

MATHEMATICS APPLICATIONS

NAME _____ SCHOOL _____

DATE STARTED _____ DATE COMPLETED _____

PREREQUISITES: Knowledge of arithmetic (including whole numbers, fractions, decimals, percents), exponents and simple algebra (find x , isolate a variable). Advanced Math Discipline course recommended.

HOW TO DO THIS COURSE: Do the steps one at a time, in order. When you finish a step, put your initials and the date on the sign-off line on the right. A split line means get a pass (and an initial) from another student (or your Academic Supervisor if it says that). Essays are written in a math notebook, to be reviewed by the Academic Supervisor.

PURPOSE: Learn to understand and apply the mathematics needed to sort out common situations.

ESTIMATED TIME: 30 hours.

MATERIALS NEEDED TO DO THIS COURSE

Study booklet, *Mathematics Applications*, with these data sheets (DS):

657	664 (drills)	658	659	880	6929	660	661	662
1044	1046	663	5426	5427	665 (drill answers)			

Exams: 666, 8263 (answers), 7022 (review), 8264 (answers)

Other materials: Electronic calculator, U.S. Federal Income Tax return form and instructions for a recent year, a W4 form, an Almanac or other source of statistics, food package that shows percentages of the different nutrients (step D.4). Math notebook (for answering Essays and written Drills.)

REGARDING EXERCISES

As explained in the Advanced Math Discipline course, there are certain things to keep in mind when studying and drilling any advanced math subject. This box is a reminder about some of those points, for the student, the checker and the Academic Supervisor. It also covers some points about doing checkouts on math exercises. (These points may be applied to checkouts on demonstrations and practical applications as well, where that will help ensure the student's understanding.)

Student's responsibility: Before asking for a checkout,

1. Drill each new ability to the point where you fully understand the math, applying the drill procedure laid out in DS #6929 Doing Advanced Math Exercises (from Advanced Math Discipline course).
2. If restudy is needed, follow the study guidelines laid out in DS #6833 Using Advanced Math Discipline (Advanced Math Discipline course).

Student checker's responsibility:

1. Before giving a math checkout, you must have already been passed on the material (and preferably you should have completed the course), so it will be easy for you to tell if the student understands it.
2. When giving checkouts, first make sure that all the student's answers are correct (you must be able to recognize right answers in forms other than provided on the answer pages, because math answers can often be written in more than one form.)
3. If the student has the right answers, select a few examples and check the write-ups to see how the work was done, to make sure the math principles were applied correctly. While learning mathematics, this is as important as having the right answers.

Academic Supervisor: In addition to ensuring that the above are done, the supervisor must:

1. Verify by direct observation that the above actions for both student and checker are followed through reliably.

2. Help make math a live subject for the student, and assist the student as needed in relating the material studied to practical situations.

A. INTRODUCTION AND BASICS

1. READ: The “Regarding Exercises” box at the beginning of this study guide. _____
2. DEMONSTRATE: The drill procedure referred to in “Regarding Exercises” and its purpose. **Supervisor pass.** _____
3. READ: DS #657 Math Applications. _____
4. ESSAY: Think of and write down five of your own examples of math situations or questions that you would be interested in being able to understand or resolve. _____
5. DRILL: Drill #1 in DS #664 Drills for Math Applications. Note to supervisor: Any areas of difficulty that show up in this drill should be dealt with by review and repair before the student goes on.) **Supervisor pass.** _____
6. READ: DS #658 Calculator Basics. Have a calculator in hand while reading this data sheet, and do all of the computations which are given in simulated calculator keys as you read them. _____
7. DRILL: Drill #2 in DS #664 Drills for Math Applications. Use this drill to get more familiar with the operation and features of your calculator, for later use. Checked per “Regarding Exercises.” _____
8. READ: DS #659 Mental Arithmetic to heading “General Approach to Mental Math.” _____
9. DRILL: Drill #3.a in DS #664 Drills for Math Applications. Repeat this drill until it is easy for you to do the exercises mentally. For the pass, have someone give you some of these or similar addition problems, then you give the answer by doing the addition mentally.

