (16)$$

Thus, sensor selection is cast as an optimization problem over all possible measurement locations to minimize the reconstruction error. Intuitively, the best sensors for reconstruction are also most descriptive of the shim gap data.

The brute force search over all $n \choose p$ possible sensor selections of $γ_i$ in (16) is combinatorial and computationally intractable even for moderately large $n$ and $p$. A common solution is to relax the binary 0-1 indicators for subset selection to fractional weights over the entire space of measurements. In such formulations, the problem is convex and can be solved using standard optimization techniques and semidefinite programs in $O(n^3)$ runtime. Instead, we advocate a greedy matrix volume maximization scheme using the pivoted matrix QR factorization. This alternative viewpoint treats the selected measurements as an optimal row selection of the linear operator $CΦ_r$, which designs sampled features $CΦ$, to be as orthogonal to each other as possible. Since the row selected columns $Φ_i$ are no longer orthonormal, greedy row selection methods attempt to maintain near-orthonormality of the features $CΦ_i$ for a numerically well-conditioned inverse.

The QR factorization is the first step in computing the SVD and many other eigendecomposition and linear solution methods. The pivoted QR factorization expresses its argument as a product of unitary and upper-triangular matrices $U$ and $R$, as well as the desired permutation matrix, $C^T$

$$AC^T = QR. \quad (17)$$
The pivoted QR is widely used to select optimal interpolation points for quadrature (Seshadri et al., 2016), polynomial interpolation (Sommariva and Vianello, 2009), model reduction (Drmac and Gugercin, 2016) and Matlab software solvers for underdetermined linear systems. This procedure selects the optimal “gap interpolation points” in a basis of shim gap principal components.

We now describe how the pivoting procedure works to decrement the resulting determinant at each pivoting step. Recall that the determinant of a matrix, when expressed as a product of a unitary factor and an upper-triangular factor, is the product of the diagonal entries in the upper-triangular factor:

\[ |\det \mathbf{AC}^T| = |\det \mathbf{Q}| |\det \mathbf{R}| = \prod_i |r_{ii}|. \]  

(18)

The pivoted QR alters the matrix \( \Phi_r^T \) with column selection (pivoting) \( \Phi_r^T \mathbf{C}^T \) to enforce the following diagonal dominance structure on these diagonal entries (Drmac and Gugercin, 2016):

\[ \sigma_i^2 = |r_{ii}|^2 \geq \sum_{j=1}^k |r_{jk}|^2; \quad 1 \leq i \leq k \leq m. \]  

(19)

The column pivoting iteratively increments the volume of the pivoted submatrix by selecting a new pivot column with maximal 2-norm, then subtracting from every other column its orthogonal projection onto the pivot column (see Algorithm 2). In this manner the QR factorization with column pivoting yields \( r \) sensor locations (pivots or permutation indices) that best characterize the \( r \) basis modes \( \Phi_r \):

\[ \Phi_r^T \mathbf{C}^T = \mathbf{QR}. \]  

(20)

This computes the best row selection \( \mathbf{C}\Phi_r \) for a well-conditioned inverse in a uniquely determined linear system. However, in the overdetermined case (\( p > r \)) there are more measurements than principal components. The solution in this case is the Moore-Penrose pseudoinverse, the best least-squares regressor to the measurements. Our method uses the pivoted QR factorization of \( \Phi_r \Phi_r^T \) to select more sensors than principal components, where the column pivots are selected from \( n \) candidate measurement indices based on the observation (denoting \( \Theta = \mathbf{C}\Phi_r \))

\[ \det \Theta^T \Theta = \prod_{i=1}^r \sigma_i (\Theta \Theta^T). \]  

(21)

Accordingly, optimized measurement indices are given by the column permutation matrix \( \mathbf{C}^T \) from the pivoted QR factorization of \( \Phi_r \Phi_r^T \),

\[ (\Phi_r^T \mathbf{C})^T = \mathbf{QR}. \]  

