Exploration of Existence and Uniqueness Issues of Dynamic Traffic Assignment Equilibrium

Chris Melson
Tarun Rambha
Dr. Stephen Boyles
The University of Texas at Austin
2012 INFORMS Annual Meeting --- Tuesday, October 16th

Outline of Presentation

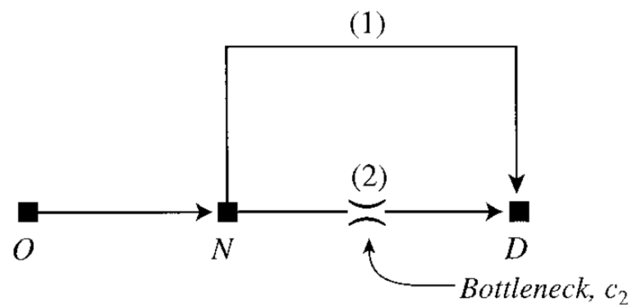
- Introduction
- Earlier Research and Motivation
- Examples
  - No Equilibrium
  - Infinite Number of Equilibrium
- Possible Solution
  - Piecewise linear fundamental diagram
- Future Work

Introduction

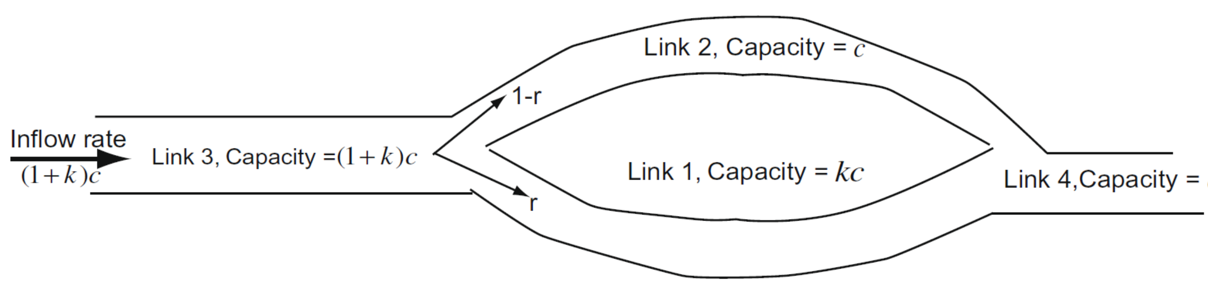
- Dynamic traffic assignment (DTA) provides hope for accurately modeling traffic
  - Addresses issues of static traffic assignment: time-varying demand, queue formation, congestion spillback, etc.
  - Needed in order to model time-dependent demand policies and most ITS technologies
- Simulation-based DTA models do not provide a universal solution or guarantee that equilibrium exists
  - Equilibrium is heuristically approximated
  - Multiple equilibrium are possible
  - Equilibrium may not exist

Earlier Work

- (Daganzo, 1998)
  - Addressed the importance of queue spillback prevention and its chaotic behavior (small perturbations of c_2 can dramatically effect the state of the network)



- (Nie, 2010)
  - Showed that four distinct user equilibria can develop
  - Categorized equilibria by stability and efficiency properties



(a) A Diverge-Merge (D-M) Network
k is the ratio of the capacities of link 1 and link 2.
r is the proportion of traffic heading for link 1.

Motivation

- Contribute to the limited research regarding DTA equilibrium issues and propose game theory as a potential solution method
  - First example
- Previous research has focused on the nature of DTA equilibrium at the merge or as a result of a downstream obstruction
  - Second example showcases the complications at the diverge

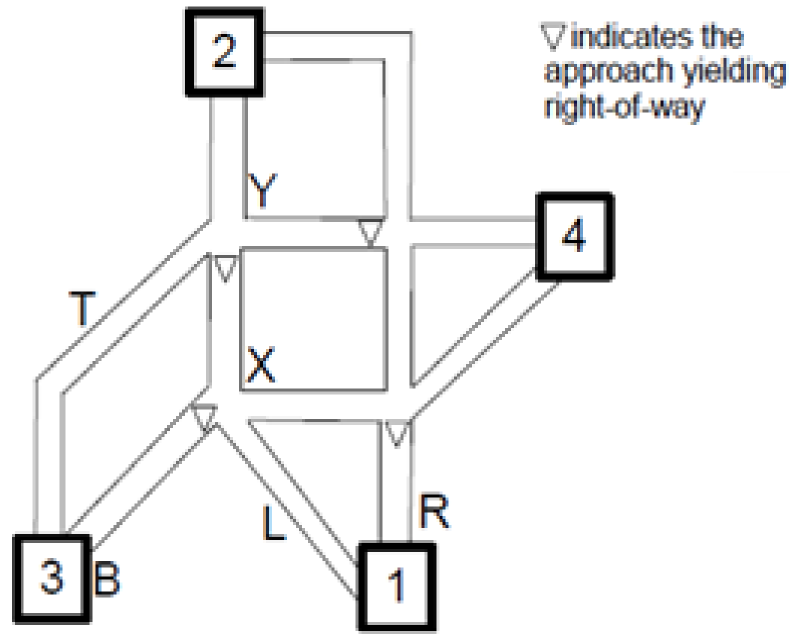
DTA as a Large-Scale Economic Game

- A game is made up of three elements:
  - I number of players [individual drivers]
  - Set of actions A_i for each player i [paths available to each driver]
  - Resulting utility of each action $u_i : A \rightarrow \mathbb{R}$ [path travel times]

- Any game with a finite number of players and a finite set of strategies is guaranteed to have a mixed-strategy Nash Equilibrium
  - Players are rational
  - Players act independently of one another
  - (Nash, 1951)

No Equilibrium Case

- Horizontal/Vertical Links: 1 minute travel time
- Diagonal Links: 1.5 minute travel time
- Yield Time: 1 minute
- Player 1 travels from Origin 3 to Destination 4
- Player 2 travels from Origin 1 to Destination 2
- Players will continually switch paths



	L	R
T	(4.5, 4.5)	(4.5, 4)
B	(5, 3.5)	(4, 4)

No Equilibrium Case (cont.)

