

Algebra 2A

NAME _____ DATE _____

PURPOSE

Learn to use algebra to solve equations.

HOW TO DO THIS COURSE: Do the steps in order, initialing and dating each when done. Where there are two sign-off lines, get the step checked and initialed on the second line by another student or, if stated, by your academic supervisor.

ESTIMATED TIME: 70 hours.

BOOKS AND REFERENCES:

Algebra 2A Companion Text, Heron Books

Algebra and Trigonometry: Functions and Applications, Foerster; Addison Wesley 1990, 1998

Algebra and Trigonometry: Functions and Applications Teacher Edition 1990

NOTES:

- Use a math notebook for all your work on this course, labeling each problem set so your supervisor can easily review your work at any time.
- Answers to drills in *Algebra and Trigonometry: Functions and Applications* are found in that book.
- The supervisor decides if drills with solid sign-off lines are checked by the student himself, another student or the supervisor.
- There are glossaries at the back of the companion text and the back of the textbook.
- You are encouraged to find other resources when you need, including math courses, tutors and calculators found online.

A. CHAPTER 1 PRELIMINARY INFORMATION

1. READ: Notes above. _____
2. READ: *Algebra 2A Companion Text* (referred to as A2A) Chapter 1 Basic Rules of Algebra. _____
3. DEMONSTRATION: Demonstrate these to another student.
axiom _____
field axiom _____

4. READ: *Algebra and Trigonometry: Functions and Applications* (referred to as A&T) Section 1-2 The Field Axioms. _____
5. DEMONSTRATION: Do large sketches on a whiteboard to demonstrate *closure*.
- a) show whether the set of even numbers is closed under addition. _____
- b) show whether the set of odd numbers is closed under addition. _____
- c) repeat this for examples given on page 5 of A&T until you understand *closure* easily. _____
6. DEMONSTRATION:
- commutativity _____ identity _____
- associativity _____ inverses _____
- distributivity _____ _____
7. DEFINE: Using the A2A glossary and other resources as desired, define rational number _____ integer _____ _____
8. DRILL: A&T, Exercise 1-2 *Do These Quickly*. _____
9. DRILL: A&T, Exercise 1-2, problems 1-9. _____
10. READ: A&T, Section 1-3 Variables and Expressions _____
11. DEMONSTRATION:
- a) On a whiteboard, write an example and explain each of these to another student:
- variable _____ domain _____
- expression _____ order of operations _____
- constant _____ absolute value _____
- evaluate _____ equivalent expressions _____
- simplifying an expression _____ symbols of inclusion _____

b) Do the following on a whiteboard using the data in the “Definitions” box (A&T bottom of p. 10):

