Game Theory and Dynamic Traffic Assignment Equilibrium

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Abstract

The exact nature of dynamic traffic assignment (DTA) equilibrium is not fully known in simulation-based models. Universal solutions for general networks may not exist and multiple equilibria are possible. This is problematic for transportation practitioners since projects are evaluated at a unique equilibrium state. By formulating traffic assignment as a large-scale game, techniques and literature from game theory can be applied to address these equilibrium issues. Two network examples are presented: one displays a scenario where no DTA equilibrium exists and the other showcases a scenario with multiple equilibria. The first network is shown to have a mixed strategy Nash equilibrium. In the second network, the amount of multiple equilibria is reduced by applying the trembling hand refinement from game theory.

Background

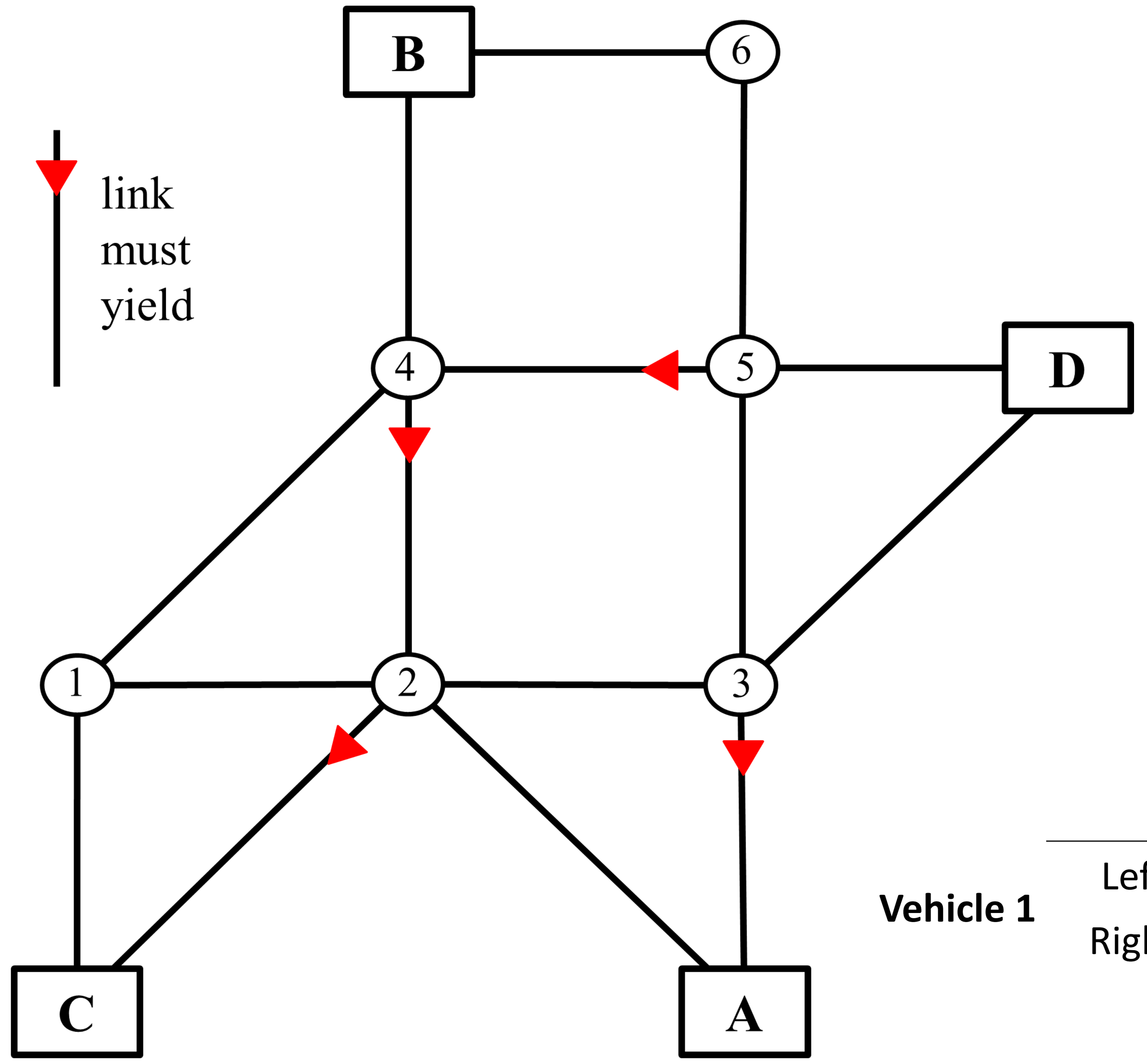
A game is characterized by three elements: (1) set I consisting of all entities/players, (2) a set of actions/strategies A_i for every $i \in I$, and (3) the utility or satisfaction u_i player i will expect from the given set of strategies, $u_i\colon A \to \mathbb{R}$. The most widely used and recognized notion of an equilibrium state is the concept of Nash equilibrium. Nash equilibrium can be categorized into two basic types of equilibrium: pure and mixed strategy. Pure strategy Nash equilibrium can be defined as the stable state where no player can improve his/her utility by changing strategies. It is expressed formally below, where a_{-i} indicates the actions of all players except player i and a_i represents all other strategies available to player i besides a'_i.