	L	R
T	(4.5, 4.5)	(4.5, 4)
B	(5, 3.5)	(4, 4)

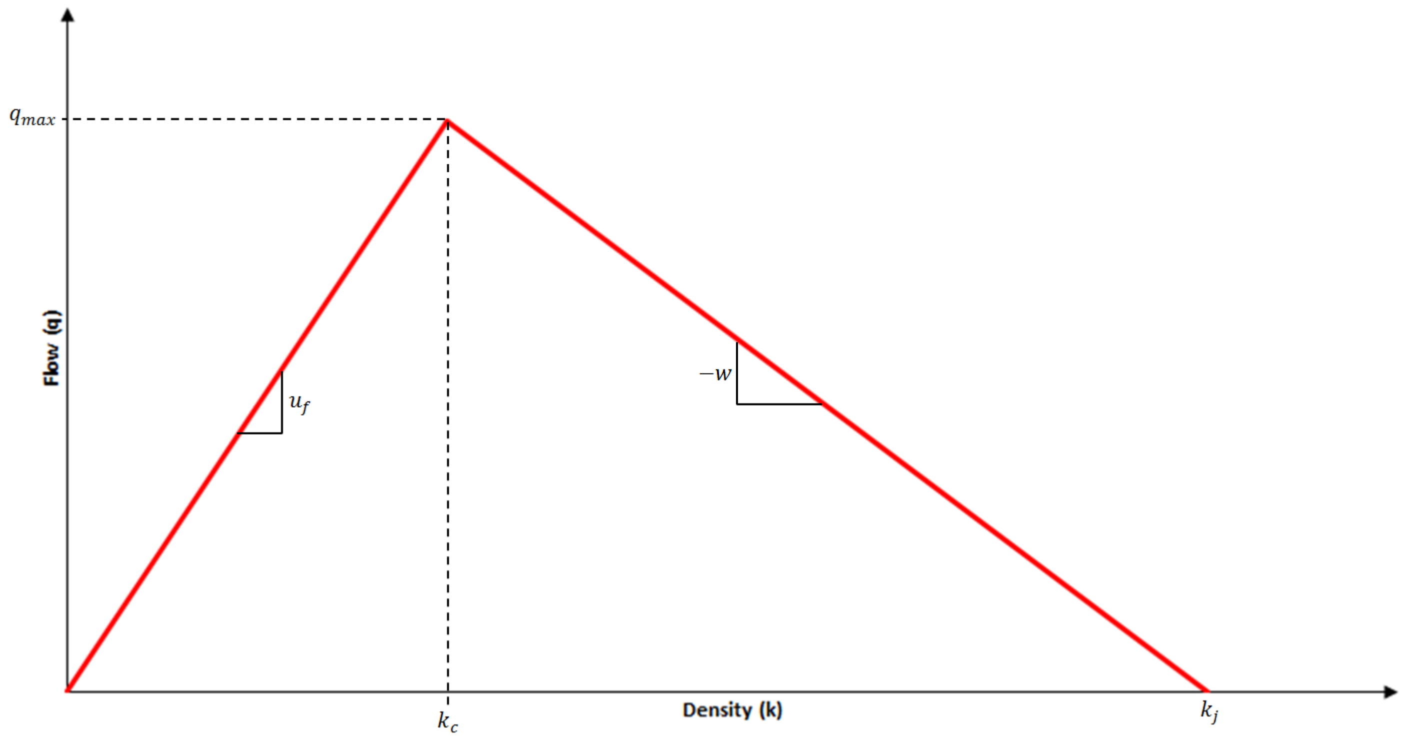
- No pure strategy Nash equilibrium

$$u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \ \forall \, i \in I$$

- Mixed-strategy Nash equilibrium
  - Player 1 will choose Path T and Path B 50% of the time
  - Player 2 will choose Path L and Path R 50% of the time

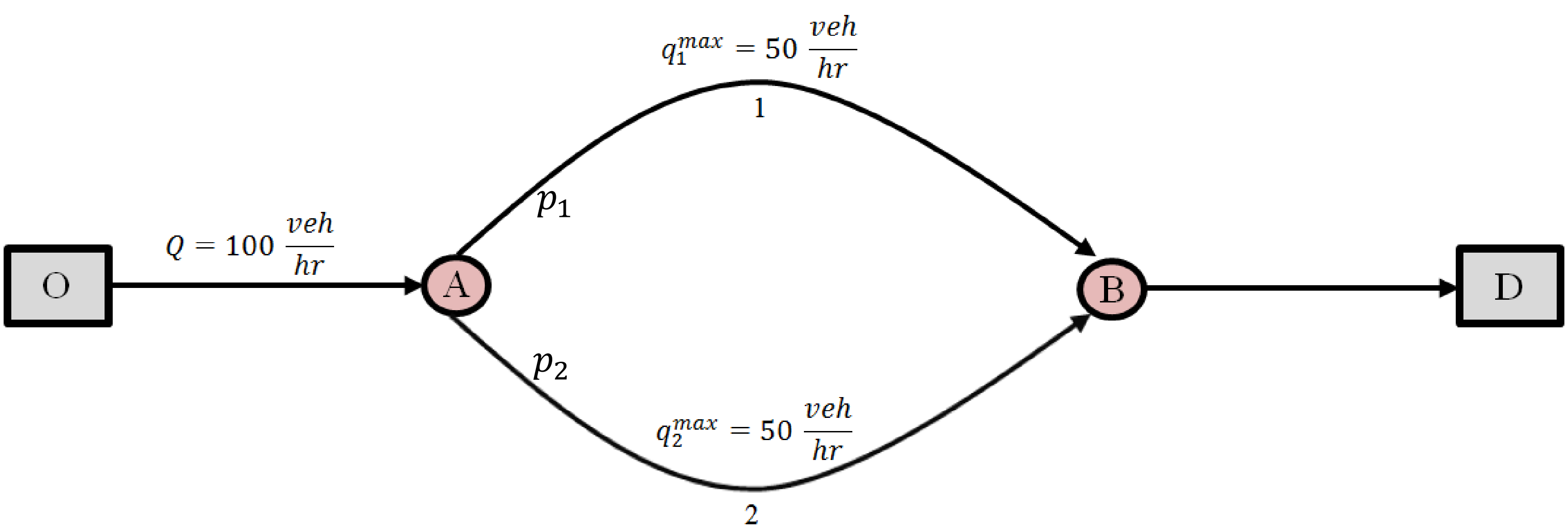
Infinitely Many Equilibrium Case

- Triangular or trapezoidal fundamental diagram
  - $[0, k_c]$ - traffic travels at free-flow speed