5 – 3x with addition instead

4x ÷ 3 three ways

5x²y⁴ without exponents

12. DRILL A&T Exercise 1-3 *Do These Quickly*.

13. DRILL: A&T, Exercise 1-3, odd-numbered problems 1-41.
Supervisor pass.

14. READ: A&T Section 1-4 Polynomials.

15. DEMONSTRATION:

polynomial _____ monomial _____ coefficient _____

degree _____ binomial _____ term _____

trinomial _____

16. DEMONSTRATION: Write three examples of polynomials and three examples of expressions which are not polynomials.

17. DRILL: A&T Exercise 1-4 *Do These Quickly*.

18. DRILL: A&T Exercise 1-4, odd-numbered problems 1-33.

19. DEFINE:

member _____ \emptyset _____ \therefore _____

\in _____ $\{\}$ _____

extraneous solution _____

20. READ: A&T Section 1-5 Equations.

21. DEMONSTRATION:

equation _____ equivalent equations _____

solving an equation _____ extraneous solutions _____

solution set _____

22. DEMONSTRATION: Each step of solving the equation $|2x + 4| = 6$ in the domain of real numbers. _____
23. DRILL: A&T Exercise 1-5 *Do These Quickly*. _____
24. DRILL: A&T Exercise 1-5, odd-numbered problems 1-31, plus #34. _____
25. DEFINE: ray. _____
26. READ: A&T Section 1-6 Inequalities. _____
27. DEMONSTRATION:
inequality _____ multiplication property of order _____ _____
28. DEMONSTRATION: Using number line(s), sketch some examples until you understand well enough to write the rule from memory:
a) each of the three possibilities for the multiplication property of order given in the box on page 28 _____
b) each part of the rules in the “Conclusion” box (including “if c is a non-negative constant”) on page 30 _____ _____
29. DRILL: A&T Exercise 1-6 *Do These Quickly*. _____
30. DRILL: A&T Exercise 1-6, odd-numbered problems 1-27. _____
31. DRILL: A&T Exercise 1-6. Examine problem 29 and see whether or not there is a value for x that will make it true. Examine problem 30 and see if there is a value for x that *won't* make it true. For problem 31, first change $3 \leq |x - 2| < 5$ into this: $|x - 2| \geq 3$ AND $|x - 2| < 5$. Leave that capital “AND” in the middle and then apply the blue box on page 30 to both sides, and create the number line graph. _____
32. DRILL: A&T Exercise 1-6. problem 32. **Supervisor pass.** _____
33. DEFINE:
lemma _____ corollary _____ proof _____ _____

34. READ: A&T Section 1-7 Properties Provable from the Axioms. Later you'll be asked to name the properties used in the steps of some proofs. You may find it useful to use a section of your notebook to write the field axioms, the properties proven in this section, and definitions such as the definition of subtraction. Also note the theorem proving techniques.

35. DEMONSTRATION:

reflexive property _____ trichotomy _____

symmetry _____ hypothesis _____

transitivity _____ converse _____

36. DRILL: First label the hypothesis, then write the converse of each of these statements. Then state whether or not the converse is true. If you don't think it is true, give an example that shows it isn't true. Answers are attached to this learning guide.

a) If a car is a Volkswagen, then it is an import. _____

b) If the sun is down, then the sky is dark. _____

c) If $x = 3$, then $2x = 6$ _____

d) If $x = 3$, then $x^2 = 9$ _____

e) If $x = 0$, then $xy = 0$ (y is "any number") _____

f) If $x = 0$, or $y = 0$, then $xy = 0$ _____

g) If an object is a square, then it has four right angles. _____

h) If an object is a square, then it has four sides of the same length. _____

i) If an object is a square, then it has four right angles and four sides, all of equal length. _____

37. DRILL: A&T Exercise 1-7 *Do These Quickly*.

38. DRILL A&T Exercise 1-7, problems 1-4 and 7. (Hint: problem 7 can be done by mimicking what was done in Example 2 on page 35 for the addition property of equality.)

39. DRILL: A&T Exercise 1-7, problems 13, 14, 15, 17, 18, 22, 24. For these problems, notice the change that occurred from one step to the next one and supply the names of the properties that justify the given

steps. Use the names of the axioms, definitions or properties already recorded in your notebook (or found in the text), and continue to add to the list in your notebook as you prove new properties.

40. READ: A&T Section 1-8 Chapter Review and Test, introductory paragraphs up to Review Problems.

41. DRILL: A&T Section 1-8 Review Problems, problems R1, R2b, R3, R4.
Supervisor pass.

B. CHAPTER 2 FUNCTIONS AND RELATIONS

1. READ: A&T Section 2-1 Graphs of Equations with Two Variables.

2. DRILL: A&T Exercise 2-1, problems 1-5.

3. READ: A&T Section 2-2 Graphs of Functions.

4. DEMONSTRATION: Use desmos.com to plot all the points in Example 1 on page 54. You can plot a point in desmos with the standard way of writing an ordered pair, such as $(-5, 5)$. You must type the parentheses and put each ordered pair in a separate box. After you have entered all 11 points, enter the equation in its own box and examine the graph.

