2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Free-Response Questions Set 1

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23} \text{ mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11} \, (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9} \, (\text{N}\cdot\text{m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \, (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7} \, (\text{T}\cdot\text{m})/\text{A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}{}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

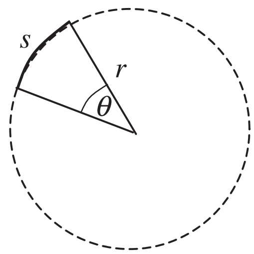
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



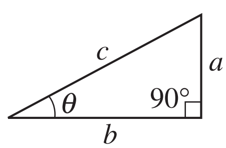
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln\left|x+a\right|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

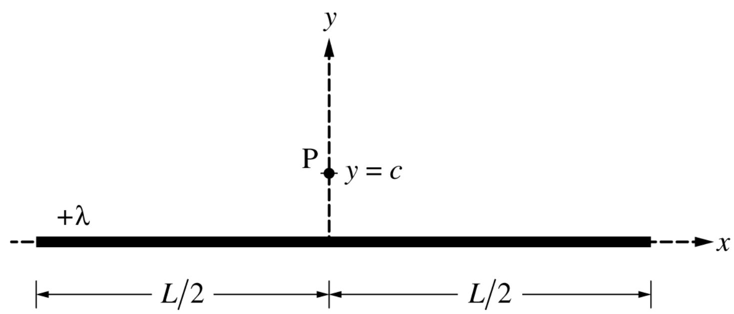
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

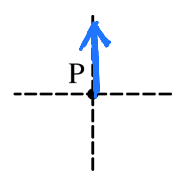


Note: Figure not drawn to scale.

1. A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

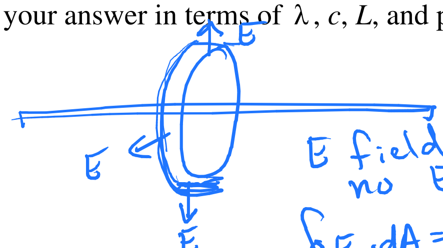
i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.



E field points away from + charge.

ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder. thin cylinder with radius c

iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.



E field is radially outwards
no E field parallel to the charge cylinder

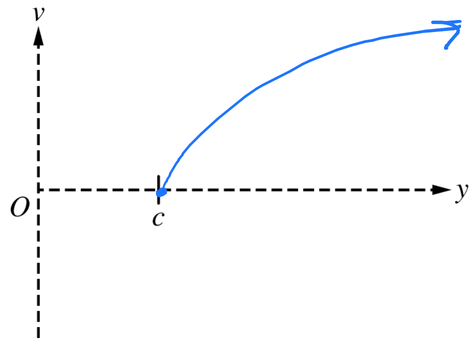
$$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$$

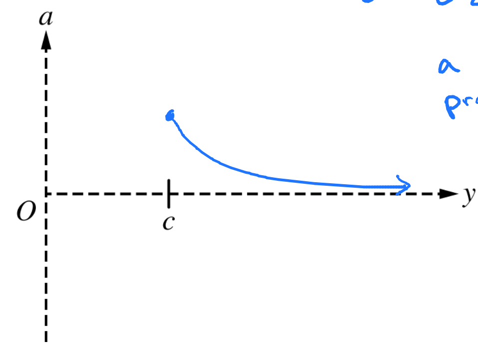
$$= E \cdot 2\pi c \cdot dx = \frac{\lambda dx}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi c \varepsilon_0}}$$

(b) A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of position y and the acceleration a as a function of position y for the proton.

