SOLID GEOMETRY

NAME ______________________________ SCHOOL ___________________________

DATE STARTED______________________ DATE COMPLETED_________________

PREREQUISITE: Plane Geometry course.

HOW TO DO THIS COURSE: Do the steps one at a time, in order. When you finish a step, put your initials and the date on the sign-off line on the right. A split line means to get a pass (and an initial) from another student (or your Academic Supervisor if it says that). Essays are turned in to the Academic Supervisor. A number of technical terms are defined in the glossary for the course. Refer to it as a first action when you encounter unfamiliar technical terms.

PURPOSE: Learn to correctly measure and construct solid figures and be able to apply this to common situations.

ESTIMATED TIME: 70–95 hours.

MATERIALS NEEDED FOR THIS COURSE

Data Sheet (DS) #2570 Geometry Kit, used separately.

Study booklet, *Solid Geometry*

Exams: 7183, 8123 (answers), 7184 (review), 8124 (answers)

Solid Geometry Academic Supervisor Manual

Other materials: Drawing pencils, straight edge, compass, protractor, blank (unlined) paper and quadruled (lines both ways) paper, heavy paper or light cardboard (for making fold-outs), drawing table, T-square, drawing instruments, solid geometry kit (physical models of various solids as described in DS #2570), soccer ball, beakers and graduated cylinders, taut-wire cheese knife for slicing clay models accurately, round toothpicks, rubber cement, ½" Styrofoam balls (or small marshmallows).

REGARDING EXERCISES

There are certain things to keep in mind when studying and drilling any advanced math subject. This box is a reminder about some of those points, for the student, the checker and the Academic Supervisor. It also covers some points about doing checkouts on math exercises. (These points may be applied to checkouts on demonstrations and practical applications as well, where that will help ensure the student's understanding.)

Student's responsibility: Before asking for your work to be checked:

1. Drill each new ability to the point where you fully understand the math.
2. Show all steps of all of your work.
3. Check your answers frequently in the *Solid Geometry Academic Supervisor Manual* to ensure you are doing the problems correctly.
4. If you miss a problem, do not erase the problem. Do the problem again next to the problem you missed, or on a new page.
5. If needed, restudy the section and then do the even-numbered problems to be sure you are doing well on the section and ready to move on.

Student checker's responsibility:

1. Before checking math work, you must have already been passed on the material so it will be easy for you to tell if the student understands it.

2. First make sure that all the student's answers are correct (you must be able to recognize right answers in forms other than those provided on the answer pages; math answers can often be written in more than one form.)
3. If the student has the right answers, select a few examples and check the student's work to ensure the steps were done correctly.
4. Sign and date the top of the page in ink. Write your name so it can be read by the student's supervisor if needed.

Academic Supervisor: In addition to ensuring that the above are done, the supervisor must:

1. Verify by direct observation that the above actions are followed by both student and checker.
2. Keep a copy of the *Solid Geometry Academic Supervisor Manual* on hand for your own reference. The worked-out solutions to study guide steps are there to help you get up to speed as needed so you can sort out any difficulties that arise. (The worked-out solutions may be made available for the students to use when difficulties are encountered. They should work through them.)
3. Help make math a live subject for the student, as discussed in DS #8983 Solid Geometry Academic Supervisor Notes in the front of the manual, and assist the student as needed in relating the material studied to practical situations.

A. INTRODUCTION

1. READ: The "Regarding Exercises" box above. ________
2. READ: Data Sheet (DS) #2554 Solid Geometry. ________
3. DEMONSTRATION: Show how a moving surface would generate a solid. ________
4. DEMONSTRATION: Pick up a physical solid and visualize a geometric solid that would occupy the same space. ________
5. ESSAY: State in your own words what sorts of useful things you expect to learn by studying solid geometry. ____ ____
6. READ AND DEMONSTRATION: DS #2555 Terms Pertaining to Geometric Solids. As you read, demonstrate each term. ________