5. DRILL: A&T Exercises 2-2 *Do These Quickly*.

6. DRILL: A&T Exercise 2-2, odd-numbered problems 1-19.

7. READ: A&T Section 2-3 Functions in the Real World.

8. DRILL: A&T Exercise 2-3 *Do These Quickly*.

9. DRILL: A&T Exercise 2-3, problems 1, 3, 5, 6, 8, 9, 11, 18, 19, 21, 22, 23, 26, 28, 41, 43. Compare your sketch with the answers in the book as you do each one. Depending on how you were envisioning the situation, your sketch might be more or less correct.

10. READ: A&T Section 2-4 Graphs of Functions and Relations.

11. DEMONSTRATION:
 - a) relation _____
 - b) the difference between a relation and a function. _____
12. DEMONSTRATION: With a sketch, show how to use the “vertical line test.” _____
13. DRILL: A&T Exercise 2-4 *Do These Quickly*. _____
14. DRILL: A&T Exercise 2-4, odd-numbered problems 1-10 and all problems 11-26. **Supervisor pass.** _____

C. CHAPTER 3 LINEAR FUNCTIONS

1. READ: A2A Chapter 2 An Example of a Linear Function. _____
2. DEMONSTRATION: linear function _____
3. READ: A&T Section 3-1 Introduction to Linear Functions.
(Note: “a polynomial *in* the variable x ” means that “ x ” is the only variable in the polynomial.) _____
4. DRILL: A&T Exercise 3-1, problem #1: On paper, determine three points that would satisfy equation 1a and type those points into desmos.com. Then type “ $y = mx + b$ ” into another box. When it asks “add sliders?” click “all.” Then slide the values of m and b to 1 and 5. The line should touch all three points. Then delete the ordered pairs from equation 1a and enter 3 ordered pairs that will satisfy equation 1c. Move the sliders to match those three points. Finally, repeat this process for equation 1e. For Problem #2, play around with the sliders to answer questions a-d. _____
5. READ A&T Section 3-2 Properties of Linear Function Graphs. _____
6. DEMONSTRATION:

y-intercept _____	x-intercept _____	slope _____
slope formula _____	rise _____	run _____
slope-intercept form _____		

7. DRILL: A&T Exercise 3-2 *Do These Quickly*. _____
8. DRILL: A&T Exercise 3-2, odd-numbered problems 1-23. **Supervisor pass.** _____
9. READ AND DEMONSTRATION: A2A Chapter 3 Graphing on a Calculator. You'll need a TI-83 Plus or TI-84 Plus graphing calculator. _____
10. READ: A&T Section 3-3 Other Forms of the Linear Function Equation. In this form of linear equation the convention is for the constants to be integers, not fractions. So if you arrive at $.5x + 2y = \frac{1}{4}$, you should multiply both sides by 4 so all the constants are integers: $2x + 8y = 1$. _____
11. DEMONSTRATION:
point-slope form _____ $Ax + By = C$ form _____
12. DEMONSTRATION: Show why the point-slope form of the linear equation is called that, and how you can use it to quickly plot a function. _____
13. DRILL: A&T Exercise 3-3, *Do These Quickly*. _____
14. DRILL: A&T Exercise 3-3, odd-numbered problems 1-13. You may use a graphing calculator or desmos.com to plot the graphs. _____
15. READ: A&T Section 3-4 Equations of Linear Functions from Their Graphs. _____
16. DRILL: A&T Exercise 3-4, odd-numbered problems 1-29. (Hint for 27 and 29: find the slopes between one set of coordinates and the next and see if the slopes are the same.) _____
17. (Optional) DRILL: A&T Exercise 3-4, problems 31 and 34. _____
18. READ: A&T Section 3-5 Linear Functions as Mathematical Models. _____
19. DEMONSTRATION: mathematical model. _____
20. READ: A2A Chapter 4 The Use of a Function as a Mathematical Model. _____
21. DRILL: Evaluate your ability to do each of the following with linear functions. Restudy and drill any areas of uncertainty.
 - a) Write the general equation _____
 - b) Determine a particular equation _____

