2019

AP® CollegeBoard

AP® Physics C: Mechanics

Free-Response Questions
Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11} \left(\text{N}\cdot\text{m}^2\right)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\left(\text{N}\cdot\text{m}^2\right)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9 \left(\text{N}\cdot\text{m}^2\right)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta\vec{p}$

$\vec{p} = m\vec{v}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_N\right|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{\max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_G\right| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

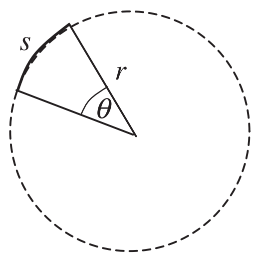
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



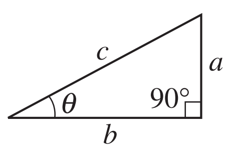
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

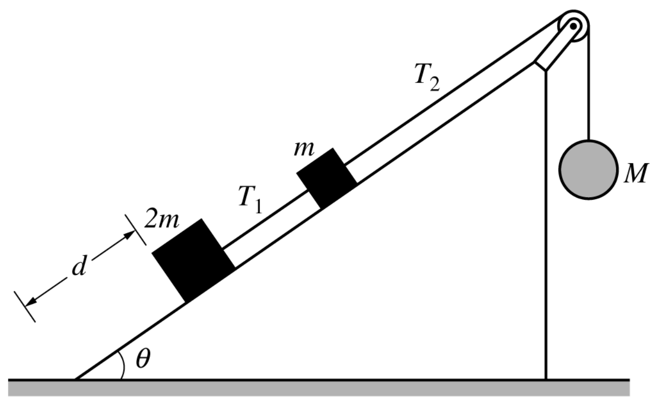
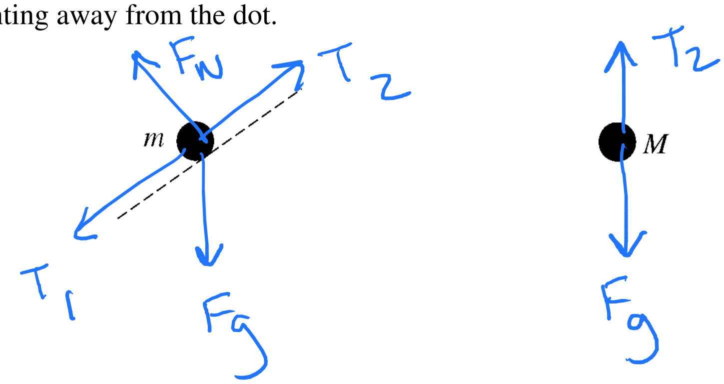


Figure 1

1. Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

   (a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



(b) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

See next page

i. The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.

ii. The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

(c) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

__✓__ Decrease to zero ____ Stay the same

ii. Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

__✓__ Up the ramp _____ Down the ramp _____ Zero

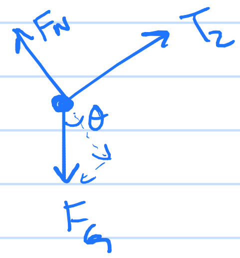
iii. Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp __✓__ Down the ramp _____ Zero

(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

b) i) combin m & 2m into 1 FBD



$$F_{net} = T_2 - F_g \sin\theta = 0$$

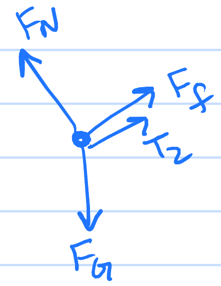
$$\boxed{T_2 = 3mg\sin\theta}$$

ii) FBD for sphere

$$T_2 = Mg$$

$$M = \frac{T_2}{g} = \frac{3mg\sin\theta}{g} = \boxed{3m\sin\theta}$$

d)



add F_f to the FBD for combined mass 3m

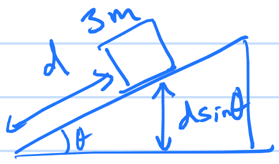
$$F_g\sin\theta = T_2 + F_f$$
$$F_N = F_g\cos\theta$$