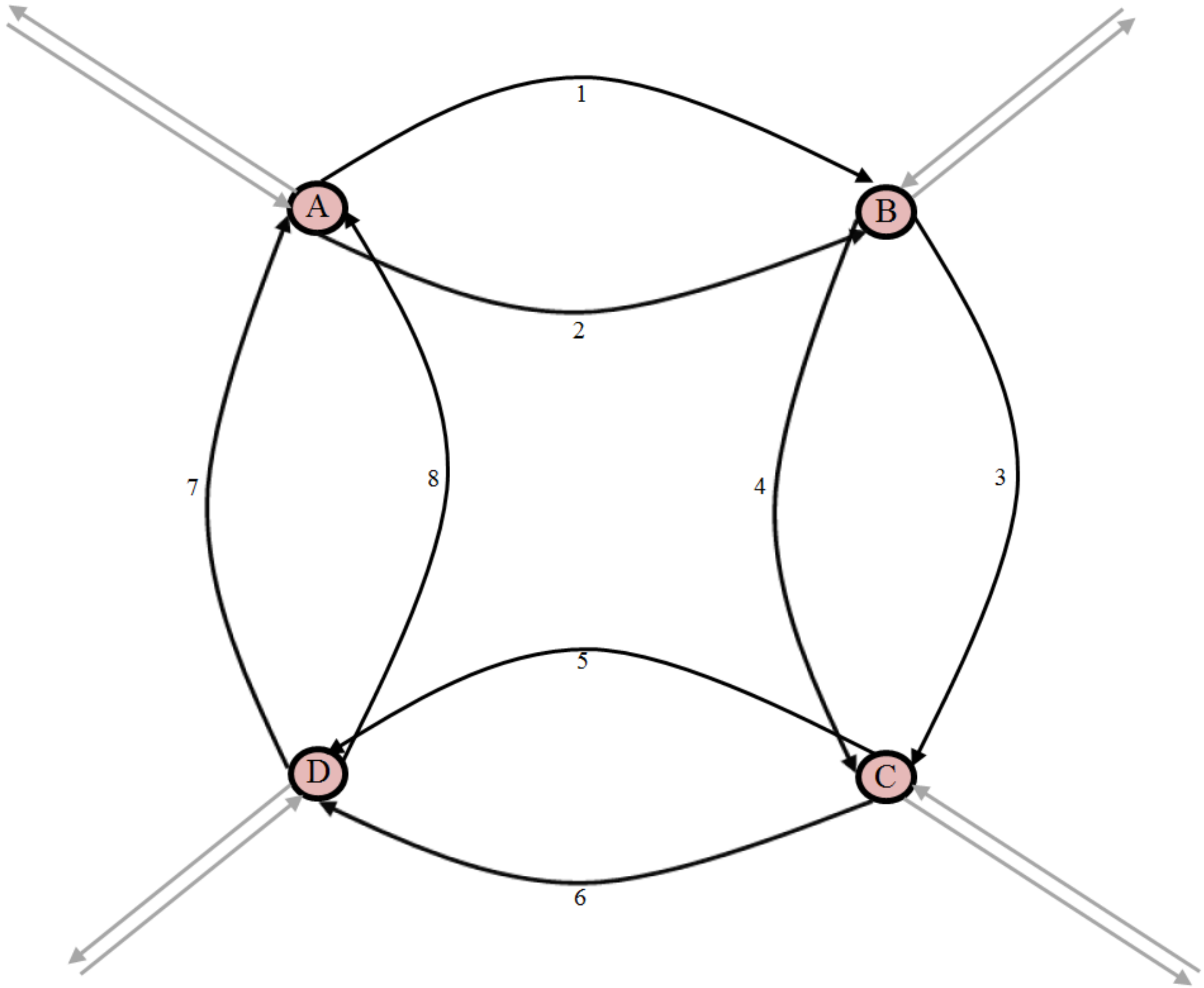


Infinitely Many Equilibrium Case (cont.)

- User Equilibrium: all used paths connecting the same origin and destination have equal and minimal travel time
  - At UE users cannot switch paths and save travel time
- One unique system optimal solution: $p_1 = p_2 = \frac{1}{2}$
- Infinitely many user equilibrium: p_1 and p_2 can vary from [0,1]