B. MEASUREMENT AND CONSTRUCTION

1. READ: DS #2556 Intersecting and Perpendicular Planes, to the heading "Projection and Axes." ________
2. DEMONSTRATION: Work out for yourself why each of the following is true:
   a) Two straight lines that are parallel in space are coplanar. ___
   b) A line intersecting a plane is perpendicular to at least one line in the plane. ___
   c) A line intersecting a plane, which line is perpendicular to more than one line in the plane at the point of intersection, is perpendicular to all

the lines in the plane which share the point of intersection, and is therefore (by definition) perpendicular to the plane. ___

d) Two planes that are parallel in space do not intersect. ___

e) Four points not all in one plane determine four planes, and the planes enclose a geometric solid. ___

f) Three parallel lines that are not all in one plane can be used to determine the intersections of three planes (think of a triangular tube). __________

3. READ: DS #2556, section “Projection and Axes.” __________

4. DEMONSTRATION: Work out for yourself why each of the following is true:

   a) Any line in space can be projected onto any plane. ___

   b) Three mutually perpendicular lines determine three mutually perpendicular planes. ___ __________

5. DEMONSTRATION: Show how the projections of an object onto three mutually perpendicular planes can be used to depict the dimensions of the object on paper. __________

6. READ: DS #2557 Introduction to Four Views, up to the heading “The Drawing Board.” __________

7. DEMONSTRATION: Hold an object with straight edges in front of you and rotate it various ways, visualizing which would be the most useful way to draw it in isometric or oblique drawings. Visualize the drawings if you can, and notice how they would differ from what your eye is seeing (try to see the difference perspective makes). __________

8. READ: DS #2557, section “The Drawing Board.” __________

9. DRILL: Get yourself set up with a drawing board and drafting equipment as described in the data sheet. Tape a piece of paper to the board and use your T-square, triangles and scale to do the following:

   a) Draw some horizontal parallel lines using the T-square (remember to hold the head of the T-square firmly against the side of the board). ___

   b) Draw some vertical parallel lines using the T-square and a triangle. ___

   c) Draw some 45° parallel lines using the T-square and a triangle. ___

   d) Draw some 45° parallel lines sloping the other way. ___

   e) Draw some 30° parallel lines using the T-square and a triangle. ___

   f) Draw some 30° parallel lines sloping the other way. ___

   g) Draw some 60° parallel lines using the T-square and a triangle. ___

h) Draw some 60° parallel lines sloping the other way. ___

i) Make an isometric drawing of a rectangular object with perpendicular dimensions of 1"× 2"× 3". (Make a model in clay first if you wish.) ___

j) Make an oblique drawing of the same object. ___ ____ ____

10. READ: DS #2557, section "Three Views." _________

11. DEMONSTRATION: Make a block of clay using the three views in the data sheet for a plan. _________

12. READ: DS #2557, section "Making a Three View Drawing." _________

13. DEMONSTRATION: Tape a blank sheet of paper to your drawing board and follow the steps in the data sheet to reproduce the example drawing. ____ ____

14. DRILL: Exercise #1 in DS #7185 Exercises for Solid Geometry (in the back of the study booklet). Checked per "Regarding Exercises." ____ ____

15. ESSAY: If you need to, do some research on perspective and the use of perspective in drawing. Then write an essay explaining the use of perspective in drawing, and why it is at times *not* used in drawings from which something will be built. ____ ____

C. GEOMETRIC SOLIDS

1. READ: DS #2558 Polyhedrons, to the heading "Pyramids." _________

2. DEMONSTRATION: Show why the angles around any vertex of a polyhedron must add up to less than 360°. _________

3. DEMONSTRATION: Construct these regular solids using round toothpicks (about 2½" long) and ½" diameter Styrofoam balls (half-inch balls of clay or small marshmallows may be used but do not work quite as well).

   a) regular tetrahedron ___

   b) regular hexahedron (cube) ___

   c) regular octahedron ___

   d) regular dodecahedron ___

   e) regular icosahedron ___ _________

4. READ: DS #2558, sections "Pyramids" and "Prisms." _________

5. DEMONSTRATION: Make a frame in the shape of a cube using round toothpicks and ½" balls of clay. Tilt the cube in various ways (keeping

the parallel edges parallel as you do) until you have an oblique parallelepiped. __________

6. DEMONSTRATION: Make a long strip of clay with a square right section. Cut the strip at various angles to demonstrate how you can get rectangular and parallelogram prisms. __________

