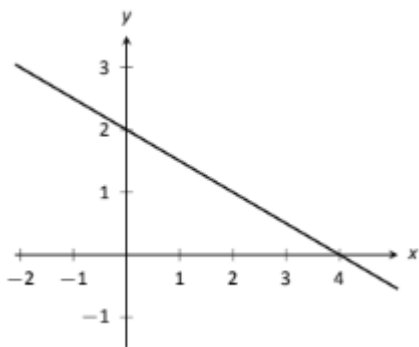


6. Mixed Applications of Integrals and Derivatives

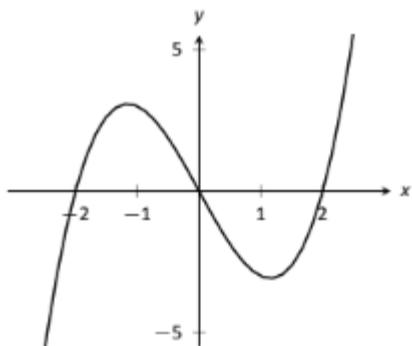
6.1 Interpreting Graphs

In the following problems, a graph of a function $f(x)$ is given. Using the graph, sketch $f'(x)$

1.



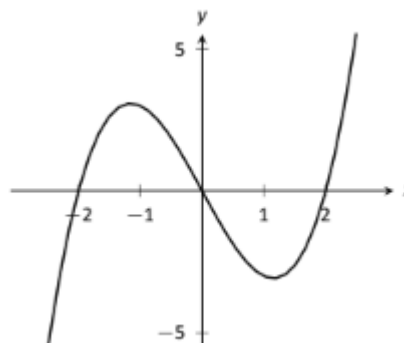
2.



In the following problem, a graph of a function $g(x)$ is given. Using the graph, answer the following questions:

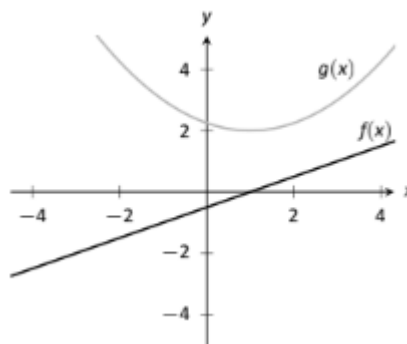
- Where is $g(x) > 0$? $g(x) < 0$? $g(x) = 0$?
- Where is $g'(x) > 0$? $g'(x) < 0$? $g'(x) = 0$?

3.

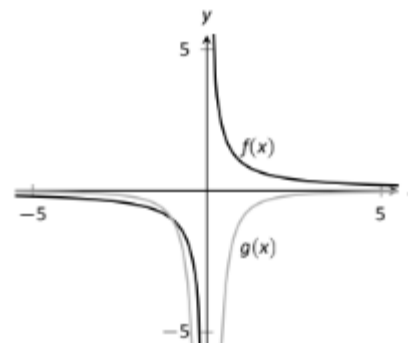


In the following problems, identify which function is the derivative of the other?

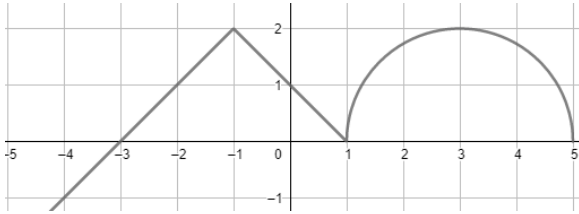
4.



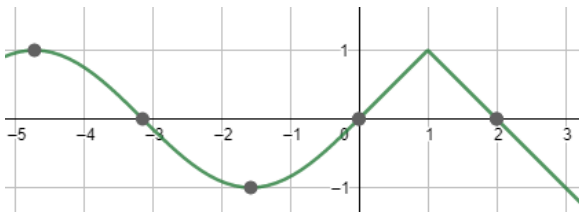
5.



6. Consider the following graph of $f'(x)$. It's constructed from two lines, and a semi-sphere with radius 2.



- If $f(-3) = 2$, compute the value of $f(1)$ and $f(5)$.
 - What is the maximum and minimum value of $f(x)$ over the interval $[-4, 5]$?
 - Find the points of inflection for $f(x)$.
 - Where is $f(x)$ increasing and decreasing?
7. Consider the following graph of $f'(x)$. It's constructed from a portion of the function $\sin(x)$ and two straight lines.



- If $f(0) = 4$, compute $f(-3)$ and $f(2)$.
- What is the maximum and minimum value of $f(x)$ over the interval $[-2\pi, 3]$?
- Find the points of inflection for $f(x)$.
- Where is $f(x)$ increasing and decreasing?
- Where is $f'(x)$ not differentiable?
- If $g(x) = f(x)^2$, what is the value of $g'(0)$?

6.2 Motion Problems

Given the following velocity function $v(t)$ and initial value for an object moving in a straight line, find the displacement $x(t)$ and acceleration $a(t)$.

- $v(t) = -32t + 20, x(0) = 0$
- $v(t) = 10, x(0) = 10$
- $v(t) = \cos t, x(0) = -5$
- $v(t) = \sqrt{t} + t^2, x(0) = -2$

$$5. v(t) = \frac{\ln t}{t}, x(1) = 10$$

Given the following acceleration function $a(t)$ and initial values for an object moving in a straight line, find:

- the displacement $x(t)$ and velocity $v(t)$.
 - When the object is speeding up and slowing down.
 - When the object changes directions.
 - The maximum position of the object.
- $a(t) = -32, v(0) = 20, x(0) = 0$
 - $a(t) = 10t, v(0) = -50, x(0) = 100$
 - $a(t) = \sin t, 0 \leq t < 2\pi, v(0) = -1, x(0) = 5$
 - A particle moves along a straight line with velocity $v(t) = t^2 - 4$. It starts at time $t = 0$ at the origin, $x(0) = 0$. Find its position $x(4)$ at time $t = 4$, and the total distance traveled during the time interval $[0, 4]$. Keep in mind that the distance traveled may or may not be the same as its final position.

Created by Allen Tsao (Bothell STEM Coach)

Questions are derived from [APEX Calculus textbook](#) and [OpenStax Calculus Volume 1](#).