Coaching instructions (use as needed for this and other mental math drills below): If the student has difficulty getting a pass, he may be coached through the drill with the following steps:

- a) The coach gives two numbers. The student may write the numbers down, but nothing else. He should say the mental steps out loud as he goes through them to get the answer. For example, if the coach gives “ $27 + 39$ ” the student might say, “Round 39 to 40, $27 + 40$ is 67, 1 fewer is 66.” Coach checks the answer with a calculator. Continue drilling this way until it is easy for the student. _____

- b) Coach continues as in a), but student writes nothing down and does the mental math silently. Continue drilling this way until the student gives the answers quickly. _____
10. **PRACTICAL APPLICATION:** Now that you’ve practiced some mental arithmetic, turn back to the beginning of the data sheet and see if you can solve the first example mentally in under 30 seconds. _____
11. **READ:** DS #659, sections “General Approach to Mental Math” and “Subtraction.” _____
12. **DEMONSTRATE:** Go through the examples in the “Subtraction” section and spot the 1.–2. steps of the general approach in each one. _____
13. **DRILL:** Drill #3.b in DS #664 Drills for Math Applications. Repeat this drill until it is easy for you to do the exercises mentally. For the pass, have someone give you some of these or similar subtraction problems, then you give the answer by doing the subtraction mentally. If you need more practice at mental subtraction, try picking 2- or 3-digit numbers at random, then subtracting them from 100 or 1000. This is good practice for the “trick” mentioned in the data sheet. _____
14. **READ:** DS #659, sections “Multiplication” and “One Digit By Two or More Digits.” _____
15. **DRILL:** Drill #3.c in DS #664 Drills for Math Applications. Repeat this drill until it is easy for you to do the exercises mentally. For the pass, have someone give you some of these or similar multiplication problems, then you give the answer by doing the multiplication mentally. _____
16. **READ:** DS #659, section “By 10” to heading “Division.” _____
17. **DRILL:** Drill #3.d in DS #664 Drills for Math Applications. Repeat this drill until it is easy for you to do the exercises mentally. For the pass, have someone give you some of these or similar multiplication problems, then you give the answer by doing the multiplication mentally. _____
18. **READ:** DS #659, sections “Division” and “Summary.” _____
19. **DRILL:** Drill #3.e in DS #664 Drills for Math Applications. Repeat this drill until it is easy for you to do the exercises mentally. For the pass, have someone give you some of these or similar division problems, then you give the answer by doing the division mentally. _____
20. **ESSAY:** Tell why the numbers in 3.e.8 are familiar and use that to explain why an estimate can be a good answer. Make up an example of a practical situation where you have the problem in 3.e.12 and explain how you would make a good mental estimate of the answer for that situation. _____

B. MATH APPLICATIONS IN PERSONAL FINANCE

1. READ: DS #880 Savings and Interest. _____
2. DEMONSTRATE:
 - a) interest _____
 - b) compound interest _____
 - c) the effect of inflation on savings _____
3. READ: DS #6929 Doing Advanced Math Exercises. (Follow the drill steps given in this data sheet when doing exercises in the remainder of this course.) _____
4. READ: DS #660 Interest Formulas, section “Computation of Simple Interest.” Note: It will be especially important to apply Advanced Math Discipline as you read this data sheet. _____
5. DEMONSTRATE: The formulas for simple interest I and the total T. _____
6. DRILL: Drill #4.a in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
7. DEMONSTRATE: Estimate mentally the simple interest on \$650 at 9% for 3 years. Write your estimate here. _____ Then check it. _____
8. READ: DS #660 Interest Formulas, section “Compound Interest (Compounded Annually).” _____
9. DEMONSTRATE: The formula for compound interest. _____
10. DRILL: Drill #4.b in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
11. DEMONSTRATE: Estimate mentally the interest on \$700 at 10% for 3 years, compounded annually. Write your estimate here. _____ (Remember, the purpose of practicing mental arithmetic is so you can quickly get useful answers “on your feet” when you need to. It is okay to jot down a few intermediate results if that helps.) _____
12. READ: DS #660 Interest Formulas, section “Compound Interest (Compounded Other than Annually).” _____
13. DEMONSTRATE: The formula for compound interest compounded other than annually. _____
14. DRILL: Drill #4.c in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____