- c) Find y if you know x _____
- d) Find x if you know y _____
- e) Plot a graph of a linear equation rapidly _____
- f) Rapidly determine the linear equation that goes with a given graph
or with two given ordered pairs _____

22. Drill A&T Exercise 3-5 *Do These Quickly*. _____

23. DRILL: A&T Exercise 3-5, problems 2, 5, 12, 14 and one more of
your choice. _____

24. DRILL: Choose another problem from Exercise 3-5 and do it with no
reference to anything else, as your supervisor observes. You pass when
you can go through a problem without help. **Supervisor pass.** _____

25. READ: A&T Section 3-6 Chapter Review and Test, introductory
paragraphs up to Review Problems. _____

26. DRILL: A&T Section 3-6 Review Problems, Exercises R1-R5. (Note on
R5: a loop is a sequence of steps that is repeated over and over until a
certain result is achieved.) _____

27. PRACTICAL APPLICATION: A&T Section 3-6 Concepts Problem.
Notes: Taxable Income is the portion of income you make that is used
to calculate how much tax you owe. Tax payable is the amount of tax
you have to pay based on that.

(TI - \$17,850 means Taxable Income minus \$17,850)

For step e, make a graph on paper. Keep in mind that the slope will
change for each different part of the domain.

Supervisor pass. _____

D. CHAPTER 4 SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES

1. (Optional) READ: A2A Chapter 5 Graphing Pairs of Linear Equations. _____

2. READ: A&T Section 4-1 Introduction to Linear Systems, including the
material on page 110. _____

3. DEMONSTRATION:
 system of equations _____ simultaneous equations _____ _____
4. DRILL: A&T Exercise 4-1, problems 1-6 _____
5. (Optional) READ: A2A Chapter 6 Simultaneous Solutions to Pairs of Equations. _____
6. (Optional) READ: A2A Chapter 7 Solving Problems with Pairs of Linear Equations. _____
7. READ: A&T Section 4-2 Solution of Systems of Linear Equations. _____
8. DEMONSTRATION:
 Equivalent systems of equations _____
 Inconsistent equations _____
 Dependent equations _____
 Independent equations _____ _____
9. DRILL: A&T Exercise 4-2 *Do These Quickly* omitting Q3 and Q6. _____
10. DRILL: A&T Exercise 4-2, odd-numbered problems 1-37. (Notice the additional instructions and hints that are given with each group of problems.) **Supervisor pass.** _____
11. READ: A&T Section 4-4 $f(x)$ Terminology, and Systems as Models. (A&T Section 4-3 is intentionally omitted.) _____
12. DEMONSTRATION: $f(x)$ _____
13. DRILL: A&T Exercise 4-4 *Do These Quickly*, omitting Q6 and Q7. _____
14. DRILL: A&T Exercise 4-4, odd-numbered problems 1-29. _____
15. READ: A2A Chapter 8 Drill for Three-Dimensional Graphs, section "Theory" (up to section "Drill"). _____
16. DEMONSTRATION: ordered triple _____

17. DRILL: Follow the set-up instructions and do the steps given in the “Drill” section of the above chapter. Work with a coach as directed, and have the coach check your answers to the questions as you go along, before you refer to the answers at the end of the chapter. _____
18. READ: A&T Section 4-6 Systems of Linear Equations with Three or More Variables¹ (A&T section 4.5 is intentionally omitted.) _____
19. DRILL: A&T Exercise 4-6 *Do These Quickly*, omitting Q4. _____
20. DRILL: A&T Exercise 4-6. Do odd-numbered problems 1-7 by hand (using a simple calculator, if you wish) until you get two problems correct. _____
21. READ: A2A Chapter 9 Solving Systems of Equations by Augmented Matrices. _____
22. DRILL: A&T Exercise 4-7, problems 1 and 3, by hand. _____
23. READ: A2A Chapter 10 Solving Equations with a Graphing Calculator. _____
24. DRILL: A&T Exercise 4-7, do problems 5 and 7 using a graphing calculator. _____
25. READ: A&T Section 4-8 Solution of Higher-Order Systems by Augmented Matrices. _____
26. DRILL: A&T Exercise 4-8 *Do These Quickly*, omitting Q7. _____
27. DRILL: A&T Exercise 4-8, problem 17 using a graphing calculator or a matrix website. _____