(22)

The QR pivoting algorithm (Algorithm 2) is one of the most powerful matrix volume maximization schemes of modern computing, and is widely implemented and optimized in most standard scientific computing packages and libraries, including Matlab, LAPACK, NumPy, among others. Bounds for QR pivots in relation to the optimal row selection are derived in Drmac and Gugercin (2016), and shown to be tighter than competing methods. We have extensively compared QR pivoting with competing convex optimization & DEIM methods to confirm its superior performance (Manohar et al., 2017a). Similar methods have been employed in variable selection for both linear and nonlinear classification tasks (Brunton et al., 2016; Manohar et al., 2017b; Guo et al., 2018).

### 3.4. Computational considerations

The pivoted QR algorithm presented in Algorithm 2 offers several advantages over competing sensor placement methods. In the \( p = r \) case, pivoted QR has \( O(nr^2) \) runtime, in contrast to standard convex optimization methods that require matrix factorizations of \( n \times n \) matrices leading to \( O(n^3) \) runtime within each iteration. Here, the state dimension \( n \) is the limiting factor in the computation since \( r \ll n \). When \( p > r \), our oversampled pivoted QR algorithm requires a single \( O(n^3) \) QR factorization and results in a hierarchical list of \( n \) optimized pivots, with the first \( p \) pivots optimized for sampling \( \Phi_r \) for any \( p > r \). Alternative optimizations recompute the factorization for each new \( p \), and there are no bounds on the distance of the convex relaxation solution to the optimal sensor selection, due to rounding
Algorithm 2 QR factorization with column pivoting of $A \in \mathbb{R}^{n \times m}$

Greedy optimization for placing $p$ sensors $\gamma$ from gap principal components.

Usage: \texttt{QRpivot}( $\Phi^T_r$, $p$ ) (if $p = m$)
\texttt{QRpivot}( $\Phi, \Phi^T_r$, $p$ ) (if $p > m$)

1: \textbf{procedure} \texttt{QRpivot}( $A$, $p$ )
2: \quad $\gamma \leftarrow \emptyset$
3: \quad for $k = 1, \ldots, p$ do
4: \quad \quad $\gamma_k = \arg \max_{j \in \gamma} \|a_j\|_2$
5: \quad \quad Find Householder $\tilde{Q}$ such that $\tilde{Q} \cdot \begin{bmatrix} a_{kk} & \cdots & a_{nk} \\ \vdots & \ddots & \vdots \\ a_{nk} & \cdots & 0 \end{bmatrix} = \begin{bmatrix} * & 0 & \cdots & 0 \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & * \end{bmatrix}$ \hspace{1cm} $\triangleright \ast$'s represent nonzero diagonal entries in $R$
6: \quad \quad Remove from all columns the orthogonal projection onto $a_{\gamma_k}$
7: \quad \quad $A \leftarrow \text{diag}(I_{k-1}, \tilde{Q}) \cdot A$
8: \quad \quad $\gamma \leftarrow \gamma \cup \gamma_k$
9: \quad end for
10: \textbf{return} $\gamma$
11: \textbf{end procedure}

procedures applied to the fractional weights to obtain the selection. In addition, QR runtime can be significantly reduced by terminating the procedure after the first $p$ pivots are obtained. The operation can be accelerated further using randomized techniques, for instance, by the random selection of the next pivot (Drmac and Gugercin, 2016) or by using random projections to select blocks of pivots (Martinsson et al., 2017; Martinsson, 2015; Duersch and Gu, 2015).

3.5. PIXI-DUST for shim gaps

Assuming that shim gaps are well characterized by $r$ low-rank features $\Phi_r$, we seek the optimal $p$ gap measurement locations that best characterize the remaining $n - p$ shim gaps. Subsequently the $n - p$ unknown gaps for a new aircraft are predicted using only $p$ measured gap values at the optimal locations. These optimal locations are obtained in the training stage using the QR column pivoting method above, effectively selecting measurement locations from regions of strong spatial correlation.