$$u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \qquad \forall\, i \in I$$

A game can have multiple pure strategy Nash equilibria or none at all. However, any game with a finite set of players and a finite set of actions is guaranteed to have a mixed strategy Nash equilibrium. In a mixed strategy solution, players are allowed to randomize among their various actions - to choose a probability distribution over their strategy set as to maximize their expected utility.

No Equilibrium Example Network

Traffic assignment can be formulated as an economic game, where each driver is a player whose strategies are the routes that they can choose. The utility of each driver is the resulting travel time, which they try to minimize in accordance with user equilibrium. Consider the network below. Vehicle 1 travels from A to B. Vehicle 2 travels from C to D.



Travel Times:
Horizontal/Vertical Links: 1 min
Diagonal Links: 1.5 min
Yield Delay: 1 min

Vehicle 1 Path Set:
Left: A → 2 → 4 → B
Right: A → 3 → 5 → 6 → B

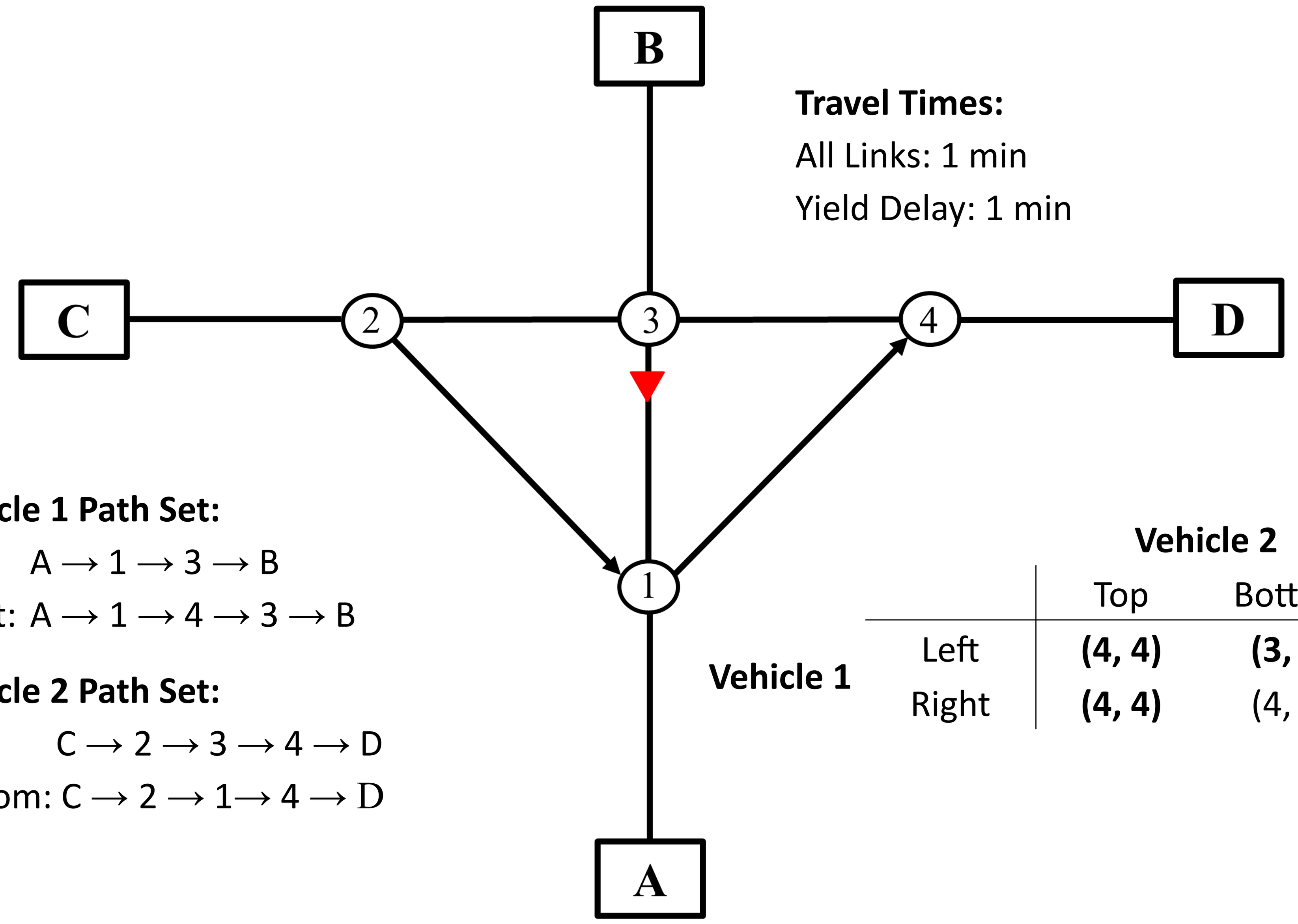
Vehicle 2 Path Set:
Top: C → 1 → 4 → 5 → D
Bottom: C → 2 → 3 → D