E. CHAPTER 5 QUADRATIC FUNCTIONS AND COMPLEX NUMBERS

1. READ: A2A Chapter 11 Quadratic Equations. _____

¹ One edition of A&T has an error in the example of combining the first and second equations. Instead of $+ 6z$ and $+ 5z$, it shows $- 6z$ and $+ 5y$.

2. READ: A&T Section 5-1 Introduction to Quadratic Functions, including the material on the preceding page. _____
3. DEMONSTRATION: Show a quadratic function. _____
4. DRILL: A&T Exercise 5-1, problems 1-6. _____
5. READ AND DEMONSTRATION: A2A Chapter 12 Completing the Square to heading “Demonstration.” Copy the table in section “Demonstration” into your notebook and fill it out as instructed. Then read the rest of the chapter. _____
6. READ: A&T Section 5-2 Graphs of Quadratic Functions. _____
7. DEMONSTRATION:

parabola _____	completing the square _____
vertex _____	vertex form _____
axis of symmetry _____	

8. DRILL: A&T Exercise 5-2 *Do These Quickly*. _____
9. DRILL: A&T Exercise 5-2, odd-numbered exercises 1-29 and 33. Note: Handling equations that have a coefficient for the x^2 term *other than one*, such as $3x^2$ in problem 21, require an additional step to complete the square. This step is demonstrated in Example 1 on page 178. _____
10. READ: A2A Chapter 13 Working with Radicals _____
11. READ A2A Chapter 14 The Quadratic Formula _____
12. READ: A&T Section 5-3 x -Intercepts, and the Quadratic Formula. _____
13. DEMONSTRATION:

discriminant _____	quadratic formula _____
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14. DRILL: A&T, Exercise 5-3 *Do These Quickly*. _____
15. DRILL: A&T Exercise 5-3, every other odd problem 1-45. _____

16. DRILL: With a coach who has completed this drill, do A2A Chapter 15 Whiteboard Drill: Deriving the Quadratic Formula. You and the coach should read the purpose and drill description before you start. _____
17. DRILL: On paper, derive the quadratic equation using a, b and c rather than 5, 13 and 7. In other words, start with $ax^2 + bx + c = 0$ and solve it for x, then simplify. When you can do this easily from memory, have your supervisor watch you do it from start to finish. **Supervisor pass.** _____
18. READ: A&T, Section 5-4 Imaginary and Complex Numbers. _____
19. DEMONSTRATION:
 imaginary number _____ complex number _____
 unit imaginary number _____ complex conjugates _____
20. DRILL: A&T, Exercise 5-4 *Do These Quickly*, omitting Q8. _____
21. DRILL: A&T, Exercise 5-4, odd-numbered problems 1-19 except #7. **Supervisor pass.** _____
22. READ: A&T, Section 5-5 Evaluating Quadratic Functions. _____
23. DRILL: A&T, Exercise 5-5 *Do These Quickly*. _____
24. DRILL: A&T, Exercise 5-5, odd-numbered problems 1-13. _____
25. READ: A&T, Section 5-6 Equations of Quadratic Functions from Their Graphs.² _____
26. DRILL: A&T, Exercise 5-6 *Do These Quickly*. Omit Q10. _____
27. DRILL: A&T, Exercise 5-6, odd-numbered problems 1-15. See if you can use a matrix to make this drill go faster. _____
28. READ: A&T, Section 5-7 Quadratic and Linear Functions as Mathematical Models. _____

² *Algebra & Trigonometry* may have a typo at the end of Example 1. If the example answer contains the line $y = .05x^2 + 3x - 11$, it should instead say $y = 0.5x^2 + 3x - 7$.