15. DEMONSTRATE: Estimate mentally the interest on \$600 at 4% for 3 years, compounded semi-annually. Write your estimate here.

16. READ: DS #660 Interest Formulas, section “Compound Interest with Savings Added Annually.” _____
17. DEMONSTRATE: The formula for compound interest with savings added annually for $n = 3$. Be sure you understand how this equation is developed. _____
18. DRILL: Drill # 4.d in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
19. DEMONSTRATE: Estimate mentally the total interest on savings deposits of \$100 a year at 5% for 3 years, compounded annually. Write your estimate here. _____
20. READ: DS #660 Interest Formulas, sections “Compound Interest with Savings Added Monthly” and “Real Life.” _____
21. DEMONSTRATE: The formula for compound interest with savings added monthly for $n = 2$. _____
22. DRILL: Drill #4.e in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
23. READ: DS #661 Borrowing Money to heading “Installment Loans.” _____
24. DEMONSTRATE: The formula for interest on a simple loan. Give an example of when you might use it. _____
25. READ: DS #661 Borrowing Money, sections “Installment Loans” and “Finance Charges, Disclosure.” _____
26. ESSAY: Explain in your own words why the two rules for borrowing money make sense. _____
27. READ: DS #661 Borrowing Money, section “Loan Formulas, and Computing Amount of Payments” and “Some Other Types of Loan.” _____
28. DEMONSTRATE: The formulas for computing interest on installment loans for some small n . Make it a real example of a loan you might take out. _____
29. DRILL: Drill #5 in DS #664 Drills for Math Applications. Use the chart in the data sheet. Checked per “Regarding Exercises.” _____
30. READ: DS #662 Taxes. _____

31. PRACTICAL INVESTIGATION:

- a) Find out what the property tax rate is in your area, how it is stated and where it is paid. ____
- b) Find out what the sales tax is in your area and on what types of items it must be paid. (If there is no sales tax where you live, find out about another place that has one and what it is there.) ____

32. DRILL: Drill #6 in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.”

33. DEMONSTRATE: You are at the checkout counter of a store with a \$10 bill in your hand and an item priced at \$9.87 you want to purchase. Estimate mentally how much change you must have in your pocket to cover a sales tax of 4%. 5%. 6%. Write your estimates here.

34. PRACTICAL APPLICATION: Get a copy of a W-4 income tax withholding form and fill it out. Do whatever research you find necessary in order to understand how to do it. Assume you earn an annual salary of \$12,500 and will be paid monthly. (You can substitute your own real income data if you wish. This step is optional for students who have previously filled out and filed a real W-4 form of their own.)

35. PRACTICAL APPLICATION: Get a copy of the federal income tax form and instructions for the tax year most recently ended. (This step is optional for students who have previously filled out and filed a real federal income tax form of their own.)

a) For each of the following, read the instructions and enter the data in the proper place on the form. (You may substitute real data for your own personal income tax if you have it.)