7. DEMONSTRATION: Find examples of these polyhedrons in the geometry kit (or refer to DS #2570 Geometry Kit to make them):

   a) quadrilateral pyramid ___

   b) regular triangular pyramid ___

   c) oblique prism ___

   d) oblique parallelepiped ___

   e) pentagonal pyramid ___

   f) square pyramid ___

   g) regular hexahedron ___

   h) right prism ___

   i) rectangular parallelepiped ___ __________

8. PRACTICAL APPLICATION: Draw three views of each of these models from the geometry kit:

   a) quadrilateral pyramid ___

   b) regular triangular pyramid ___

   c) pentagonal pyramid ___

   d) rectangular parallelepiped ___ _____ _____

9. READ: DS #2558, sections "Geodesic Solids" and "Pythagorean Theorem for Solids." __________

10. DEMONSTRATION: Get a soccer ball and see how the surface is composed of hexagons and pentagons. Do some research to find illustrations and uses of geodesic domes and "buckyballs." Imagine how you might use the Pythagorean theorem for solids to model a geodesic dome on a computer. __________

11. (Optional) PRACTICAL INVESTIGATION: Identical cubes can be packed tightly together, leaving no spaces between them, but this is not true of all polyhedrons. Do some research on polyhedrons found in nature, paying attention to this point (various crystal structures would be a good place to start, but don't get too wrapped up in mineralogy terms). Consider how you might use the Pythagorean theorem for solids to model

a crystal structure on a computer. Write a report telling what you found out, and how studying solid geometry helped you understand it. ____ ____

12. READ: DS #2559 Surface Areas of Polyhedrons. __________

13. DRILL: Exercise #2 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

14. READ: DS #2560 Volumes of Parallelepipeds. __________

15. DEMONSTRATION: Make an oblique parallelepiped out of clay or Styrofoam (or use the paper foldout model from DS #2570 Geometry Kit). Transform it into a rectangular parallelepiped by following the steps described in the data sheet. Observe that the same transformation could be accomplished by making two mutually perpendicular cuts at the start. __________

16. DRILL: Exercise #3 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." Optional: Parallelepipeds exercise 3c) 4) (on the lower right). (DS#2549 Special Triangles is included in your study booklet for reference as you do these exercises.) ____ ____

17. READ: DS #2561 Finding Volume by Displacement. __________

18. DRILL: Exercise #4 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

19. READ: DS #2562 Volumes of Prisms. __________

20. PRACTICAL APPLICATION: Make an oblique prism from clay, and calculate its volume by measuring it (use metric units) and applying the formula, then transform it into a right prism and again calculate its volume by applying the formula. Compare the results to see that the formula gives the same answer in both cases. Check the volume of the clay by the displacement method (squash it into some different shape before you do this). Account for any discrepancies among your calculated and measured values. ____ ____

21. READ: DS #2563 Surface Areas of Pyramids. __________

22. DEMONSTRATION: Slant height of a pyramid. __________

23. DRILL: Exercise #5 in DS #7185 Exercises for Solid Geometry. (Refer to DS#2549 Special Triangles if needed.) First, do exercises a), c), e) and g), and check your answers. If you do them all easily it is okay to go on to the next course step. If you have any difficulty with them, restudy and redo them; then do the exercises b), d) and f). Checked per "Regarding Exercises." ____ ____

24. READ: DS #2564 Volumes of Pyramids, to the heading "Example: Volume of a Tetrahedron." Do the demonstration below as you read. __________

25. DEMONSTRATION (as you read the section "Formula for the Volume of a Pyramid"):

    a) Do steps 1–10 from the data sheet, starting with a clay model of a regular tetrahedron. After step 3, calculate the volume of the prism. ___

    b) As you do steps 4–10 verify for yourself that the reasoning and conclusions are correct. Verify by the displacement method if you like that the three pieces you end up with have equal volumes. ___ ____ ____

26. READ: DS #2564, section "Example: Volume of a Tetrahedron." __________

27. DRILL: Work through the two ways of finding the height until you feel you can apply them on your own to any pyramid. ____ ____

28. READ: DS #2564, section "Volume of Any Pyramid." __________

29. DEMONSTRATION:

    a) Verify for yourself that the reasoning and calculations in the tetrahedron and square pyramid examples are correct (especially the calculations of height). ___

    b) Decide for yourself if the conclusion that the formula $V = 1/3$ area of base $\times$ height applies to any pyramid is correct. ___ ____ ____

