2. Derivatives

2.1 Using the Definition of the Derivative

Use the definition of the derivative to compute the derivative of the given function.

1. $f(x) = 6$
2. $f(t) = 4 - 3t$
3. $h(x) = x^3$
4. $r(x) = \dfrac{1}{x}$
5. $f(x) = \dfrac{x}{5 - x}$
6. $f(x) = \dfrac{1}{\sqrt{x-1}}$
7. $f(x) = 2x + \dfrac{1}{\sqrt{2x+3}}$

2.2 Power Rule

Compute the derivative of the given functions:

1. $f(x) = 7x^2 - 5x + 7$
2. $m(t) = 9t^5 - \dfrac{1}{8}t^3 + 3t - 8$
3. $f(x) = \dfrac{2}{\sqrt{x}}$
4. $f(x) = 2x^3\sqrt{x}$
5. $g(x) = \sqrt[3]{x^5}$
6. $f(x) = \dfrac{1}{2x^6} + \dfrac{x}{\sqrt[4]{x^9}}$
7. $g(x) = (2x-5)^3$
8. $f(x) = (2-3x)^2$

2.3 Derivatives of Other Functions

1. $f(r) = 6e^r$
2. $f(x) = 2\ln x - x$
3. $f(x) = \ln\left(3x^2\right)$
4. $f(t) = \ln(17) + e^2 + \sin\pi/2$
5. $f(x) = 2^x + 3^x$
6. $f(\theta) = 9\sin\theta + 10\cos\theta$
7. $h(t) = e^t - \sin t - \cos t$

2.4 Product Rule and Quotient Rule

In the following problems,

(a) use the product rule to differentiate the product.
(b) manipulate the function algebraically and use the power rule to compute the derivative.
(c) Show that the results of (a) and (b) are equal

1. $f(x) = x\left(x^2 + 3x\right)$

2. $h(s) = (2s-1)(s+4)$

Compute the derivative of the given function

3. $f(x) = x\sin x$

4. $f(x) = e^x \ln x$

5. $f(x) = \left(3x^2+8x+7\right)e^x$

6. $g(t) = 4t^3 e^t - \sin t \cos t$

7. $f(x) = e^x \sin(x) x^3$

8. $f(x) = x\sin x$; find $f''(x)$

In the following problems,

(a) use the quotient rule to differentiate the product.

(b) manipulate the function algebraically and use the power rule to compute the derivative.

(c) Show that the results of (a) and (b) are equal

9. $f(x) = \dfrac{x^2+3}{x}$

10. $h(s) = \dfrac{3}{4s^3}$

Compute the derivative of the given function

11. $g(x) = \dfrac{x+7}{x-5}$

12. $h(x) = \cot x - e^x$

13. $f(x) = \left(16x^3+24x^2+3x\right)\dfrac{7x-1}{16x^3+24x^2+3x}$

14. $f(x) = \dfrac{\sin x}{\cos x + 3}$

15. $\dfrac{\cos x}{x} + \dfrac{x}{\tan x}$

16. $f(x) = x^2 e^x \tan x$

17. $f(x) = \csc x$; find $f''(x)$

2.5 Chain Rule

Compute the derivative of the following functions

1. $f(x) = \left(4x^3 - x\right)^{10}$

2. $g(\theta) = (\sin\theta + \cos\theta)^3$

3. $f(x) = \left(\ln x + x^2\right)^3$

4. $f(x) = \left(x + \dfrac{1}{x}\right)^4$

5. $f(x) = 3^{x^3 - 1x}$

6. $g(x) = \tan(5x)$

7. $g(t) = \sin\left(t^5 + \dfrac{1}{t}\right)$

8. $p(t) = \cos^3\left(t^2+3t+1\right)$

9. $f(x) = \ln\left(x^2\right)$

10. $g(r) = 4^{\cos(r)}$

11. $g(t) = 15^2$

12. $h(t) = \dfrac{2^t+3}{3^t+2}$

13. $f(x) = \dfrac{3^{x^2}+x}{2^{x^2}}$

14. $f(x) = \left(x^2+x\right)^5\left(3x^4+2x\right)^3$

15. $f(x) = \sin(3x+4)\cos(5-2x)$

16. $f(x)=\dfrac{\sin(4x+1)}{(5x-9)^3}$

17. $h(t)=\sin^{-1}(2t)$

18. $g(x)=\tan^{-1}(2x)$

19. $g(t)=\sin t\cos^{-1} t$

20. $h(x)=\dfrac{\sin^{-1} x}{\cos^{-1} x}$

21. $f(x)=\tan\left(\tan^{-1} x\right)$

22. $f(x)=2^{\tan^{-1} x}\cdot 3^{e^x}$

23. $f(x)=e^{\sin^{-1}(\tan x)}$

2.6 Tangent and Normal Lines

Find the equations of the tangent and normal lines to the graph of the function at the given point.