Unknown shim gaps for a new aircraft are predicted (approximated) given only its gap measurements $y \in \mathbb{R}^r$ at a few optimized locations. Prediction is accomplished with a linear regression on the trained features (11). We solve for the unknown feature coefficients $a$ using only the given gaps at optimal locations in the index set $I$, then predict all remaining gaps using (10), i.e., $a = (C \Phi)^\dagger y = \Phi(I,:)^\dagger y$, and the prediction (11). Finally, we assess the success of the prediction with the percentage of points that differ from the true gap values by less than $0.005^\circ$, which is the desired measurement tolerance for a laser scanner, or equivalently, the number of indices $i$ out of $n$ total that satisfy $|x_i - \hat{x}_i| < 0.005$.

Finally, it is useful to decompose a large aircraft structure into several smaller shim components, which are each analyzed independently. As we see, this improves the prediction performance and reduces the number of measurements by ensuring that the data remains tightly correlated across aircraft.

4. Results

We demonstrate the performance of the PIXI-DUST architecture on production data from a challenging part-to-part assembly on a Boeing aircraft. This data set consists of 10076 laser gap measurements of the part assembly for 54 different production instances of the same aircraft type. Measurement locations are aligned between the instances, making the data amenable to SVD. We build a low-order model of the shim distribution using RPCA and then design optimized measurement locations based on these data-driven features. We train the model on 53 aircraft and then validate on the remaining aircraft; this process is repeated for all 54 possible training/test partitions to cross-validate the results. Thus, a data matrix $X \in \mathbb{R}^{10076 \times 53}$ of training data is constructed, in which each column contains all of the shim gaps.
The low-rank decomposition of $X$ via RPCA is shown in Fig. 2. The singular value distribution indicates dominant low-rank structure, and the principal components illustrate the coherent correlated patterns observed in the shim gap data. RPCA has a tunable parameter $\lambda$ that controls the strength of outlier rejection or the number and magnitude of nonzero entries in $S$. In practice, $\lambda$ is set to an optimal value determined by the dimensions of the matrix, which in our case is $\lambda = 1/\sqrt{\max(10076, 53)}$.

### 4.1. Prediction results for entire wing to body join

First, we investigate sensor optimization and shim prediction considering the entire data set as single large structure. In the next section, we decompose the data into several individual shim segments.

Based on the PIXI-DUST algorithm, $r = 31$ RPCA modes and $p = 100$ optimized measurements are identified using oversampled QR pivoting. Potential overfitting may occur when fitting more than 100 measurements to these 31 features, since some measurements may be contaminated by noise not reflected in the feature space. Thus, increasing the number of measurements further does not yield substantial improvements in prediction accuracy.

The $p = 100$ optimized sensors consist of less than 1% of the original 10076 gap measurements. From these few measurements, the remaining shim gaps are predicted with high accuracy. Figure 3 shows the median pointwise absolute error between the predicted and true shim values. The prediction is successful with high probability across the 53 cross-validation cases. The absolute error distribution for all points across all tests, shown in Figure 4, reveals that 87% of all points are predicted within $\pm 0.005\degree$. The few gap locations with error larger than the required $\pm 0.005\degree$ will likely not contribute significantly to a manufactured shim, as these are averaged out in splining and fabrication. The few points that are outside of this prediction tolerance will be outweighed by the numerous successfully predicted gaps in the fabrication of the shim, as around 1000 laser scan points are used to define each shim.

### 4.2. Prediction results for segmented shims

The number of incorrectly predicted gaps can be reduced by training our method on each individual shim part separately, as there are seven individual shim segments in the assembly considered. Considering each shim segment separately helps to ensure that data is tightly correlated across aircraft, improving
Figure 3: Median gap prediction error. The median prediction error at almost all spatial locations falls below the manufacturing measurement tolerance of 0.005 inches. The median at each point is taken across 53 instances of cross-validated prediction. The subsequent figure plots the probability distribution of prediction error.