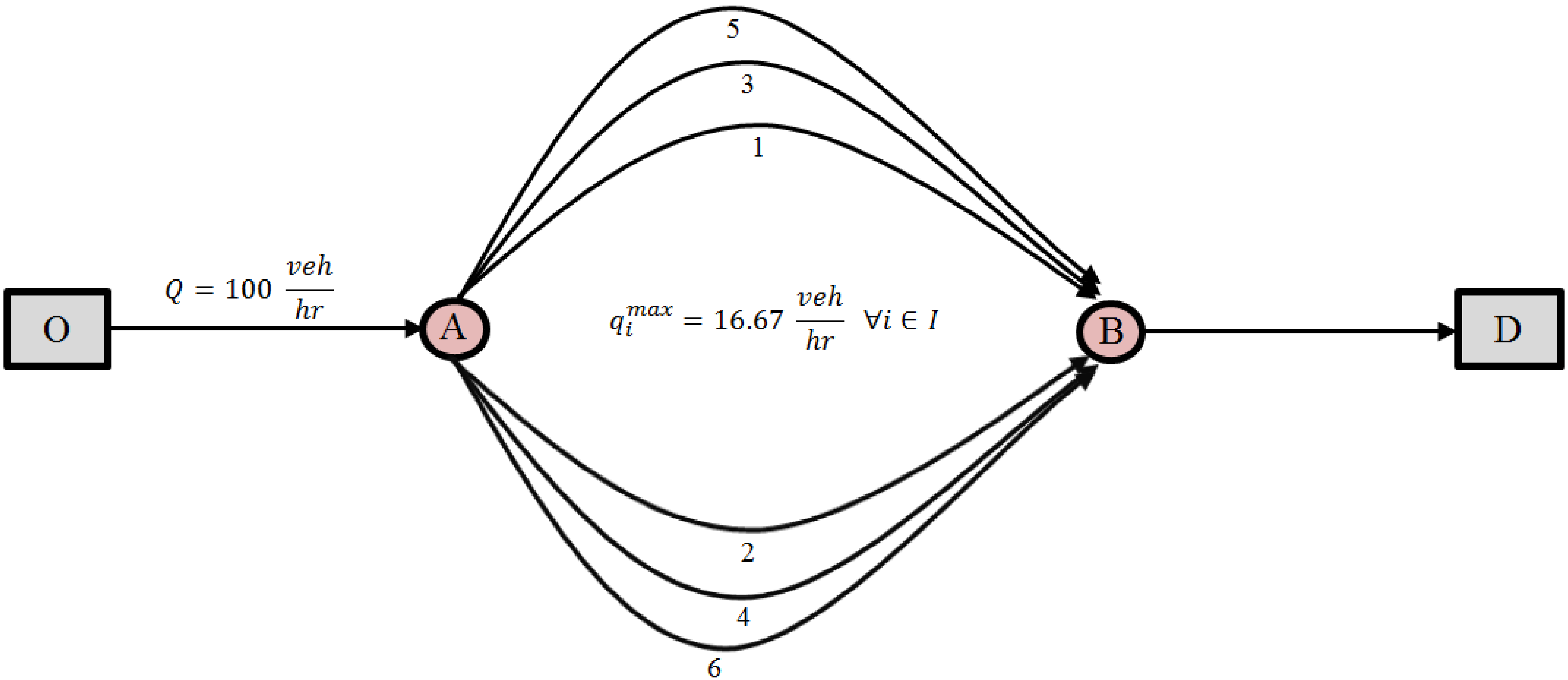


Effect on Surrounding Network



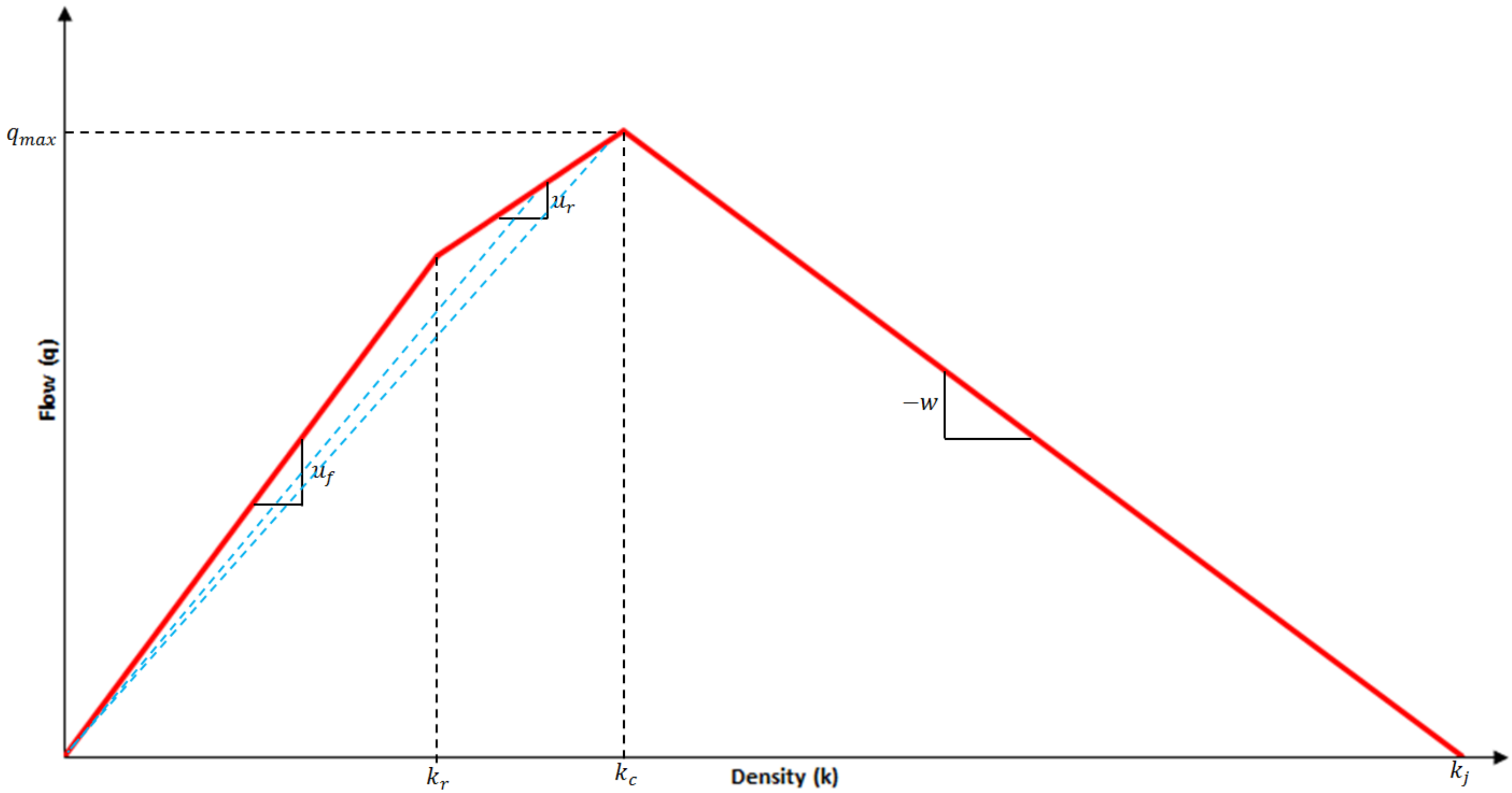
Infinite Price of Anarchy

- $Price\ of\ Anarchy = \frac{TSTT\ at\ UE}{TSTT\ at\ SO}$



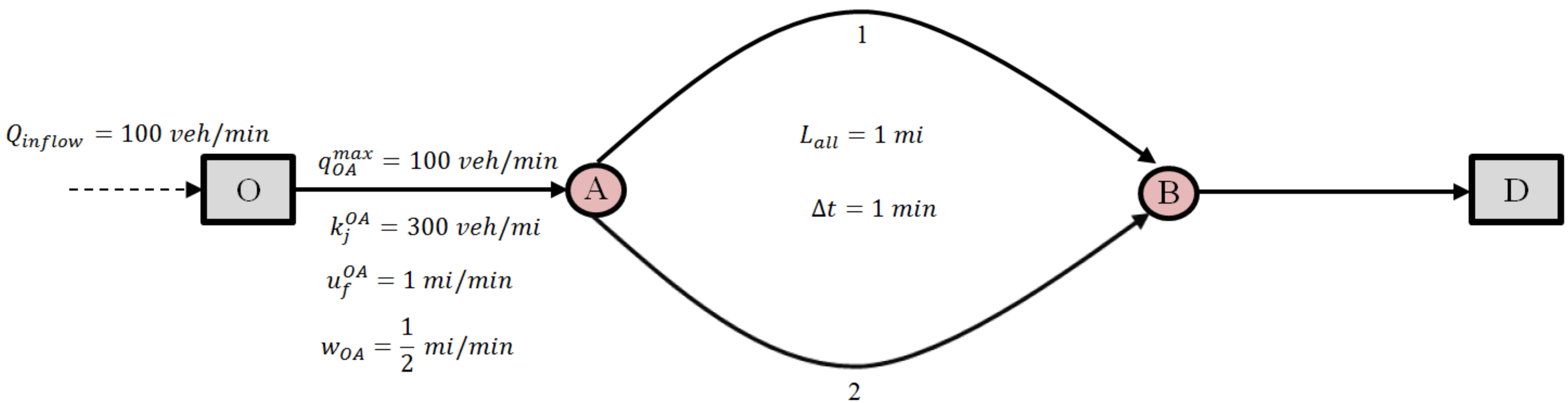
Piecewise Linear Fundamental Diagram

- $[0, k_r]$ - traffic travels at free-flow speed
- $[k_r, k_c]$ - traffic speeds vary; speeds are truly a function of density
- Unique travel time when link is operating at capacity