$$T_2 = 3mg\sin\theta - \mu_s F_N$$
$$= 3mg\sin\theta - \mu_s 3mg\cos\theta$$

$$T_2 = Mg = 3mg\sin\theta - \mu_s 3mg\cos\theta$$

$$\boxed{M = 3m(\sin\theta - \mu_s\cos\theta)}$$

e) KE = PE - energy lost to friction



work done by friction

$$\frac{1}{2}(\cancel{3m})v^2 = 3mgd\sin\theta - F_f \cdot d$$

$$= \cancel{3m}gd\sin\theta - \mu_k \cancel{3m}g\cos\theta \cdot d$$

$$\frac{1}{2}v^2 = gd(\sin\theta - \mu_k\cos\theta)$$

$$\boxed{v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}}$$

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

   (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s. next page

   (b) Calculate the speed of the rocket at $t = 6.0$ s.

   (c)

      i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

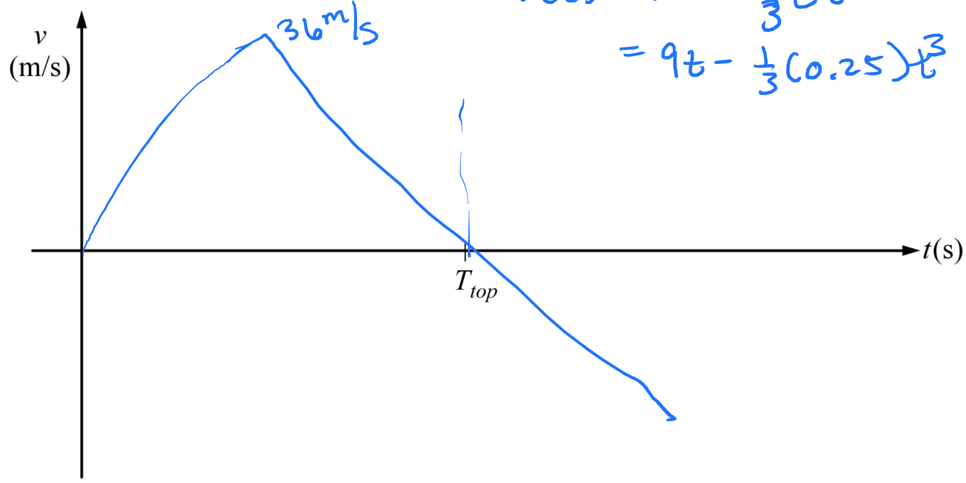
      ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

   (d) Calculate the maximum height reached by the rocket relative to its launching point.

   (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

$v(t) = Kt - \frac{1}{3}Lt^3$

$= 9t - \frac{1}{3}(0.25)t^3$



GO ON TO THE NEXT PAGE.

a) $\int F\,dt = \int_0^6 m(k-Lt^2)\,dt = \left(kt - \frac{1}{3}Lt^3\Big|_0^6\right)m$

$= \left(6k - \frac{1}{3}L(216)\right)(0.5\,kg)$

$= \left[6(9) - \frac{1}{3}(0.25)(216)\right](0.5)$

$= 18\ N\cdot s$

b) $\int F\,dt = m\Delta v = 0.5\,kg\,v = 18$

$\boxed{v = 36\,m/s}$

c) $KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.5\,kg)(36\,m/s)^2$

i) $= \boxed{324\,J}$

ii) $a(t) = k - Lt^2$

$v(t) = \int a(t)\,dt = kt - \frac{1}{3}Lt^3 + C$

$v(0) = 0 \Rightarrow C = 0$

$\Delta x = \int_0^6 v(t)\,dt = \frac{1}{2}kt^2 - \frac{1}{12}Lt^4\Big|_0^6$

$= \frac{1}{2}(9)(6)^2 - \frac{1}{12}(0.25)(6)^4$

$= 135\,m$

$PE = mg\Delta x = \boxed{661.5\,J}$

d) engines shut off and $v_i = 36\,m/s$

$v_f^2 = v_i^2 + 2a\Delta x$

$0 = 36^2 - 2g\Delta x$

$\Delta x = \frac{36^2}{2g} = 66.1\,m$

added onto the 135m in first 6 sec.