30. PRACTICAL APPLICATION: The data sheet examples found the height, base area and volume of a tetrahedron and an equilateral quadrangular pyramid with edges $e = 1$ unit. These results can be generalized for all such pyramids with edge e by substituting e for 1 unit in the calculations. Do this to show that the following formulas are correct (write these up clearly):

    a) The height h of a regular tetrahedron is given by $h = (e\sqrt{2})/\sqrt{3}$. ___

    b) The base area A_b of a regular tetrahedron is given by $A_b = (e^2\sqrt{3})/4$. ___

    c) The volume V of a regular tetrahedron is given by $V = (e^3\sqrt{2})/12$. ___

    d) The height h of an equilateral quadrangular pyramid is given by $h = e/\sqrt{2}$. ___

    e) The base area A_b of an equilateral quadrangular pyramid is given by $A_b = e^2$. ___

    f) The volume V of an equilateral quadrangular pyramid is given by $V = (e^3\sqrt{2})/6$. ___ ____ ____

31. READ: DS #2564, section "Example: Volume of a Regular Octahedron." Start the demonstration below as you read. __________

32. DEMONSTRATION: Get a model of an octahedron and make sure you understand how the height of the pyramids making it up was determined. Use the solution in the data sheet to calculate the volume of the model. If you wish, check your result by using the displacement method. _________

33. DRILL: Exercise #6 in DS #7185 Exercises for Solid Geometry. First, do exercises a), c), e) and g), and check your answers. If you do them all easily it is okay to go on to the next course step. If you have any difficulty with them, restudy and redo them; then do the exercises b), d) and f). Checked per "Regarding Exercises." ____ ____

34. READ: DS #2564, section "Ratios in Solids." _________

35. DEMONSTRATION: Ratios in solids:

    a) Get a model pyramid and mark a line on one side parallel to the base. Extend the line around the remaining sides, keeping it parallel to the base on all sides. Show by measuring on the model that item 1 of the ratios for similar pyramids in the data sheet is correct, and that the smaller pyramid defined by the lines marked on the model is similar to the bigger one. ___

    b) Get two similar pyramids and measure their heights and their base areas. Show by comparing these measurements that item 2 of the ratios for similar pyramids in the data sheet is correct. ___

    c) Show that item 3 of the ratios for pyramids in the data sheet is correct by comparing the areas of the sides of two similar pyramids. ___

    d) Compute the volumes of two similar pyramids and use this data to show that item 4 of the ratios for pyramids in the data sheet is correct. ___

    e) Work out for yourself why if the above are true then these ratios are also true:

        1) The ratio of any two corresponding dimensions cubed equals the ratio of the volumes. ___

        2) If you cube the square roots of the areas of two corresponding faces, the ratio of these values equals the ratio of the volumes. ___

    f) Demonstrate item 5 of the ratios for pyramids this way: Picture a pyramid with a square base. Four similar pyramids with ½ the height would just fit on the same base. A fifth similar pyramid would perch on top of them with a corner of its base at each apex. A sixth similar pyramid would fit inverted under the top one. Work out how the rest of the volume of the big pyramid equals two more of the small ones. (You can make sketches or use clay to help you do this.) ___ ____ ____

36. ESSAY: It is a physical fact that the strength of a structural member (whether it is a steel I-beam in a building or a bone in the body of an

animal) is proportional to its cross-sectional area, whereas its weight is proportional to its volume. Research this fact if you need to, then write an essay that uses ratios in solids to clearly demonstrate why (for example) if a daddy-long-legs spider grew as big as a house (or even as big as a mouse) while keeping the same proportions in its parts, it would probably not be able to stand up. ____ ____

Note: Items #37–45 below are optional—they are there for students who want an extra challenge. They provide additional application of many concepts already covered and can lead to a deeper understanding, but are not essential to a basic knowledge of geometry.