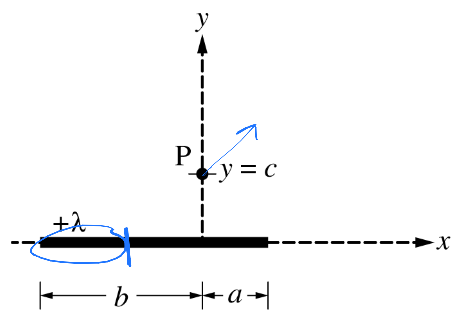
$F = qE = q\frac{\lambda}{2\pi y \varepsilon_0} = ma$



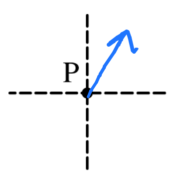


a is inversely proportional to y

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



more charge on the left side which creates more E field

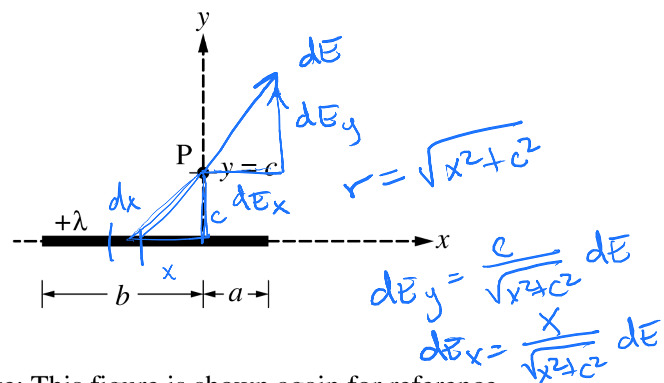
(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ✓ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

not easily because the E field angle is not known so the field will exit any Gaussian surface that goes around the cylinder



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$|E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

Vertical component: $$|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component __✓__ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

1) the charge is in the x direction so the integral should be in terms of dx.

2) the integrand should only have x + divided by $(c^2+x^2)^{3/2}$

$$\boxed{|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{c}{(c^2+x^2)^{3/2}}\, dx}$$

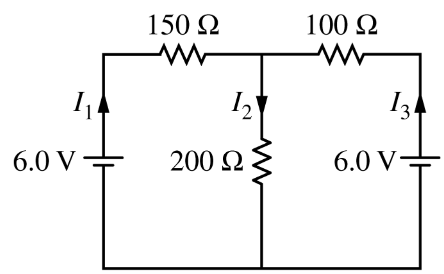


Figure 1

2. The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

ii. Calculate the current in the $200\ \Omega$ resistor.

iii. Calculate the power dissipated by the $200\ \Omega$ resistor.

$6V - 150 I_1 - 200 I_2 = 0$ KVL

$I_2 = I_1 + I_3$ KCL

$6V - 100 I_3 - 200 I_2 = 0$ KVL

(see next page for algebra on solving for I_1, I_2, I_3

$\boxed{I_2 = 0.231A}$ $\boxed{P_2 = 0.107W}$

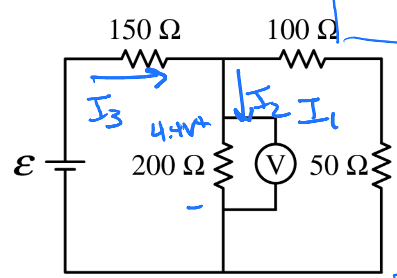


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) Calculate the current through the $50\ \Omega$ resistor.

(c) Calculate the voltage $\mathcal{E}$ of the battery.

b) current through 50Ω = current through 100Ω
voltage across both 100Ω & 50Ω is 4.4V

so $I_1 = \frac{V}{R} = \frac{4.4V}{100+50} = \boxed{0.0293A}$

c) $I_1 = 0.0293A$

$I_2 = \frac{4.4V}{200\Omega} = 0.022A$

$I_3 = I_1 + I_2 = \boxed{0.0513A}$

V across 150 $= I_3 150 = 7.7V$

so $\mathcal{E} = 7.7 + 4.4$

$\boxed{\mathcal{E} = 12.1V}$

GO ON TO THE NEXT PAGE.

$$6 = 150I_1 + 200I_2 = 450I_1 + 200(I_1 + I_3)$$

$$6 = 350I_1 + 200I_3$$

$$6 = 200I_2 + 100I_3 = 200(I_1 + I_3) + 100I_3$$

$$6 = 200I_1 + 300I_3$$

divide by 50

$$\frac{6}{50} = 7I_1 + 4I_3 \qquad \times 3$$

$$\frac{6}{50} = 4I_1 + 6I_3 \qquad \times -2$$

$$\frac{18}{50} = 21I_1 + 12I_3$$

$$\frac{-12}{50} = -8I_1 - 12I_3$$

$$\frac{6}{50} = 13I_1 \qquad I_1 = 0.00923A$$

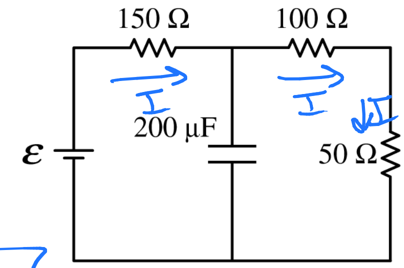
$$I_3 = \frac{1}{6}\left(\frac{6}{50} - 4I_1\right) = 0.0139A$$

