Wavelet Pooling for Convolutional Neural Networks using Unitary Gates

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Abstract

In this paper, we explore the idea of constructing novel wavelet transforms using unitary gates which are analogous to quantum-circuit gates used to represent a type of tensor network called Multi-Scale Entanglement Renormalization Ansatz (MERA), predominantly used in condensed matter physics and theoretical physics for representing quantum many-body states. Particularly, we explain the idea of incorporating novel wavelet pooling and un-pooling operations designed using local unitary gates to replace the max-pooling layers in the convolutional neural networks (CNN). We demonstrate the application of the newly designed wavelet transforms for the task of image super-resolution, note that our goal is to provide a formal theoretical framework, hoping this work will serve as an ingress for consilience between machine learning and theoretical physics.

1. Introduction

The realization [1] that quantum many-body problem often encountered in physics can be elucidated using neural networks into a tractable computational form has led to many exciting applications in machine learning, statistical physics and condensed matter physics. Tensor networks which were primarily introduced as a numerical tool to simulate quantum systems, represent wave functions as a network of interconnected tensors, this representation has allowed to deal with the exponential growth of hilbert space, when describing a sufficiently complex quantum system with many interacting particles. The most popular framework to simulate quantum many-body systems is Multi-Scale Entanglement Renormalization Ansatz (MERA) introduced by [11], which has found applications in machine learning, statistical physics, quantum mechanics and even to describe space-time geometry. [1, 5, 9, 10, 7] Our work is primarily inspired by [2, 3], authors in this work have demonstrated the existence of a fundamental connection between wavelet transforms used in signal processing and MERA, using this connection they propose novel wavelet transforms (WT) and a framework to design WT using unitary-gates, the fundamental building blocks of quantum circuits. In this work, we leverage this idea to propose a wavelet pooling operation for the convolutional neural networks as an alternative to neighborhood pooling operations commonly used. We also provide a scheme to use these newly designed wavelet transforms in variety of image enhancement/restoration setting, more particularly we explore the idea of image super-resolution.

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2. Unitary Gates powered Wavelet Transforms

The idea of constructing wavelet transforms using unitary gates is based on the premise that, wavelet transforms provide a multi-resolution analysis capturing correlations of the signal at various scales, this is analogous to the MERA formulation which encodes long-scale relations of quantum many-body systems using circuits designed from unitary gates. Authors in [3] show that the scaling sequence $\boldsymbol{h}$ and wavelet sequence $\boldsymbol{g}$ can be uniquely encoded using N depth unitary circuits. The algorithm to encode the wavelet transforms hinges on choosing the angles θ_k that characterize the circuit and allows for a unique mapping of the scaling sequence $\boldsymbol{h}$. Let $\mathcal{V}$ be a ternary unitary gate, its called ternary because it can be decomposed as [3],

$$\mathcal{V} = V_N U_{sw} V_{N-1} U_{sw} ... \tag{1}$$

where, V is the sum of reflection matrices given by,

$$v(\theta) = \frac{1}{2}\begin{bmatrix} cos(\theta)+1 & \sqrt{2}sin(\theta) & cos(\theta)-1 \\ -\sqrt{2}sin(\theta) & 2cos(\theta) & -\sqrt{2}sin(\theta) \\ cos(\theta)-1 & \sqrt{2}sin(\theta) & cos(\theta)+1 \end{bmatrix} \tag{2}$$

and U_{sw} is an invariant matrix called swap gate. Thus, a ternary gate consists of multiple unitary gates strewed with swap gates in-between. The unitary gates can be thought of as logic gates used in digital electronics which can be combined in a variety of ways to achieve complex logic.

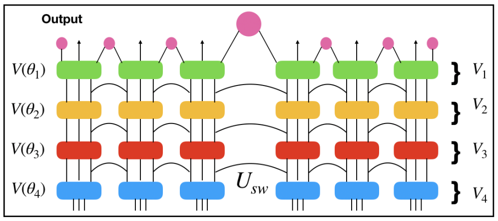

Figure 1. A graphical representation of unitary gates depicting ternary circuits of depth 4. The Ternary gates are interspersed with swap gates which are 2x2 matrices invariant under reflection, the layers $V_{1,2,3,4}$ are the result of the sum of 3x3 matrices obtained using equation 2.