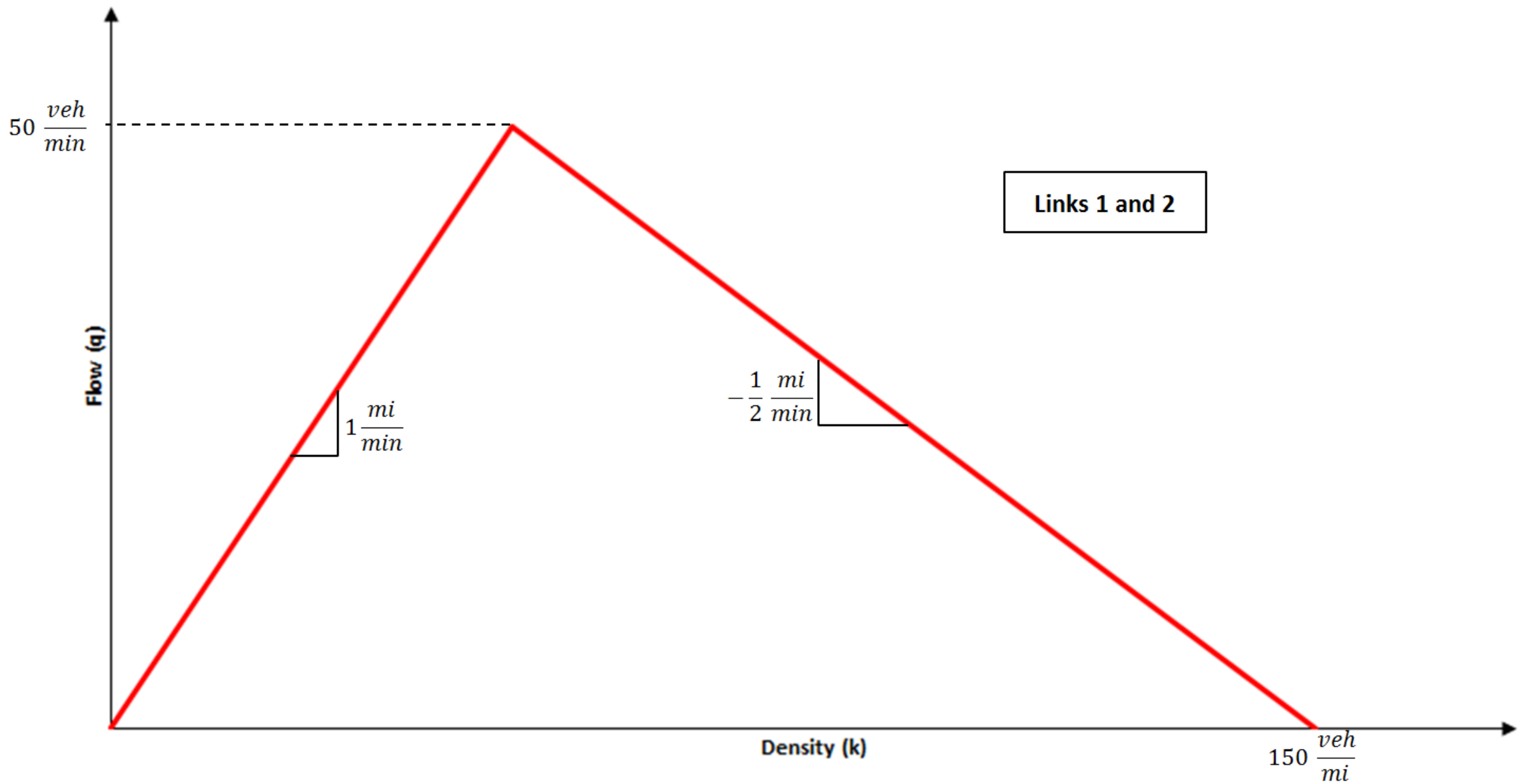
Numerical Example

- Compare Case I (links 1 and 2 have triangular fundamental diagrams) and Case II (links have piecewise linear diagrams) when $p_1 = 1$ and $p_2 = 0$