29. DRILL: Review A2A Chapter 4 The Use of a Function as a Mathematical Model. Then evaluate your own ability to do each of the following with quadratic functions. Clear up any uncertainties you have by reviewing the material and drilling.

- a) Write the general equation _____
- b) Determine a specific equation _____
- c) Find y if you know x _____
- d) Find x if you know y _____
- e) Plot a graph of a quadratic equation rapidly _____
- f) Rapidly determine the quadratic equation that goes with a given graph (or with three given-ordered pairs). _____

Get a check from another student on the above skills for quadratic functions.

30. DRILL: A&T, Exercise 5-7 *Do These Quickly*.

31. DRILL: A&T, Exercise 5-7, problems 1, 3, 5 and three of your choice selected from these: 2, 4, 6, 7, 8, 15-20.

32. READ: A&T Section 5-8 Chapter Review and Test, introductory paragraphs up to Review Problems.

33. DRILL: A&T Section 5-8 Review Problems R1-R7.

F. FINAL APPLICATIONS

1. PRACTICAL APPLICATION:

- a) Discover a linear (or approximately linear) relationship in your environment. It is preferable that you find such a relationship that you can observe directly, but it is permissible for you to find one by statistical research (as in, for example, the *World Almanac*). ____
- b) Gather several data points and plot them, and determine the equation of the linear function that represents (or nearly represents) the relationship. Use that equation to predict more data, and observe further (or gather more data) to see how accurate those predictions are. ____

c) Write a clear report of your findings, including your graph and equation, with independent and dependent variables and range and domain noted. ____

Supervisor pass. _____

2. PRACTICAL APPLICATION: Repeat the above practical application for a quadratic function. **Supervisor pass.** _____

I have completed the steps of this course. I understand what I studied and can use it.

Student _____ Date _____

The student has completed the steps of this course and knows and can apply what was studied.

Academic supervisor _____ Date _____

The student has passed the exam for this course.

Examiner _____ Date _____

FOR FACULTY

ADDITIONAL RESOURCES

Exam and answers

Materials list

Answers to Hypothesis and Converse Drill

The hypothesis is underlined in the statement; the conclusion that becomes the new hypothesis is underlined in the converse.

	Statement	Converse	True-False	Counter-example
a)	If <u>a car is a Volkswagen</u> , then it is an import.	If <u>a car is an import</u> , then it is a Volkswagen.	False	Toyota
b)	If <u>the sun is down</u> , then the sky is dark.	If <u>the sky is dark</u> , then the sun is down.	False	eclipse
c)	If <u>$x = 3$</u> , then $2x = 6$	If <u>$2x = 6$</u> , then $x = 3$	True	
d)	If <u>$x = 3$</u> , then $x^2 = 9$	If <u>$x^2 = 9$</u> , then $x = 3$	False	$x = -3$
e)	If <u>$x = 0$</u> , then $xy = 0$ (y is "any number").	If <u>$xy = 0$</u> , then $x = 0$.	False	$x = 5$, $y = 0$
f)	If <u>$x = 0$, or $y = 0$</u> , then $xy = 0$	If <u>$xy = 0$</u> , then $x = 0$ or $y = 0$	True	
g)	If <u>an object is a square</u> , then it has four right angles.	If <u>an object has four right angles</u> , then it is a square.	False	rectangle
h)	If <u>an object is a square</u> , then it has four sides of the same length.	If <u>an object has four sides of the same length</u> , then it is a square.	False	rhombus
i)	If <u>an object is a square</u> , then it has four right angles and four sides, all of equal length.	If <u>an object has four right angles and four sides, all of equal length</u> , then it is a square.	True	