37. (Optional) READ: DS #2565 The Frustum of a Pyramid, to the heading "Volume of a Frustum." _________

38. (Optional) DEMONSTRATION: Get a model of a regular frustum (or make one from clay) and apply the formula to find the lateral surface area. Then do the same for a frustum that is not regular. _________

39. (Optional) READ: DS #2565, section "Volume of a Frustum." _________

40. (Optional) DEMONSTRATION:
    a) Make a large clay pyramid, calculate its volume and then check the volume by the displacement method. ___
    b) Slice off the top to make a frustum and calculate the volumes of the frustum (using the frustum formula) and the smaller pyramid. ___
    c) Check those volumes by the displacement method and by adding them to see how close you come to the volume of the original pyramid. ___ _________

41. (Optional) DRILL: Exercise #7 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

42. (Optional) READ: DS #2565, section "Average Area." _________

43. (Optional) DEMONSTRATION: Test the general result found in Example 1 in the data sheet. Get or make a pyramid, then make a prism the same height as the pyramid and with base edges 0.58 of the base edges of the pyramid. Compare the volumes by using the displacement method. _________

44. (Optional) DRILL: For each of the frustums in Exercise #7a of DS #7185, use the volume and height to find the base edge length of an equivalent square prism. Check to see if the ratios are in the range predicted in the data sheet (using equivalent square bases for frustums 1, 3 and 5). Checked per "Regarding Exercises." ____ ____

45. (Optional) ESSAY: Look around your environment and find something that has the shape of a frustum (it may be part of a more complex shape). Tell what you found and why you think it has that shape rather than another. ____ ____

D. CURVED SURFACES

1. READ: DS #2566 Cylinders. ________

2. DEMONSTRATION: For each of the following statements about cylinders, recall or find the equivalent facts about prisms. Note what translations are being made from prisms to cylinders. Write each statement below as a formula that you could apply to a cylinder.
   a) The volume of a cylinder = area of base × height. ___
   b) The volume of a cylinder = area of a right section × length of an element. ___
   c) The lateral surface area of a cylinder = perimeter of a right section × length of an element. ___
   d) The lateral surface area of a right cylinder = perimeter of base × height. ___ ________

3. DEMONSTRATION: Find or make models to show that:
   a) An oblique section of a right circular cylinder is an ellipse. ___
   b) A right section of an oblique circular cylinder is an ellipse. ___
   c) An oblique section of a right elliptical cylinder *may* be circular. ___
   d) An oblique section of an oblique elliptical cylinder *may* be circular. ___ ____ ____

4. PRACTICAL APPLICATION:
   a) Compute the volume of a tin can, then see how much water it will hold. ___
   b) Work out how to make a paper model of a right circular cylinder with d = 3" and h = 3". Find its total surface area and volume. ___ ____ ____

5. DRILL: Exercise #8a, problems #1, 2, 4 and 7, in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

6. READ: DS #2567 Cones, to the heading "Frustum of a Cone." ________

7. DEMONSTRATION: Show how a right circular cone is like a regular pyramid, and how and why certain pyramid formulas can also apply to cones. ________

8. DEMONSTRATION: Get a model of a right circular cone (or make one out of clay) and cover its lateral surface with a sheet of paper. Carefully cut away the excess paper so what is left is congruent with the lateral surface of the cone. Unwrap the paper from the surface and lay it flat on another sheet of paper. It will be a sector of a circle. Show how the area of the sector A_s relates to the area of the full circle. (Using r and h_s as defined in the data sheet, this can be expressed as $A_s = \pi h_s^2 \times r/h_s$.) Use this result to show that the area of the lateral surface approaches the area of the base as the height of the cone approaches zero. _________

9. DEMONSTRATION: Get a model of a right elliptical cone (or make one out of clay) and cover its lateral surface with a sheet of paper. Carefully cut away the excess paper so what is left is congruent with the lateral surface of the cone. Unwrap the paper from the surface and lay it flat on another sheet of paper. How does this change to an elliptical base affect the shape of the lateral surface of the cone? (Hint: It may be hard to see the difference in your model, but consider what happens to the slant height at different points when the base is an ellipse.) _________

10. DRILL: Exercise #9 a) through c) in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

Note: Items #11–13 below are optional—they are there for students who want more application. Students who chose to do the steps above on DS #2565 The Frustum of a Pyramid should do these steps as well.