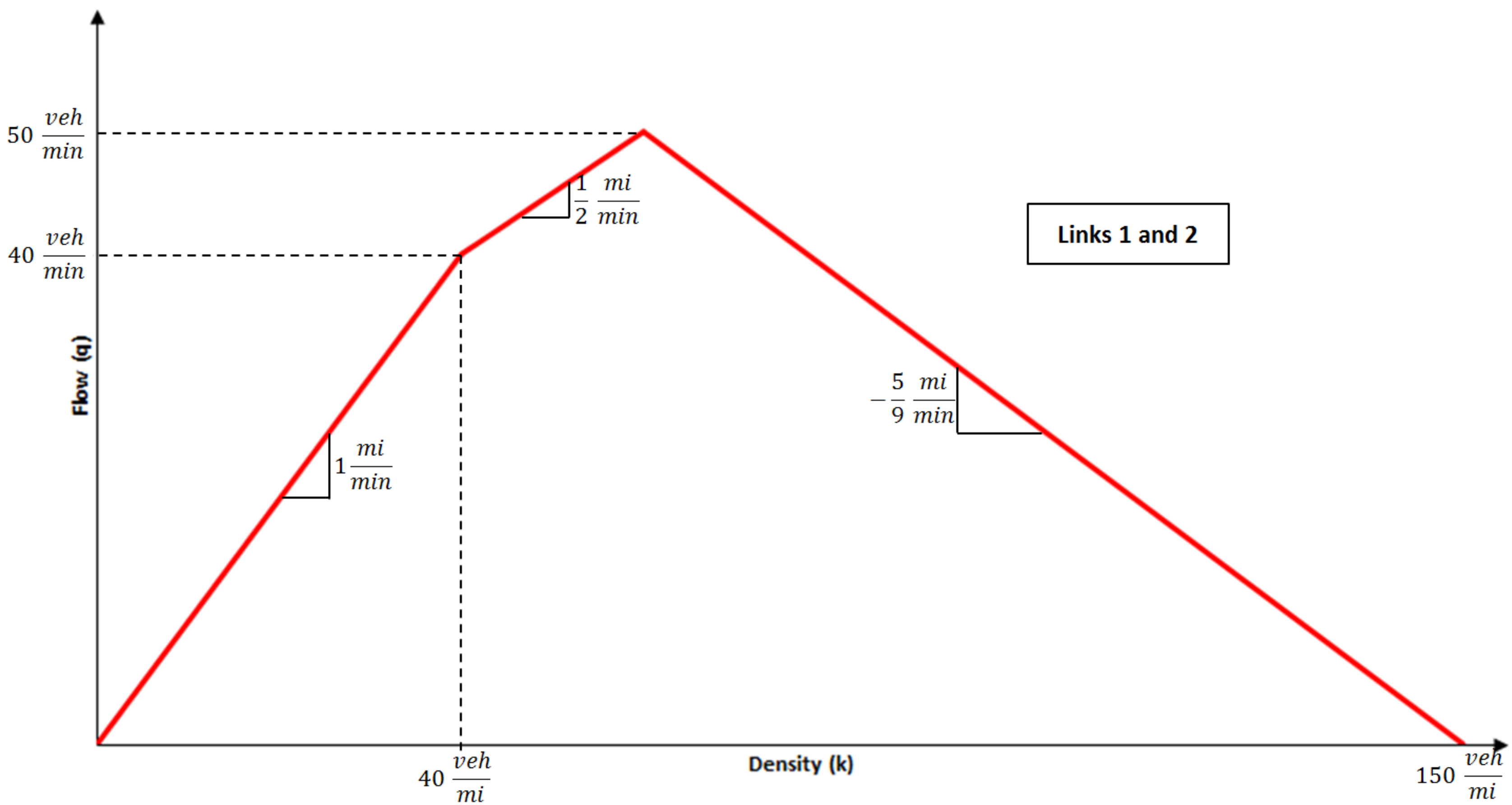
Case I

- Links 1 and 2 are identical and have the following fundamental diagram:



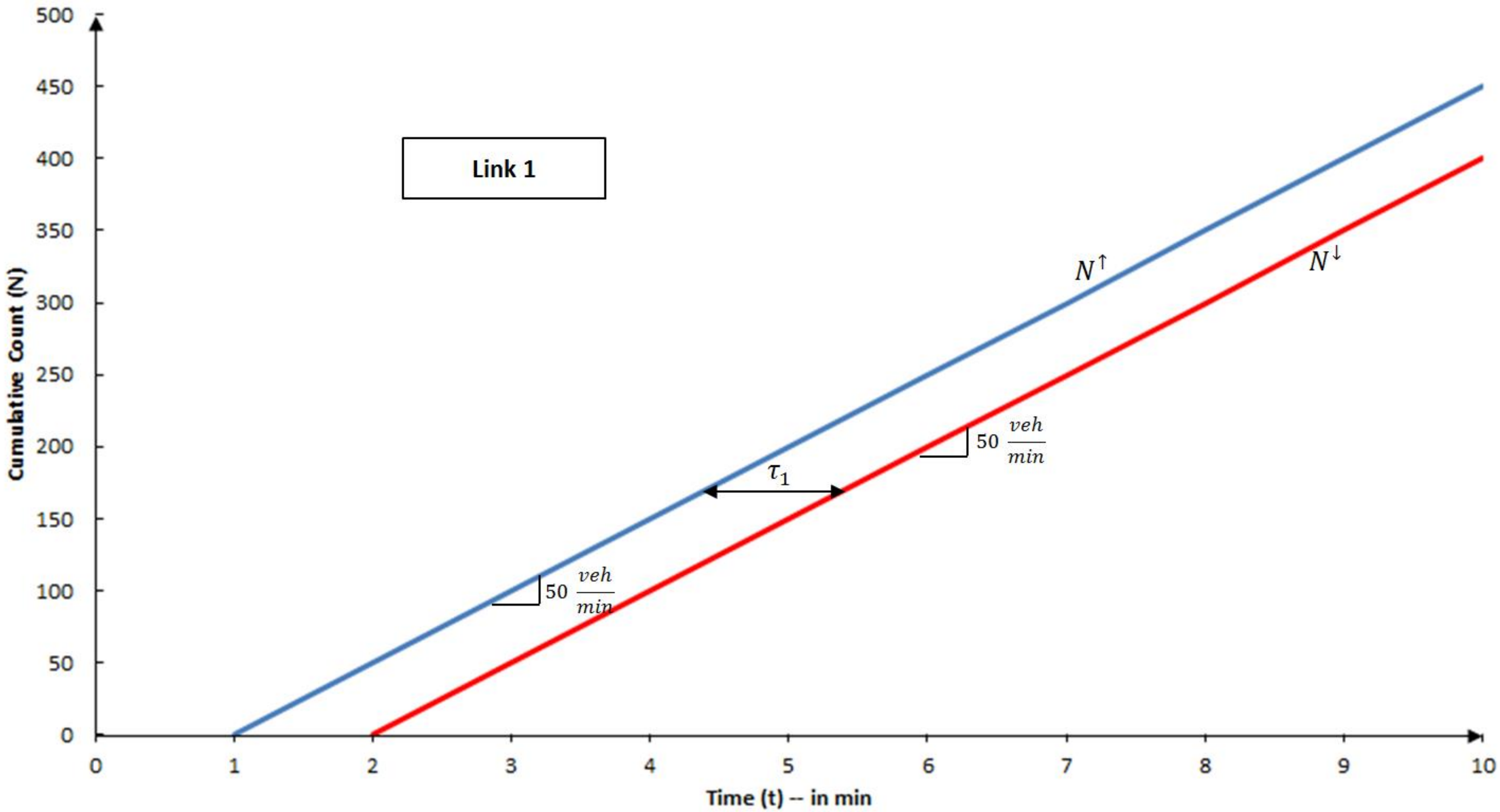
Case II

- Links 1 and 2 are identical and have the following fundamental diagram:



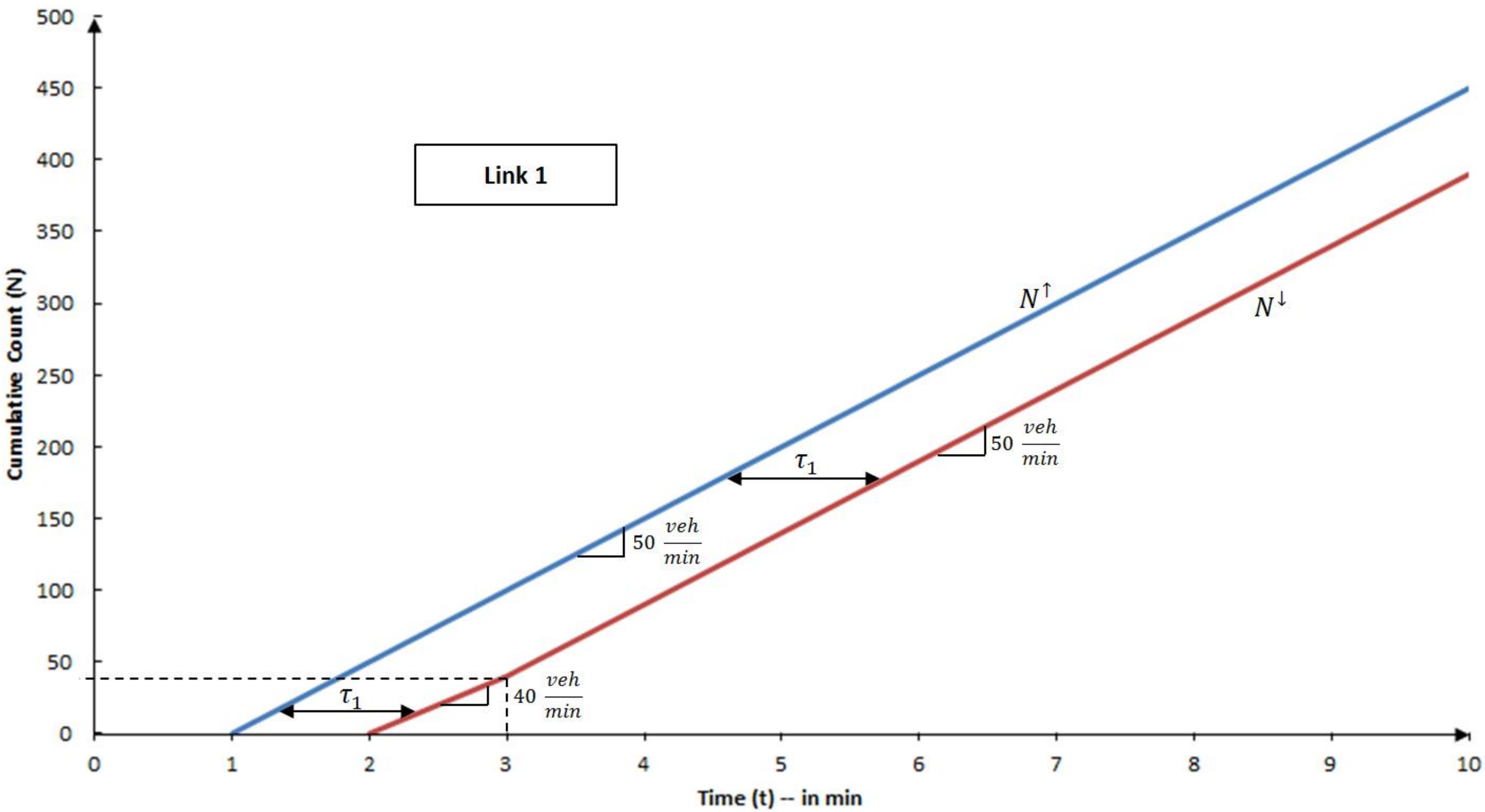
Case I Results

- $\forall \; n \in [0, N], \;\; \tau_1 = 1.0\, min = \; \tau_f$
  - $\tau_1 = \tau_2$
- $[p_1 = 1, p_2 = 0\ ]$ satisfies user equilibrium