- 1) Filing Status: single
- 2) Exemptions: one (yourself)
- 3) Dependents: none
- 4) W-2 taxable income: \$12,500
- 5) Taxable Interest (income from savings): \$237
- 6) Dividend Income (from stocks): \$143
- 7) Taxable Refunds: \$76 (from previous state tax)
- 8) Amount of Income Tax Withheld: \$1,452

b) Following the instructions, compute your tax based on the above. Assume other income lines on the form, such as business income, capital gains, etc., do not apply to you. Take the standard deduction. Assume you have no other taxes or tax credits to report. Decide if you must pay more or are owed a refund. **Supervisor pass.**

C. UNITS AND FORMULAS

1. READ: DS #1044 Units and Conversion to heading “Combined Units.” _____
2. DEMONSTRATE:
 - a) basic unit _____
 - b) conversion ratio _____
 - c) inverse _____
3. READ: In the exercises for Drill #7 and Drill #8 that follow, using methods and formulas covered in the data sheets, the math is quite simple. When you do each problem, imagine you are in a situation where you must solve it “on your feet.” Do the math mentally before you write anything down. Get at least a useful estimate, if not the exact answer, to each problem and note that. Then check your mental math by doing it in writing. _____
4. DRILL: Drill #7.a. and b in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____
5. READ: DS #1044 Units and Conversion, section “Combined Units” to the end. _____
6. DEMONSTRATE: Show why “combined units” are called that. _____
7. DRILL: Drill #7.c in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____
8. READ: DS #1046 Mathematical Formulas. _____
9. DEMONSTRATE: What a mathematical formula is and how you would select and use one to solve a problem. _____
10. READ: DS #663 Using Formulas to heading “Mileage Formula.” _____
11. DEMONSTRATE: How the rate formula is used in the data sheet examples. _____
12. DRILL: Drill #8.a in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____
13. READ: DS #663 Using Formulas, section “Mileage Formula.” _____
14. DEMONSTRATE: How the mileage formula is used in the data sheet examples. _____
15. DRILL: Drill #8.b in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____

16. READ: DS #663 Using Formulas, section “Electrical Energy.” _____
17. DEMONSTRATE: How the electrical power formula is used in the data sheet examples. _____
18. DRILL: Drill #8.c in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____
19. READ: DS #663 Using Formulas, section “Temperature Conversions.” _____
20. DEMONSTRATE: How the temperature conversion formula is used in the data sheet examples. _____
21. DRILL: Drill #8.d in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____
22. READ: DS #663 Using Formulas, section “Proportions” to the end. _____
23. DEMONSTRATE: How mathematical proportions are used in the data sheet examples. Be sure you understand exactly how the square factor affects the result in example 2. _____
24. DRILL: Drill #8.e in DS #664 Drills for Math Applications, using the instruction in step 3 above. Checked per “Regarding Exercises.” _____

D. STATISTICS AND PROBABILITY

1. READ: DS #5426 Statistics to heading “Averages.” _____
2. DEMONSTRATE: Statistical trends (upward and downward). _____
3. DRILL: Drill #9 in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
4. PRACTICAL APPLICATION: Find a food package with a label that shows percentages of the different nutrients. Make a graph that shows this effectively. _____
5. READ: DS #5426 Statistics, section “Averages.” _____
6. DEMONSTRATE: median _____
7. DRILL: Drill #10 in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
8. PRACTICAL APPLICATION: Investigate statistics available over a long period (20 to 30 years if possible) on one of the topics listed here or another that you are interested in (from an Almanac or some other source of data). Do a properly scaled line graph, and decide if it shows a trend. _____

- Winning times for a racing event
- Baseball batting or pitching averages
- Football rushing or passing yardage
- Average air or ocean temperatures
- Average daily rainfall
- Average ultraviolet light at ground level
- Average solar flare activity
- Average annual yield of a food crop
- Stock market prices
- Population (of a city, state, country or the world)
- Annual government expenditures in some category
- Per capita occurrence of some event (anything from movie rentals to food choices to illnesses or crimes)

