1. Limits and Continuity

1.1 Computing Limits

1. $\lim\limits_{x\to 1} x^2+3x-5$

2. $\lim\limits_{x\to 0} \dfrac{x+1}{x^2+3x}$

3. $\lim\limits_{x\to -1} \dfrac{x^2+8x+7}{x^2+6x+5}$

4. $\lim\limits_{x\to 2} f(x)$, where

$$f(x)=\begin{cases} x+2 & x\le 2 \\ 3x-5 & x>2 \end{cases}$$

5. $\lim\limits_{x\to 0} f(x)$, where

$$f(x)=\begin{cases} \cos x & x\le 0 \\ x^2+3x+1 & x>0 \end{cases}$$

In the following exercises, use the following information to evaluate the given limit, when possible. If it is not possible to determine the limit, state why not.

- $\lim_{x\to 9} f(x)=6,\quad \lim_{x\to 6} f(x)=9,\quad f(9)=6$
- $\lim_{x\to 9} g(x)=3,\quad \lim_{x\to 6} g(x)=3,\quad g(6)=9$

6. $\lim\limits_{x\to 9} \left(\dfrac{f(x)-2g(x)}{g(x)}\right)$

7. $\lim\limits_{x\to 9} g(f(x))$

8. $\lim\limits_{x\to 6} g(f(f(x)))$

In the following exercises, use the following information to evaluate the given limit, when possible. If it is not possible to determine the limit, state why not.

- $\lim_{x\to 1} f(x)=2,\quad \lim_{x\to 10} f(x)=1,\quad f(1)=1/5$
- $\lim_{x\to 1} g(x)=0,\quad \lim_{x\to 10} g(x)=\pi,\quad g(10)=\pi$

9. $\lim\limits_{x\to 1} f(x)^{g(x)}$

10. $\lim\limits_{x\to 1} f(x)g(x)$

Evaluate the given limits

11. $\lim\limits_{x\to \pi/4} \cos x\sin x$

12. $\lim\limits_{x\to 0} \ln x$

13. $\lim\limits_{x\to \pi/6} \csc x$

14. $\lim\limits_{x\to \pi} \dfrac{x^2+3x+5}{5x^2-2x-3}$

15. $\lim\limits_{x\to 6} \dfrac{x^2-4x-12}{x^2-13x+42}$

16. $\lim\limits_{x\to 2} \dfrac{x^2+6x-16}{x^2-3x+2}$

17. $\lim\limits_{x\to -2} \dfrac{x^2-5x-14}{x^2+10x+16}$

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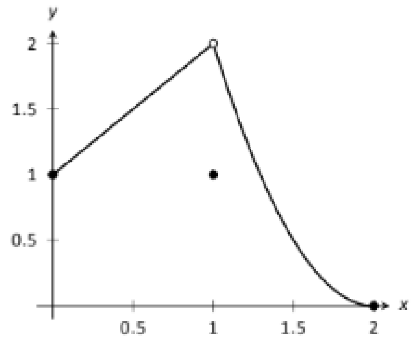
Questions are derived from [APEX Calculus textbook] and [OpenStax Calculus Volume 1].

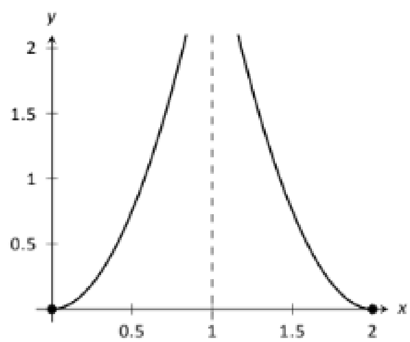
1.2 Graphical Limits

1.



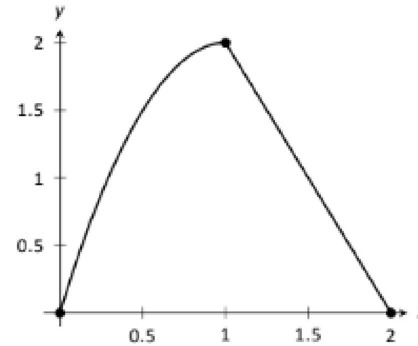
(a) $\lim_{x\to 1^-} f(x)$
(b) $\lim_{x\to 1^+} f(x)$
(c) $\lim_{x\to 1} f(x)$
(d) $f(1)$
(e) $\lim_{x\to 0^-} f(x)$
(f) $\lim_{x\to 0^+} f(x)$

2.



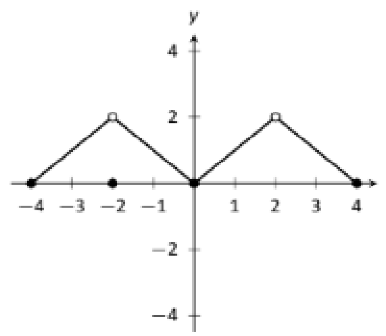
(a) $\lim_{x\to 1^-} f(x)$
(b) $\lim_{x\to 1^+} f(x)$
(c) $\lim_{x\to 1} f(x)$
(d) $f(1)$
(e) $\lim_{x\to 2^-} f(x)$
(f) $\lim_{x\to 0^+} f(x)$

3.