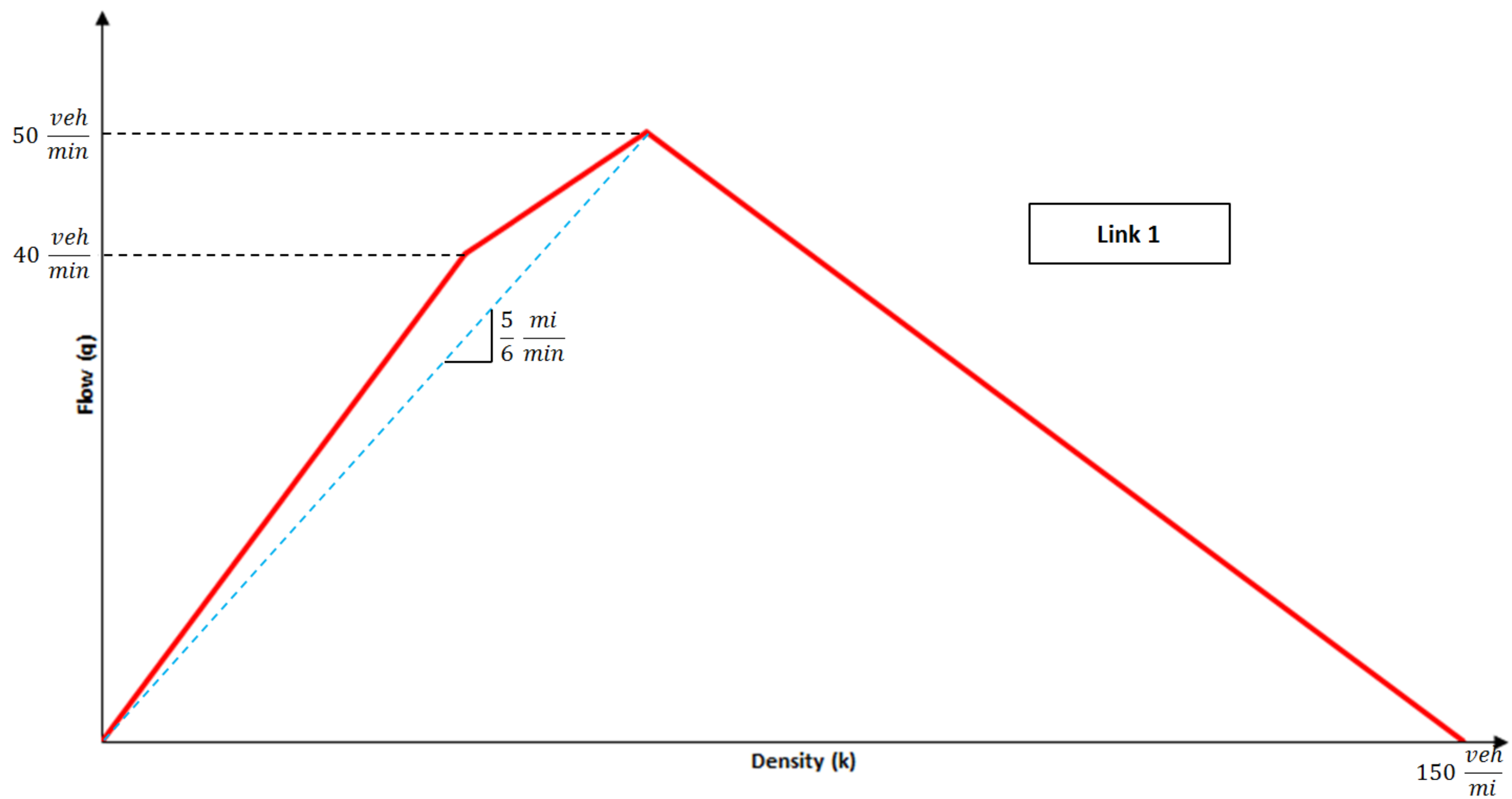
Case II Results

- $n \in [0,40], \;\; \tau_1 = \dfrac{n}{200} + 1$
- $n \in [40, N], \;\; \tau_1 = 1.2\, min$
  - $\tau_1 > \tau_2 \;\; ===> [p_1 = 1, p_2 = 0\ ]$ does not satisfies user equilibrium



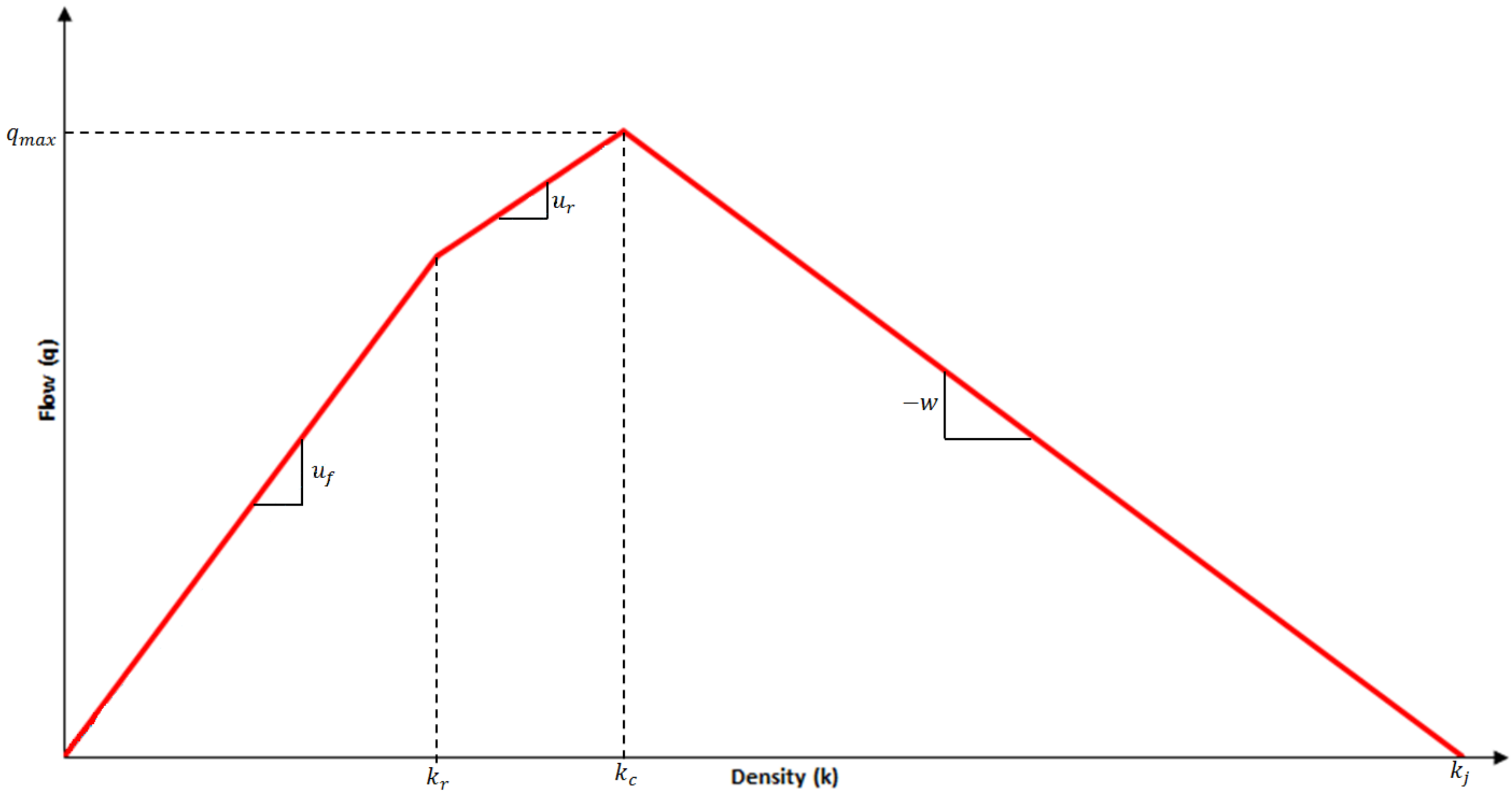
Case II Results (cont.)

- Unique link speed/travel time at capacity [$\tau_1 = 1.2\ min$]
- The piecewise linear fundamental diagram results in **one, unique user equilibrium** [$\tau_1 = \tau_2 = 1.2\ min$ and $p_1 = p_2 = \frac{1}{2}$]



Future Work

- Defining u_r and k_r



References

1. Daganzo, C.F. (1998). “Queue Spillovers in Transportation Networks with a Route Choice.” *Transportation Science* 32(1), 3–11.
2. Nie, Y. (2010). “Equilibrium Analysis of Macroscopic Traffic Oscillations.” *Transportation Research Part B* 44(1), 62–72.
3. Nash, J. (1951). “Noncooperative Games.” *Annals of Mathematics* 54, 286–295.