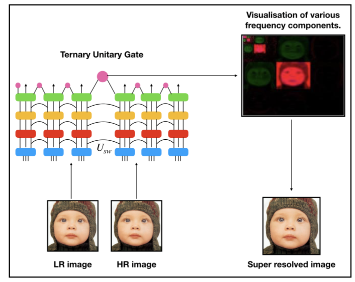

Figure 2. A formal workflow to perform image super-resolution using the ternary unitary gates. The idea is to combine the difference of frequency components of the low-resolution and high resolution images and then combining it with the input.

3. Applications

We describe how wavelets designed using these unitary gates can be used in a variety of applications.

3.1. Wavelet Pooling

The pooling operations summarize the output of the convolutional layers into a single neuronal value, the most common ones used are max and average pooling operations. The max-pooling takes the maximum of the region and selects it for the reduced feature map while the average pooling involves calculating the mean. While these simple methods are effective, they often lead to omission of important and dilution of pertinent details of the images respectively. Recent works [8, 4, 12] propose to use wavelet pooling as an alternative to max and average pooling. The idea is to use discrete wavelet transforms to transform the output of the CNN layers, since the wavelets are ideal for capturing important information at various levels. Continuing this same trend, we propose to use wavelet transforms designed using ternary unitary gates as pooling layers (TUG-wavelets), particularly we propose to use wavelet transforms with dilation=3 designed by authors in [3].The major advantage of using this wavelet transform is because it allows for well-resolved division of low, mid and high frequency components of the signal $\{s^+, b^+, b^-\}$ respectively. In the forward propagation, the wavelet co-efficients can be calculated using the equation 2, for the values of θ, authors[1] in [3] provide a comprehensive table for choosing the angle values. We design a depth 4 circuit with $\theta_k = \{0.0721, -0.8476, -0.5760, 0.5917\}$. During backpropagation the wavelet co-efficients which capture various pertinent details of the image at different scales can be combined using the inverse transform.

3.2. Image Super-Resolution

The wavelet transform designed in the earlier section has many alluring properties, allowing us to apply it in various image restoration tasks like image-denoising, image compression[2], image super-resolution etc. Following the method outlined in [6], we propose a formal scheme to perform image super-resolution using the wavelet transforms designed earlier. The TUG-wavelets have well-resolved low,mid and high frequency components which capture various information of the image like horizontal, vertical and diagonal details. We take a pair of low-resolution (LR) and high-resolution (HR) images and pass them through the TUG-wavelets $\mathcal{T}$, we obtain $\{s^+_{LR}, b^+_{LR}, b^-_{LR}\}$ and $\{s^+_{HR}, b^+_{HR}, b^-_{HR}\}$, the low frequency and high frequency components of the corresponding LR and HR images. We can train a network with a cost function -

$$\mathcal{C} = \frac{1}{2}||\Delta FC - f(\mathcal{T})||_2^2 \tag{3}$$

where, ΔFC represents the difference between the corresponding low and high frequency components of the LR and HR images. The function $f(\mathcal{T})$ represents the feed-forward procedure. Following the logic of authors in [6] we hope the network to learn the differences between the different frequency components of the LR and HR images, combining the differences with the input wavelet coefficients allow us to obtain the super-resolved images.

4. Conclusion

In this paper we have proposed a framework to demonstrate the use of novel wavelet transforms using quantum circuits.We have provided a general schema for applying the wavelet transforms in the context of replacing pooling operations of CNN and also to perform image super-resolution. We will provide more results and ablation studies in our future works, meanwhile we hope this will work will serve as

[1]The authors would like to thank Glen Evenbly for all the helpful discussions.

[2]Check out code base at https://www.tensors.net/wavelets-mera

an ingress for more consilience between deep learning and theoretical physics.

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