$\Delta x = 135 + 66.1 = \boxed{201.1\,m}$

a) F_G

$F_G = ma = m\frac{v^2}{R}$

$mg = m\frac{v^2}{R}$

$\boxed{v = \sqrt{Rg}}$

b) $U_i = mgh$ $\quad U_A = KE + PE + KE_{rot}$

$mgh = \frac{1}{2}mv^2 + mg(2R) + \frac{1}{2}I\omega^2$

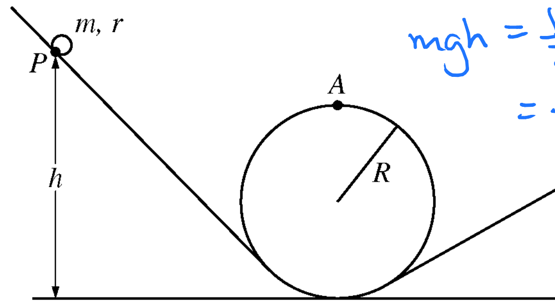
$= \frac{1}{2}mRg + 2mRg + \frac{1}{2}bmr^2\left(\frac{v}{r}\right)^2$

$= \frac{5}{2}mRg + \frac{1}{2}bmv^2$

$= \frac{5}{2}mRg + \frac{1}{2}bmRg$

$\cancel{mg}h = \cancel{mg}R\left(\frac{5}{2} + \frac{1}{2}b\right)$

$\boxed{h = R\left(\frac{5}{2} + \frac{1}{2}b\right)}$



Note: Figure not drawn to scale.

3. The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

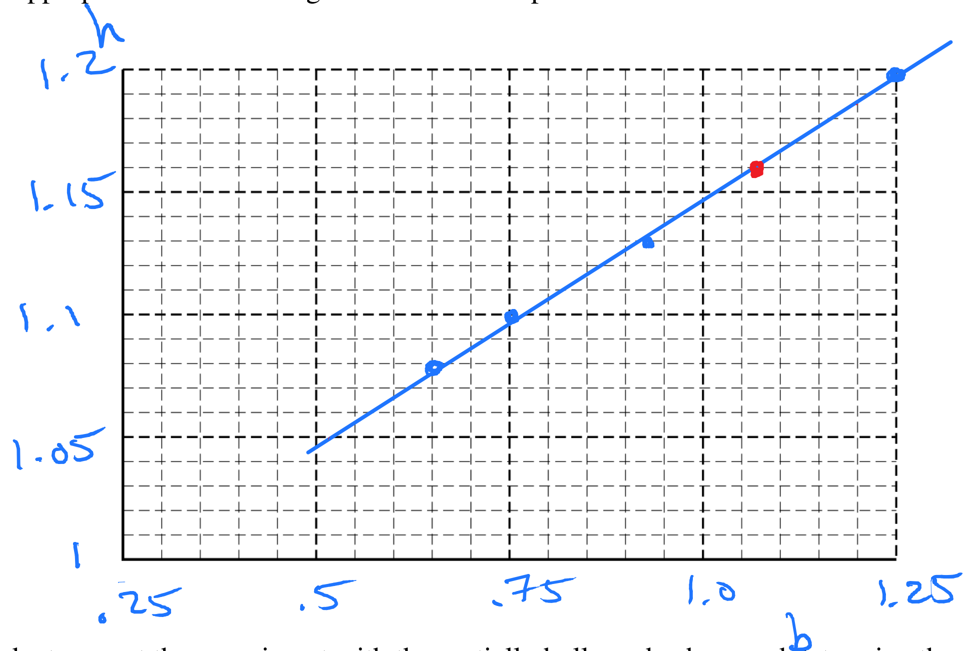
(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

$\boxed{b \cong 1.07}$

(e) Calculate R, the radius of the loop.

(f) In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ✓____ Less ____ The same

Justify your answer.

$h = R\left(\frac{5}{2} + \frac{1}{2}b\right)$

$R = \dfrac{h}{\frac{5}{2} + \frac{1}{2}b} = \dfrac{1.16}{\frac{5}{2} + \frac{1}{2}(1.07)}$

$\boxed{R = 0.382 m}$

the initial height would be $h + \frac{r}{2}$

and the height at A would be $h - \frac{r}{2}$ due to the center of mass being not at the ground.

So less PE @ A requires less PE initially

STOP

END OF EXAM