$$I_2 = I_1 + I_3 = \boxed{0.231A}$$

$$P_2 = I_2^2 R = (0.231A)^2(200\Omega) = \boxed{0.107W}$$

(d)

i. The 200 Ω resistor in the circuit in Figure 2 is replaced with a 200 μF capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the 50 Ω resistor.

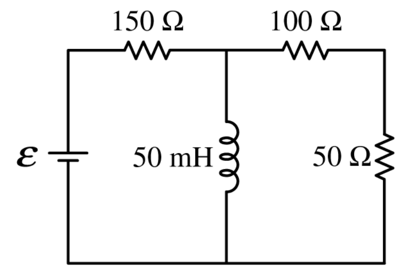


Steady state ⇒ no current through Capacitor

3 resistors in series

so $I = \frac{V}{R} = \frac{12.1V}{300\Omega} = \boxed{0.0403A}$

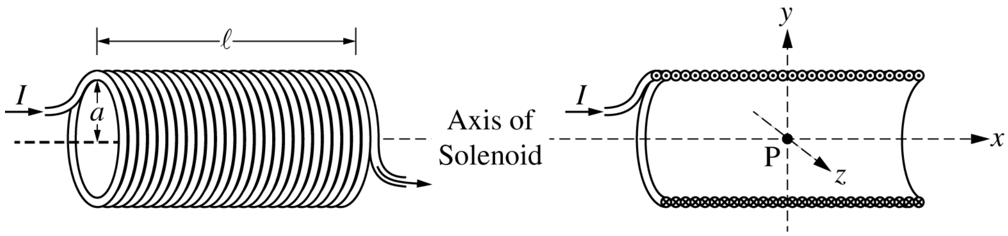
ii. The 200 Ω resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the 50 Ω resistor greater than, less than, or equal to the current calculated in part (b) ?



____ Greater than ✓____ Less than ____ Equal to

Justify your answer.

in steady state, the voltage across the inductor is 0. Thus, it is shorted so the current through the 50Ω resistor is 0.



Solenoid   Cutaway View

<u>Note:</u> Figures not drawn to scale.

3. A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) Select the correct direction of the magnetic field at point P.

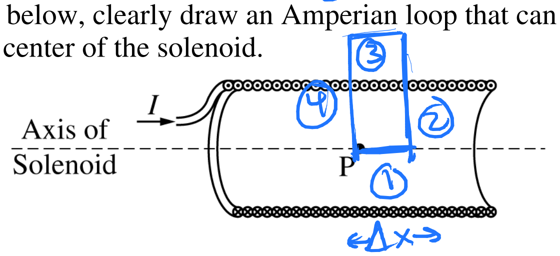
✓ $+x$-direction   ____ $+y$-direction   ____ $+z$-direction

____ $-x$-direction   ____ $-y$-direction   ____ $-z$-direction

Justify your selection. right hand rule for current & magnetic field

(b)

i. On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



Cutaway View

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I$$

section ① $\int B \cdot dl = B\Delta x$

secton ②, ④ $\int B \cdot dl = 0$ b/c B is 0 in the vertical direction

section ③ if segment 3 is far enough away, $B \simeq 0$ so $\int B \cdot dl = 0$

$$\oint B \cdot dl = B\Delta x = \mu_0 I \cdot \#\text{turns} = \mu_0 I \frac{N}{\ell} \Delta x$$