(a) $\lim_{x\to 1^-} f(x)$
(b) $\lim_{x\to 1^+} f(x)$
(c) $\lim_{x\to 1} f(x)$
(d) $f(1)$

4.



(a) $\lim_{x\to -2^-} f(x)$
(b) $\lim_{x\to -2^+} f(x)$
(c) $\lim_{x\to -2} f(x)$
(d) $f(-2)$
(e) $\lim_{x\to 2^-} f(x)$
(f) $\lim_{x\to 2^+} f(x)$
(g) $\lim_{x\to 2} f(x)$
(h) $f(2)$

1.3 Limits of Piecewise Functions

1. $$f(x) = \begin{cases} x+1 & x \le 1 \\ x^2 - 5 & x > 1 \end{cases}$$

(a) $\lim_{x\to 1^-} f(x)$ (c) $\lim_{x\to 1} f(x)$
(b) $\lim_{x\to 1^+} f(x)$ (d) $f(1)$

2. $$f(x) = \begin{cases} x^2 - 1 & x < -1 \\ x^3 + 1 & -1 \le x \le 1 \\ x^2 + 1 & x > 1 \end{cases}$$

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(a) $\lim_{x \to -1^-} f(x)$

(b) $\lim_{x \to -1^+} f(x)$

(c) $\lim_{x \to -1} f(x)$

(d) $f(-1)$

(e) $\lim_{x \to 1^-} f(x)$

(f) $\lim_{x \to 1^+} f(x)$

(g) $\lim_{x \to 1} f(x)$

(h) $f(1)$

3. $$f(x) = \begin{cases} 1 - \cos^2 x & x < a \\ \sin^2 x & x \geq a \end{cases}$$

(a) $\lim_{x \to a^-} f(x)$

(b) $\lim_{x \to a^+} f(x)$

(c) $\lim_{x \to a} f(x)$

(d) $f(a)$

4. $$f(x) = \begin{cases} x^2 & x < 2 \\ x + 1 & x = 2 \\ -x^2 + 2x + 4 & x > 2 \end{cases}$$

(a) $\lim_{x \to 2^-} f(x)$

(b) $\lim_{x \to 2^+} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $f(2)$

5. $$f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

(a) $\lim_{x \to 0^-} f(x)$

(b) $\lim_{x \to 0^+} f(x)$

(c) $\lim_{x \to 0} f(x)$

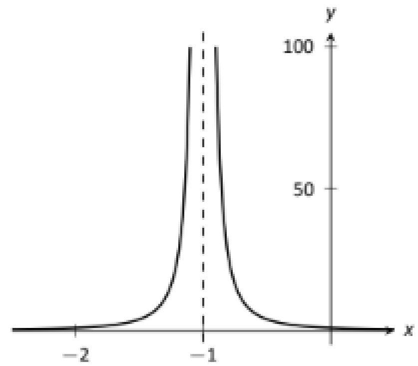
(d) $f(0)$

1.4 Limits with Infinity

$$f(x) = \frac{1}{(x+1)^2}$$

(a) $\lim_{x \to -1^-} f(x)$

(b) $\lim_{x \to -1^+} f(x)$

1. 

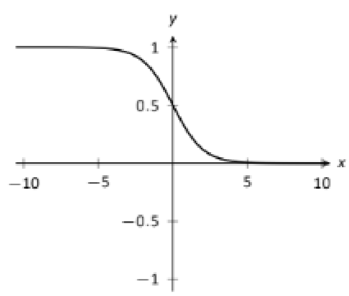
$$f(x) = \frac{1}{e^x + 1}$$

(a) $\lim_{x \to -\infty} f(x)$

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to 0^-} f(x)$

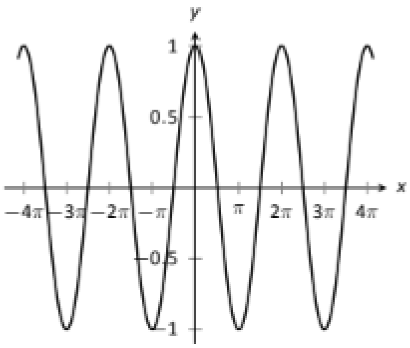
(d) $\lim_{x \to 0^+} f(x)$

2. 

$f(x) = \cos(x)$

(a) $\lim_{x \to -\infty} f(x)$

(b) $\lim_{x \to \infty} f(x)$

3. 