Figure 4: Pointwise absolute error distribution. The normalized histogram of absolute error across 534,028 total validation points (10076 gaps × 53 aircraft) is shown on the right. The cumulative distribution function (CDF) of this distribution (left) shows that 87% of the predicted gap absolute errors are less than desired tolerance of 0.005 inches, indicated by the red threshold.

Prediction. It is possible to reduce the number of total measurements and improve the overall performance using this segmentation approach.

Figure 5 displays the seven separately manufactured shim segments, as well as the sensor ensembles for each shim. Prediction results are shown in Table 1. Prediction accuracy is vastly improved, and 96-99% of the shim gap locations are predicted to within the desired ±0.005" tolerance. Furthermore, we note that that the rates of optimal measurements $r$ vary from anywhere between 2% to 6% of all points within the shim, which indicates that some shims are higher-dimensional and require more features (hence, sensors) to be fully characterized. This is also reflected in the sensor ensembles in Fig. 5.

Figure 6 shows the optimized sensor locations for the second shim segment. As the number of sensors is increased from $p = 26$ to $p = 100$, the sensor density increases near the originally identified points. Figure 7 shows the error distribution as the number of sensors is increased, indicating that there is a performance cap at a certain small number of sensors. Finally, Fig. 8 shows the error distributions of predicted shim gaps for all seven segments, comparing the performance of optimized sensors versus randomly chosen measurement locations. In all cases, optimized sensors result in significantly more

<table>
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<tr>
<th>Shim No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Accurate</td>
<td>97.90</td>
<td>98.05</td>
<td>99.82</td>
<td>99.94</td>
<td>99.99</td>
<td>99.03</td>
<td>99.97</td>
</tr>
<tr>
<td>Optimal sensors (avg)</td>
<td>26</td>
<td>26</td>
<td>25</td>
<td>26</td>
<td>25</td>
<td>26</td>
<td>25</td>
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<tr>
<td>Total points</td>
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<td>1116</td>
<td>453</td>
<td>692</td>
<td>709</td>
<td>768</td>
<td>664</td>
</tr>
</tbody>
</table>

Table 1: Segmented prediction results show vastly improved prediction accuracies, with 97-99% of gaps predicted to within the desired 0.005 inch measurement tolerance.
Figure 5: **Gap measurements.** This figure plots high-fidelity gap measurement locations segmented into 7 canonical shim regions (left). Ensembles of measurement locations (sensors) selected by our method are pictured on the right. Sensors are sized by the number of times they are selected by our method across 53 different gap measurement training sets. They are more concentrated around edges and corners, which is where we expect more systemic gap variation.

Figure 6: **Optimal sensors for single segment.** Optimal sensors of our method (red) are plotted over gap data (yellow corresponds to larger gap sizes) for shim segment #2 (Fig. 5). (left) 26 optimal measurement locations required to predict all unmeasured gaps with 98.05% accuracy (Table 1). Since this shim is well-described by as few as \( r = 26 \) principal components, the minimum required number of optimal sensors is 26. Our method can also increase the number of optimal sensors (right), which increases sensor density near the previously selected locations. Although additional sensors provide redundant information, it is shown in Fig. 7 that they yield only marginal improvements in accuracy.

Figure 7: **Pointwise error vs. number of optimal sensors.** As the number of sensors are increased, pointwise error distributions do not show much improvement beyond \( p = 60 \) sensors for shim 2, which has 1116 points in total. For instance, at \( p = 80 \) only 3 points are mispredicted with absolute error greater than 0.005 inches out of all 1116 \( \times 53 \) test points. Finally, the best prediction using all 1116 points yields only marginal improvements.
Figure 8: Absolute error distribution for optimal (blue) vs. random (orange) sensors. The above are histograms of pointwise absolute error across all validation tests using reconstruction from optimal and an equal number of random sensors (which are randomly selected from available measurement locations shown in Figure 5). The red line represents the desired measurement tolerance of 0.005 inches, and the legend indicates the percentage of points that fall within this error tolerance. The histograms indicate that optimal sensors predict nearly twice as well as random sensors.

accurate shim prediction over random sensors, with the vast majority of predicted shims within the desired tolerance.