		Vehicle 2	
		Top	Bottom
Vehicle 1	Left	(4.5, 4.5)	(3.5, 5.0)
	Right	(4.0, 4.5)	(4.0, 4.0)

The left and bottom paths are initially the shortest paths. If both vehicles choose these paths, Vehicle 2 will be delayed at node 2. Vehicle 2 opts for the top path, gaining priority at node 4 and delaying Vehicle 1. Vehicle 1, in turn, opts for the rightmost path. This frees the way for Vehicle 2 to return to the bottom path, allowing Vehicle 1 to return to the left path, and so forth *ad infinitum*. There is no user equilibrium solution. However, there is a mixed strategy Nash equilibrium: Vehicle 1 will choose the left path 50% of the time, and Vehicle 2 will choose the top path 50% of the time. This added information from game theory can aid practitioners; this fixed ratio of departure can be used to approximate traffic flows, leading to other indices of interest (e.g., crash rates).

Multiple Equilibria Example Network

There are three user equilibrium solutions associated with the network shown below: (1) Vehicle 1 chooses the left path and Vehicle 2 chooses the top path, (2) Vehicle 1 chooses the left path and Vehicle 2 chooses the bottom path, and (3) Vehicle 1 chooses the rightmost path and Vehicle 2 chooses the bottom path. These solutions correspond to three pure strategy Nash equilibrium points.



Travel Times:
All Links: 1 min
Yield Delay: 1 min

Vehicle 1 Path Set:
Left: A → 1 → 3 → B
Right: A → 1 → 4 → 3 → B

Vehicle 2 Path Set:
Top: C → 2 → 3 → 4 → D
Bottom: C → 2 → 1 → 4 → D

		Vehicle 2	
		Top	Bottom
Vehicle 1	Left	**(4, 4)**	**(3, 4)**
	Right	**(4, 4)**	(4, 4)

By applying the concept of trembling hand perfect equilibrium to the network, weakly dominated strategies can be eliminated. This includes the user equilibrium associated with Vehicle 1 using the rightmost path. Vehicle 1 should only choose the rightmost path if Vehicle 2 chooses the top path 100% of the time; the travel time on the left path will always be less than or equal to the travel time of the rightmost path. Therefore, using the trembling hand perfect equilibrium theory, an unrealistic equilibrium is removed. This reduction is vitally beneficial to transportation engineers since any collected data or measured index from a network model must represent realistic traffic conditions.