11. (Optional) READ: DS #2567 Cones, section "Frustum of a Cone." _________

12. (Optional) DEMONSTRATION: Derive the formula for the volume of a frustum of a circular cone from the general formula (the last and next to last formulas in the data sheet). _________

13. (Optional) DRILL: Exercise #9 d) through f) in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

14. READ: DS #2568 Spherical Geometry, to the heading "Geometry of Spherical Surfaces." _________

15. DEMONSTRATION: Get a globe (or any sphere), and show each of the following:
    a) The difference between a great circle and a small circle. ___
    b) The difference between a major arc and a minor arc. ___
    c) The shortest distance between two points on the globe is an arc of a great circle. ___
    d) Looking at a globe, imagine a plane flight from Vancouver, British Columbia, to Paris, France, which goes in the straightest line possible

(along a great circle). Notice what compass direction it must fly at the start and how this changes as the flight goes along. Even though Paris is due east of Vancouver, see how much further the plane would have to fly if it just flew east the whole way (use a string to compare). Notice what sort of circle the due-east course lies on. ___ _________

16. DRILL: Exercise #10 in DS #7185 Exercises for Solid Geometry. (Keep a record of the distances measured for step 20.d. below.) Checked per "Regarding Exercises." ____ ____

17. READ: DS #2568, section "Geometry of Spherical Surfaces." _________

18. DEMONSTRATION: Get a globe (or any sphere), and show each of the following:

    a) The diameter D_s of a great circle as measured on the surface is half of another great circle. ___

    b) The ratio of the circumference of a small circle to its diameter as measured on the surface depends on the size of the circle. ___

    c) The sum of the angles in a triangle as measured on the surface depends on the size of the triangle. ___ _________

19. READ: DS #2568, section "Surface Area of a Sphere." _________

20. DEMONSTRATION: Surface area of a sphere:

    a) Put a layer of clay (thin, but not so thin it won't hold together) over a hard plastic ball. Carefully scribe evenly spaced lines in the clay to divide the surface into lunes. Then peel the lunes off of the ball and lay them out flat. Notice that the lunes will not automatically lie completely flat and smooth but bump up a bit or become somewhat wrinkled (distorted). ___

    b) Compare a model of a sphere to a model of a cylinder with the same diameter and height (use a paper cylinder of the same height around the sphere). Visualize how the surface areas can be the same, and how a projection from one to the other would be distorted. ___

    c) Get a globe and see if the map is assembled from lunes. Compare the globe to various flat maps of the world. See how the different flat maps try to show the curved surface, and how regional areas are distorted in different ways. Look on each map for data about what type of projection was used to make it, and do some research to find out how each type of projection is done. ___

    d) Use at least two different flat maps of the world (such as a polar projection and a cylindrical projection). Look at the scales and try to measure the distances from London to Sydney and from Austin to

Cairo on the flat maps. Which types distort these measurements the least? The most? Decide what makes each type of map useful. ___ ____ ____

21. READ: DS #2568, section "Geodesics." ________

22. DEMONSTRATION: Be sure you can visualize how a pentagon or hexagon can form the base of a shallow pyramid, and the height can be adjusted such that all the vertexes including the apex will lie on the surface of a given sphere. Work it out with a model as needed. ________

23. (Optional) PRACTICAL APPLICATION: Research different geodesic patterns, such as the soccer ball pattern and pictures of various geodesic domes, and look for the repeating patterns of pentagons and hexagons within them. Many patterns are possible. Pictures of various "buckyball" molecules are another place to look for them.

    If you wish, you can experiment with paper foldouts of geodesic patterns. One approach is to start with a pattern of hexagons and pentagons drawn on paper, then cut it and use it as a foldout to see if it makes a dome. Then you can convert the hexagons and pentagons into pyramids. ________

24. READ: DS #2568, sections "The Volume of a Sphere" and "Parts of a Sphere." ________

25. DRILL: Study the derivation of the formula for the volume of a sphere until you understand it well and can write it yourself without reference to the materials. Checked per "Regarding Exercises." ____ ____

26. DEMONSTRATION: Cut a ball of clay into two hemispheres, then cut the hemisphere into four quadrants to make a clay model of a quadrant of a sphere. ________

27. DRILL: Exercise #12 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

28. PRACTICAL APPLICATION: The Greek geometer Archimedes worked out the ratios of spheres and cylinders somewhere around 200 BC. He was so pleased with his discovery that he asked to have a diagram of it engraved on his tomb in Sicily. When the Roman statesman Cicero was in Sicily some 175 years later he showed his respect for Archimedes and his accomplishments by having the engraving restored.