5. Conclusions and discussion

This work demonstrates the ability of data-driven sensor optimization to dramatically reduce the number of measurements required to accurately predict shim gaps in aircraft assembly. We combine sparse sensor optimization with robust feature extraction and apply the technique to historical Boeing production data from a representative aircraft. With around 3% of the original laser scan measurements, our algorithm is able to predict the vast majority of gap values to within the desired measurement tolerance, resulting in accurate shim prediction. These optimized measurements exhibit excellent cross-validated performance and may inform targeted, localized laser scans in the future. Reducing the burden of data acquisition and downstream computations has tremendous potential to improve the efficiency of predictive shimming applications, which are ubiquitous and often take place in the critical path of aircraft assembly. Thus, streamlining this process may result in billions of dollars of savings.

The proposed PIXI-DUST algorithm has been demonstrated on laser point scan data, although the method is broadly applicable to data from various metrology equipment. In the current study, reducing the number of laser scan points directly reduces measurement time, which is considerable. However, in many predictive shimming applications, vast and increasing volumes of point cloud data are being collected. As opposed to laser scans, point clouds are relatively inexpensive to acquire, and it is less clear that sparse sensing will significantly reduce acquisition time. Nonetheless, point cloud scans often yield millions or billions of data points, which result in computationally expensive downstream predictions. The sparse sensor algorithm may pinpoint key locations in this vast data, dramatically reducing shim computations, likewise reducing the time required for predictive shimming. In addition, because high-resolution scan data is available, it may be possible to use thousands of nearby points in the vicinity of an identified sparse sensor location to obtain a robust gap estimate in this region. It may also be possible to obtain super-resolution, leveraging the availability of large quantities of nearby data.

There are a number of interesting future directions for this work. In the present study, we use RPCA to remove outliers in the data, providing a robust computation of low-rank patterns. However, the sparse outliers may contain information about anomalies and defects, and machine learning may be used to automatically classify these outliers. It has also been observed in this data that aircraft tend to cluster
in the PCA feature space, indicating possible differences in manufacturing processes across line number. Mining this data and using these PCA coefficients as diagnostics for process control may be possible.

Although the present work has been applied to historical production data, we are currently working to implement this technology for online learning on future aircraft. Importantly, every step in the algorithm is relatively inexpensive, involving laptop computations on the order of minutes. The initial RPCA and pivoted QR are one-time computations, which can be accelerated by GPU implementations when datasets contain on the order of millions of points. The dimensionality reduction provided by RPCA permit each aircraft’s gap data to be compactly represented by its PCA coefficients, plus a small number of PCA features shared by all aircraft. Then, the full data can be cheaply recovered from a simply projection onto PCA features. Thus the PCA framework (even without optimal measurements) permits compact storage of large data that often result from scans. After the offline sensor optimization, online predictive shimming merely requires 1) collecting a small subset of optimized measurements and 2) performing least-squares regression. The features and predictions only improve with increasing data, and so as more aircraft are manufactured, the dominant correlation structures, and resulting sparse sensor locations, become more clear. In the first tens of aircraft, full measurements will be collected, and these measurements will then be pared down and optimized as coherent patterns emerge. It may also be possible to collect slightly more data than indicated by the optimization to validate predictions. When large deviations in predicted shims occur, it may be necessary to temporarily increase sensor resolution or pinpoint changes in the production process. It will also be important to investigate how to transfer knowledge acquired on one shim location to other similar parts.

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References