9. READ: DS #5426 Statistics, sections “Errors in the Data,” “Correlation” and “Summary.” _____
10. DEMONSTRATE: Two real examples of how statistics might be misleading or misinterpreted if the data is incomplete or not fully understood. _____
11. DRILL: Drills #11 and #12 in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
12. PRACTICAL APPLICATION: Reassess the data you used for the step 8 Practical Application above (or use data for a different topic if you prefer) and compare it to other data that might be related to it. (For example, you might compare baseball batting and pitching averages.) Evaluate the data in terms of what you have learned to see if it is accurate and unbiased, and decide if there really is any correlation. Write a report on what you find and submit this to your supervisor. **Supervisor pass.** _____
13. READ: DS #5427 Probability to heading “Computing Probabilities.” _____
14. DEMONSTRATE: How to convert probability to odds, and vice versa. _____
15. DRILL: Drills #13 and #14 in DS #664 Drills for Math Applications. Checked per “Regarding Exercises.” _____
16. READ: DS #5427 Probability, section “Computing Probabilities.” _____
17. DEMONSTRATE:
 - a) The probability of two events occurring together. _____

b) The probability of either one of two events occurring. ____

c) Dependent vs. independent events. ____

18. READ: DS #5427 Probability, the rest of the data sheet.

19. PRACTICAL APPLICATION:

a) Flip a coin 10 times (or 10 coins once), and count the number of times heads comes up. Write the ratio for this statistic as the number of heads over the total number of flips. Express this as a decimal. ____

b) Flip the same coin 25 times, and again write the ratio for the number of heads over the total number of flips, and express the statistic as a decimal. ____

c) Flip the same coin 100 times, and again write the ratio for the number of heads over the total number of flips, and express it as a decimal. ____

d) Explain in an essay how your statistical results compare to the probability concepts in the data sheet you just read, covering these points:

1. If you used your statistic from step a) to predict the probability of future heads versus tails, how reliable would it be?

2. Would statistic b) or c) be more reliable? Why?

20. ESSAY: Suppose you were given the job of gathering statistics on which to base a probability prediction. Explain what you would do to make the prediction as accurate as possible.

21. DRILL: Drills #15 and #16 in DS #664 Drills for Math Applications. Checked per "Regarding Exercises."

22. ESSAY: Tell about a way that knowledge of statistics and probability and how they are related can be useful to you.

E. FINAL APPLICATION SECTION

1. DRILL: Drill #17 in DS #664 Drills for Math Applications. Refer back to the data sheets for any formulas you need. Checked per "Regarding Exercises."

2. PRACTICAL APPLICATION: Suppose you are in your local hardware store and you notice that they are having a special today-only sale on paint. This reminds you that your house needs painting. You check, and find they have exactly the brand and color you want to use. But you weren't expecting this sale, so you haven't made any plans and don't know how many gallons you should buy to paint your house.

You know that your house is about 40 feet long by 30 feet wide and about 15 feet high, with triangular peaks on the 30-foot sides that extend up

another 10 feet. Visualizing your house and counting mentally, you determine that there are a total of 10 windows, each one about 3 feet wide by 4 feet high, and two doors, both about 3 feet wide by 7 feet high. You pick up a can and read that a gallon of paint covers about 500 square feet.

To make this application a little more realistic, do this. Memorize the above data, then note down the time and get up from your desk, leaving all your study materials there, and go stand next to a wall, pretending you are standing in the paint aisle in the store. While you are standing there, mentally estimate the number of gallons of paint you should buy. When you have the answer, go back to your desk and write the answer here, along with the time it took you to come up with it.

Gallons of paint needed: _____ Time to compute it mentally: _____

If the paint normally sells for \$15 a gallon and the sale price is \$10 a gallon, how much have you saved by being able to “think on your feet” and come up with the amount of paint you need? Amount saved:

If you decide to buy extra paint, how many more gallons can you buy and still pay less than you would have had to pay to buy enough to paint your house at the regular price? Extra gallons: _____

I have completed the steps of this course. I understand what I studied and can use it.

Student _____ Date _____

The student has completed the steps of this course and knows and can apply what was studied.

Academic Supervisor _____ Date _____

The student has passed the exam for this course.

Examiner _____ Date _____