    Your task: From the individual formulas, develop the ratios comparing a cylinder and a sphere that have the same diameter (with the height of the cylinder equal to this diameter). Compare both surface areas and volumes. Do some research on Archimedes if you wish. Then write an essay explaining these ratios and telling why you think Archimedes was so pleased with his discovery. **Supervisor pass.** ____ ____

29. READ: DS #2569 Volumes and Areas of Irregular Solids, to the heading "Very Irregular Volumes." ________

30. DEMONSTRATION: Look around your environment and choose five different objects. For each one, decide how you could divide it up into simple geometric solids that do a good job of approximating the object, and make a sketch or a list to show that. ____ ____

31. DRILL: Exercise #13 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

Note: Items #32–38 below are optional—they are there for students who want more application. Students who chose to do the steps above on DS #2565 The Frustum of a Pyramid should do these steps as well.

32. (Optional) READ: DS #2569 Volumes and Areas of Irregular Solids, section "Very Irregular Volumes," up to Example 4. ________

33. (Optional) DEMONSTRATION: Keeping in mind the application of ratios in solids, study carefully the steps given in Example 3. Once you have fully understood it, show another student how the general procedure for finding the volume of *any* solid reduces to the formula for the volume of a *particular* solid (in this case the pyramid with base area = K). ____ ____

34. (Optional) READ: DS #2569, section "Very Irregular Volumes," Example 4. ________

35. (Optional) DEMONSTRATION: Compute the volume of the sphere in Example 4 using the dimensions for sphere b or c in the multiple frustum formula. How close does this result come to 4.5π? ________

36. (Optional) DRILL: Exercise #14 in DS #7185 Exercises for Solid Geometry. Checked per "Regarding Exercises." ____ ____

37. (Optional) DEMONSTRATION: The volume of any solid may be expressed as an average area × a height. ________

38. (Optional) PRACTICAL APPLICATION: Work out the frustum formula and compute the *surface area* of the sphere illustrated in the data sheet, and compare your result to what you get using $A = 4\pi r^2$. (You may do this for any or all of spheres a, b and c. Of course c will give you the most accurate result.) ________

39. READ: DS #2569, section "A More Practical Example." ________

40. DEMONSTRATION: Show how the Pythagorean theorem is used to find the slant height of the hill from the data on the map, and how this would be

used to compute the surface area (find the area of at least one triangle this way). _________

41. PRACTICAL APPLICATION: Estimate the volume of the hill in the data sheet by using the formula for the volume of a cone. Start by approximating its base area with simple shapes such as circles or ellipses. Then make a more accurate estimate by using the grid method to get the area of the base. _________

42. PRACTICAL APPLICATION: For the same hill as above, consider what it would take to get the area of each contour layer and to calculate the volume as a series of frustums (it is not necessary to actually do these calculations). Then think about these points:

    a) Would this approach improve the accuracy of your volume estimates?

    b) If the contour intervals were less regular (i.e., if the slope varied a lot as you went up the hill), how might that affect the relative accuracy of the different methods?

    c) Compare what you think are the amounts of work involved in using the cone method versus using the frustum method.

    d) Decide for yourself what the practical trade-offs in different situations might be between accuracy and the time taken to get the result using the different methods.

    Write down a general idea of a real situation involving actual work and the expenditure of effort and money where the cone approach would likely be adequate and another situation where it would be worth it to use the frustum method. **Supervisor pass**. ____ ____

E. FINAL APPLICATION SECTION

1. PRACTICAL APPLICATION: Exercise #15 in DS #7185 Exercises for Solid Geometry. Checked per “Regarding Exercises.” **Supervisor pass.** ____ ____

2. READ: DS #7188 Mathematics and Art. _________

3. ESSAY: Give five specific examples from your study of geometry that demonstrate how geometry is related to art. ____ ____

4. ESSAY: Explain how you expect geometry to be useful to you in life. **Supervisor pass.** ____ ____

I have completed the steps of this course. I understand what I studied and can use it.

Student____ Date ____________________

The student has completed the steps of this course and knows and can apply what was studied.

Academic Supervisor______________________________ Date ____________________

The student has passed the exam for this course.

Examiner ______________________________ Date ____________________