so $\boxed{B = \mu_0 I \frac{N}{\ell}}$

Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

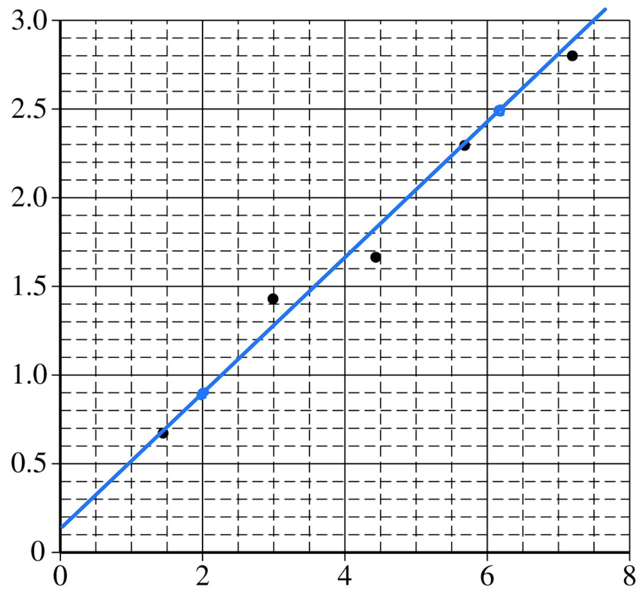
$B = \mu_0 I \frac{N}{\ell}$

$= \mu_0 \frac{V}{R} \frac{N}{\ell}$

$= \frac{\mu_0 N}{R \ell} V$

$\text{slope} = \frac{\mu_0 N}{R \ell}$

$R = \frac{\mu_0 N}{\ell \times \text{slope}}$



$\text{slope} \approx \frac{2.5\times10^{-4} - 0.9\times10^{-4}}{6.2 - 2} = 0.381\times10^{-4}$

(c)

i. On the graph above, draw a best-fit line for the data.

$R = \frac{4\pi\times10^{-7} \times 100}{0.40 \times .381\times10^{-4}} = \boxed{8.25\Omega}$

ii. Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

(d) One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

i. Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for B ?

__✓__ Yes ____ No

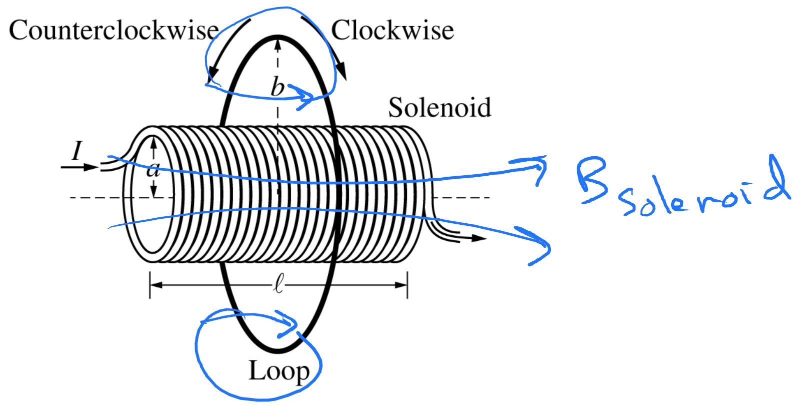
Justify your answer. the y-intercept is non-zero

ii. Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes __✓__ No

Justify your answer.

the slope would not be shifted because all measured values would have an extra 2.5×10^{-5} T



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

✓ Clockwise ____ Counterclockwise

Justify your answer.

the flux is decreasing so the loop generates a current that creates a B fied to the right

ii. Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

$$V = -\frac{d\Phi}{dt}$$

$$\Phi_B = B \cdot A = \frac{\mu_0 I \ell}{N} \cdot \pi a^2$$

$$-\frac{d\Phi_B}{dt} = -\frac{\mu_0 \ell}{N} \pi a^2 \frac{dI}{dt}$$

current decreasing

$$-\frac{\mu_0 \ell}{N} \pi a^2 \left(\frac{-I}{\Delta t}\right)$$

$$V = \frac{\mu_0 \ell}{N} \pi a^2 \frac{I}{\Delta t}$$

$$I_{ind} = \frac{V}{R_L} = \boxed{\frac{\mu_0 \ell}{R_L N} \pi a^2 \frac{I}{\Delta t}}$$

STOP

END OF EXAM