

4. Integration

4.1 Antiderivatives

Compute the following integrals

1. $\int 3x^3 dx$

2. $\int (10x^2 - 2) dx$

3. $\int 1 ds$

4. $\int \frac{3}{t^2} dt$

5. $\int \sec^2 \theta d\theta$

6. $\int (\sec x \tan x + \csc x \cot x) dx$

7. $\int 3^t dt$

8. $\int (2t + 3)^2 dt$

9. $\int x^2 x^3 dx$

10. $\int a dx$

Solve for $f(x)$ given the initial value problem.

11. $f'(x) = \sin x$ and $f(0) = 2$

12. $f'(x) = 4x^3 - 3x^2$ and $f(-1) = 9$

13. $f'(x) = 7^x$ and $f(2) = 1$

14. $f''(x) = 7x$ and $f'(1) = -1, f(1) = 10$

15. $f''(\theta) = \sin \theta$ and $f'(\pi) = 2, f(\pi) = 4$

16. $f''(x) = 0$ and $f'(1) = 3, f(1) = 1$

4.2 Riemann Sums and The Definition of the Integral

For the following problems, approximate the integral by using the given interval width and using:

- a) Left Hand Sum
- b) Right Hand Sum
- c) Midpoint Sum
- d) Trapezoid Rule

1. $\int_{-3}^3 x^2 dx$ $n = 6$

2. $\int_0^2 (5 - x^2) dx$ $n = 4$

3. $\int_0^2 (x + 3x^3) dx$ $n = 5$

4. $\int_0^2 4^x dx$ $n = 4$

5. $\int_1^2 \ln x dx$ $n = 3$

Compute the integral using the definition of the integral (i.e. setup the area as a sum of n rectangles and take the limit as $n \rightarrow \infty$).

$$6. \int_0^1 x^3 dx$$

$$7. \int_{-1}^3 (3x - 1) dx$$

$$8. \int_0^1 (3x^3 - 4x^2) dx$$

Convert the following infinite sums into an integral expression. Note: there is not a single unique answer.

$$9. \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + i \frac{3}{n}\right) \frac{3}{n}$$

$$10. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \sqrt{i \frac{2}{n}}$$

$$11. \lim_{n \rightarrow \infty} \sum_{i=1}^n \sin^2 \left(1 - i \frac{\pi}{n}\right) \frac{\pi}{n}$$

4.3 The Definite Integral

Compute the definite integral for each function.

$$1. \int_{-1}^1 x^2 dx$$

$$2. \int_0^\pi \sin x dx$$

$$3. \int_0^3 (x^3 + 2x^2 - 5x + 7) dx$$

$$4. \int_0^{2\pi} \cos x dx$$

$$5. \int_1^4 \frac{1}{\sqrt{x}} dx$$

$$6. \int_0^5 x^4 dx$$

4.4 Derivatives of Integrals (Fundamental Theorem of Calculus)

Compute the derivatives of the following integrals.

$$1. F(x) = \int_2^{x^3+x} \frac{1}{t} dt$$

$$2. F(x) = \int_x^{x^2} (t + 2) dt$$

$$3. F(x) = \int_{\ln x}^{e^x} \tan t dt$$

4.5 U-Substitution (Part 1)

Evaluate the Indefinite Integral

$$1. \int 3x^2 (x^3 - 5)^7 dx$$

$$2. \int x (x^2 + 1)^8 dx$$

$$3. \int \frac{1}{2x + 7} dx$$

$$4. \int \frac{x}{\sqrt{x + 3}} dx$$

$$5. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

6. $\int \frac{\frac{1}{x} + 1}{x^2} dx$

7. $\int \sin^2(x) \cos(x) dx$

8. $\int \cos(3 - 6x) dx$

9. $\int \sec(2x) dx$

10. $\int x \cos(x^2) dx$

11. $\int \cot x dx$

12. $\int e^{3x-1} dx$

13. $\int e^{x^2-2x+1}(x-1) dx$

14. $\int \frac{e^x}{e^x + 1} dx$

15. $\int 3^{3x} dx$

16. $\int \frac{\ln x}{x} dx$

17. $\int \frac{\ln(x^3)}{x} dx$

18. $\int \frac{x^2 + 3x + 1}{x} dx$

19. $\int \frac{x^3 - 1}{x + 1} dx$

20. $\int \frac{3x^2 - 5x + 7}{x + 1} dx$

4.6 U-Substitution (Part 2)

1. $\int \frac{7}{x^2 + 7} dx$

2. $\int \frac{14}{\sqrt{5 - x^2}} dx$

3. $\int \frac{5}{\sqrt{x^4 - 16x^2}} dx$

4. $\int \frac{3}{\sqrt{-x^2 + 8x + 9}} dx$

5. $\int \frac{x^2}{(x^3 + 3)^2} dx$

6. $\int \frac{x}{\sqrt{1 - x^2}} dx$

7. $\int \sin(x) \sqrt{\cos(x)} dx$

8. $\int \frac{3x^3 + 4x^2 + 2x - 22}{x^2 + 3x + 5} dx$

9. $\int \frac{9(2x + 3)}{3x^2 + 9x + 7} dx$

10. $\int \frac{x}{x^4 + 81} dx$

11. $\int \frac{3x - 2}{x^2 - 2x + 10} dx$

12. $\int \frac{x^2 + 5x - 2}{x^2 - 10x + 32} dx$

$$13. \int \frac{\cos(x)}{\sin^2(x) + 1} dx$$

Evaluate the Definite Integral

$$14. \int_{-\pi/2}^{\pi/2} \sin^2 x \cos x dx$$

$$15. \int_{-2}^{-1} (x + 1)e^{x^2+2x+1} dx$$

$$16. \int_2^4 \frac{1}{x^2 - 6x + 10} dx$$

Challenge Questions

$$17. \int_e^{e^2} \frac{dx}{x \ln(x) \ln(\ln(x))}$$

$$18. \int \sqrt{4 - \sqrt{x}} dx$$

$$19. \int \sin(t) \cos(2t) dt$$

$$20. \int \frac{1}{e^{-x} + e^x} dx$$