Numerically compute the following limits:

4. $\lim_{x \to \infty} f(x) = \dfrac{x^2 - 1}{x^2 - x - 6}$

5. $\lim_{x \to \infty} f(x) = \dfrac{x^2 - 11x + 30}{x^3 - 4x^2 - 3x + 18}$

Identify the horizontal and vertical asymptotes (if any) of the given function

6. $f(x) = \dfrac{2x^2 - 2x - 4}{x^2 + x - 20}$

7. $f(x) = \dfrac{x^2 + x - 12}{7x^3 - 14x^2 - 21x}$

8. $f(x) = \dfrac{x^2 - 9}{9x + 27}$

Evaluate the given limit:

9. $\lim_{x \to \infty} \dfrac{x^3 + 2x^2 + 1}{x - 5}$

10. $\lim_{x \to -\infty} \dfrac{x^3 + 2x^2 + 1}{x^2 - 5}$

Challenge Questions

11. $\lim_{x \to 1} \dfrac{1}{x - 1} - \dfrac{2}{x^2 - 1}$

12. $\lim_{x \to 0} \dfrac{\frac{1}{x+1} + \frac{1}{x-1}}{x}$

13. $\lim_{x \to \infty} \sqrt{x^2 - 4x + 1} - x$

14. $\lim_{x \to 2-} \dfrac{x^2 - 4x + 4}{|x - 2|}$

1.5 Limits with Trig Functions

1. $\lim_{x \to 0} \dfrac{\sin 3x}{x}$

2. $\lim_{x \to 0} \dfrac{\sin x}{x^2}$

3. $\lim_{x \to 0} \dfrac{\sin x}{\cos x}$

4. $\lim_{x \to 0} \dfrac{\tan x}{x}$

5. $\lim_{x \to 0} \dfrac{\sin x}{1 - x}$

6. $\lim_{x \to 0} \dfrac{x \sin x - x^2 \sin x}{x}$

7. $\lim_{x \to 0} \dfrac{\sin 2x}{\sin x}$

8. $\lim_{x \to 0} \dfrac{x}{1 - \cos^2 x}$

9. $\lim_{x \to 0} \dfrac{\sin x}{x + \sin x}$

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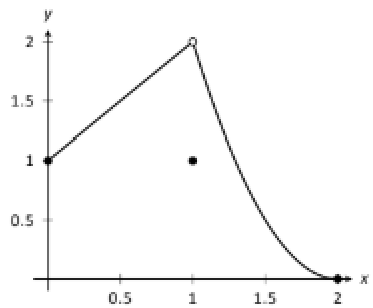
10. $\lim\limits_{x\to 0} \dfrac{x^2+\sin 3x}{2x+\tan 2x}$

1.6 Continuity

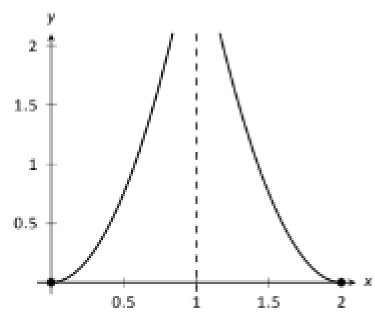
A graph of a function f is given along with a value a. Determine if f is continuous at a; if it is not, state why it is not.

$a = 1$



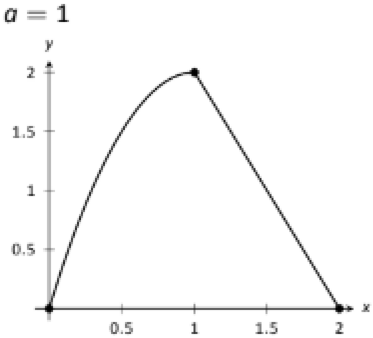
1.

$a = 1$



2.

$a = 1$

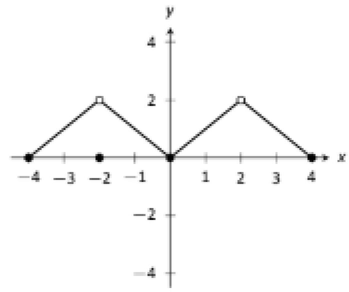


3.

(a) $a = -2$

(b) $a = 0$

(c) $a = 2$



4.

In the following problems, determine if f is continuous at the indicated values. If not, explain why.

5. $f(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin x}{x} & x > 0 \end{cases}$

   (a) $x = 0$

   (b) $x = \pi$

6. $f(x) = \begin{cases} \frac{x^2+5x+4}{x^2+3x+2} & x \neq -1 \\ 3 & x = -1 \end{cases}$

   (a) x = -1

   (b) x = 10

Give the intervals on which the given function is continuous:

7. $f(x) = x^2 - 3x + 9$

8. $g(x) = \sqrt{4 - x^2}$

9. $f(t) = \sqrt{5t^2 - 30}$

10. $g(x) = \dfrac{1}{1+x^2}$

11. $g(s) = \ln s$

12. $f(k) = \sqrt{1 - e^k}$

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1.7 Intermediate Value Theorem

1. Let f be continuous on $[1, 5]$ where $f(1) = -2$ and $f(5) = -10$. Does a value $1 < c < 5$ exist such that $f(c) = -9$? Why/why not?

2. Let f be continuous on $[-1, 1]$ where $f(-1) = -10$ and $f(1) = 10$. Does a value $-1 < c < 1$ exist such that $f(c) = 11$? Why/why not?

Challenge Questions

3. Give an interval for x, the solution to the equation $\cos x = x$, using the Intermediate Value Theorem.

4. Show that the equation $2^x = x + 3$ has a solution in the interval $2 < x < 3$.

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