1. $f(x)=x^3-x$ at $x=1$

2. $g(x)=\ln x$ at $x=1$

3. $f(x)=-2\cos x$ at $x=\pi/4$

4. $g(s)=e^5\left(s^2+2\right)$ at $(0,2e^5)$

5. $g(x)=\dfrac{x^2}{x-1}$ at $(2,4)$

6. $g(\theta)=\dfrac{\sin\theta-4\theta}{\theta+1}$ at $(0,0)$

7. $f(x)=\left(4x^3-x\right)^{10}$ at $x=0$

8. $g(\theta)=(\sin\theta+\cos\theta)^3$ at $\theta=\pi/2$

Find the x-values where the graph of the function has a horizontal tangent line.

9. $f(x)=x\sin x$ on $[-1,1]$

10. $f(x)=\dfrac{x}{x+1}$

Challenge Questions

11. The graph of $f(x) = x^2$ has two tangent lines that intersect at a given point (2,0). Find the equations of both tangent lines.

12. Find all values a such that $y=a\sqrt{x}$ is tangent to the line $y=x+1$

2.7 Implicit Differentiation

Compute the following derivatives using implicit differentiation

1. $x^4+y^2+y=7$

2. $\cos(x)+\sin(y)=1$

3. $\dfrac{y}{x}=10$

4. $x^2\tan y=50$

5. $\left(y^2+2y-x\right)^2=200$

6. $\dfrac{\sin(x)+y}{\cos(y)+x}=1$

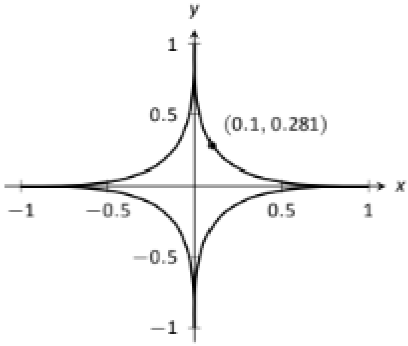
7. $\ln\left(x^2+xy+y^2\right)=1$

Find the equation of the tangent line to the graph of the implicitly defined function at the indicated points.

8. $x^{2/5}+y^{2/5}=1$

(a) At (1,0)

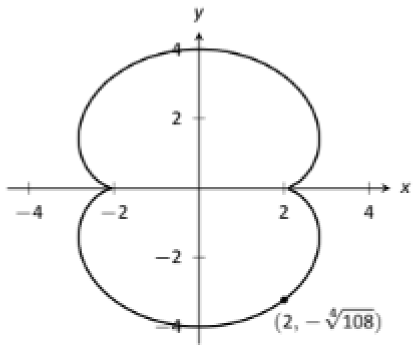
(b) At (0.1, 0.281)



9. $\left(x^2+y^2-4\right)^3=108y^2$

(a) At (0,4)

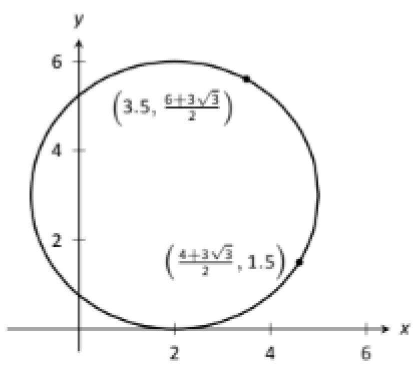
(b) At $(2,-\sqrt[4]{108})$



10. $(x-2)^2+(y-3)^2=9$

(a) At $\left(\frac{7}{2},\frac{6+3\sqrt{3}}{2}\right)$

(b) At $\left(\frac{4+3\sqrt{3}}{2},\frac{3}{2}\right)$



Find the second derivative of the following implicitly defined functions.

11. $x^4+y^2+y=7$

12. $\cos x+\sin y=1$

Find the derivatives by using logarithmic differentiation to find *dy/dx*.

13. $f(x)=x^{\sqrt[5]{x}}$

14. $f(x)=\left(e^{-x}+\sin x\right)^{x^2}$

15. $f(x)=(\ln x)^{\ln x}$

16. $f(x)=x^{x^2}$